A Montegovian Treatment of Modal Subordination

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Examples, like those given in (1) and (2) from [6], involve anaphoric links between pronouns and their antecedents not predicted by standard versions of dynamic semantics such as DRT [3,4]. These are known as examples of *modal subordination* (MS).

(1) A wolf might enter. It would growl.

(2) A wolf might enter. *It will growl.

(3) A wolf enters. ?It would growl.

While most accounts of MS have exploited DRT or other dynamic frameworks [6,3,7,8], our account uses the higher order, Montagovian framework of [1], using the functional programming technique of *continuations*. Our approach shares all the advantages of the general framework of [1], embedding dynamic logics within a classical setting where quantifiers and variables receive their standard interpretation and assignments and discourse referents are not elements of basic types (as in [2] or as in Dynamic Montague Grammar). We extend [1]'s continuation based approach to MS phenomena, showing how the modifications needed to deal with MS amount to particular choices about the lexical entries of various terms and a particular precisification of the binder properties of the monad used to model discourse semantics.

Like Montague, [1] exploits the homomorphic interpretation of syntactic types and structures into semantic types and terms. But instead of Montague's simple t for the interpretation of the sentence type s, $[\![s]\!]$, [1] has: $\gamma \rightarrow (\gamma \rightarrow t) \rightarrow t$ where γ is the type of the environment or discourse context already given; $\gamma \rightarrow t$ is the type of the discourse "to come". This is the *continuation* of the sentence that is being evaluated. In particular, if this sentence introduces "dynamically speaking" a new discourse referent x, given an environment e and a continuation k as parameter, it can provide $(x :: e)$ (with $\cdot :: \cdot$ a list constructor) as parameter to k, so that it makes the value of x available for k.

Then, we have the standard interpretations: $[\![np]\!] = (e \rightarrow [\![s]\!]) \rightarrow [\![s]\!]$ and $[\![n]\!] = e \rightarrow [\![s]\!]$. In particular, pronouns are interpreted as follows: $[\![it]\!] = \lambda P.\lambda ik.P\,(\mathtt{sel}\,i)\,i\,k$, where $i\colon \gamma, k\colon \gamma \rightarrow t$, and where $\mathtt{sel}\,e$ is a function that selects a *suitable* discourse antecedent inside e (note that here the antecedents are explicitly given in e and ordinary bound objectual variables rather than discourse referents and computed over the representation as in DRT).

For MS we need one environment for discourse entities introduced in the actual world and another for discourse entities introduced in a modal context (or in possible worlds). So instead of having only one environment, we have two of them[1]. Then, we also use two continuations: one that contains facts about the actual world, one that contains facts about live possibilities the discourse describes. The result is a pair[2]. So we interpret sentences with the type[3]: $[\![s]\!] = \gamma \rightarrow \gamma \rightarrow (\gamma \rightarrow \gamma \rightarrow t) \rightarrow (\gamma \rightarrow \gamma \rightarrow t) \rightarrow (t \rightarrow t \rightarrow t) \rightarrow t$.

Using suitable lexical entries together, we get:

$$\begin{array}{lll}
[\![\textit{A wolf might enter}]\!] & = \lambda i_1 i_2 k_1 k_2 f.f & (\Diamond(\exists x.(\mathbf{wolf}\,x) \wedge ((\mathbf{enter}\,x) \wedge (k_1\,(x :: i_1)\,i_2)))) \\
 & & (k_2\,i_1\,i_2) \\
[\![\textit{It would growl}]\!] & = \lambda i_1 i_2 k_1 k_2 f.f & (\Box((\mathbf{growl}\,(\mathtt{sel}(i_1 \cup i_2))) \wedge (k_1 i_1 i_2))) \quad (k_2\,i_1 i_2) \\
[\![\textit{It will growl}]\!] & = \lambda i_1 i_2 k_1 k_2 f.f & (k_1 i_1 i_2)((\mathbf{growl}\,(\mathtt{sel} i_2)) \quad (k_2\,i_1 i_2))
\end{array}$$

[1] We could have used a pair, or a record.

[2] Modelled with the higher-order type of functions taking functions with two arguments—i.e., the type $(t \rightarrow t \rightarrow t) \rightarrow t$.

[3] The first of the two environments and the first of the two continuations are the modal ones.

The combination rule between modalized sentences, which, using the categorial terminology of monads, states the binder property of the monad, tells us how to put two sentence types to get another. For us the binder property is sensitive to modal features of its arguments—here is the modal composition when the second sentence has a modal mood:
$[\![S_1.S_2]\!] = \lambda i_1 i_2 k_1 k_2 f.[\![S_1]\!]\, i_1\, i_2(\lambda i'_1 i'_2.[\![S_2]\!] i'_1 i'_2 k_1 k_2 \Pi_1)\, k_2\, f$
With $\Pi_1 = \lambda ab.a$ the first projection, we get[4]:
$[\![(1)]\!] \ = (\Diamond(\exists x.(\mathbf{wolf}\, x) \wedge ((\mathbf{enter}\, x) \wedge (\Box((\mathbf{growl}\,(\mathtt{sel}(x :: \mathtt{nil}) \cup \mathtt{nil})) \wedge \top))))) \wedge \top$
$[\![(2)]\!] \ = (\Diamond(\exists x.(\mathbf{wolf}\, x) \wedge ((\mathbf{enter}\, x) \wedge \top))) \wedge (\mathbf{growl}\,(\mathtt{sel}\,\mathtt{nil}))$
In the first case, the `sel` function has access to x, so (1) is predicted to be good, whereas (2) is predicted to be bad, because x is not part of the accessible environment.

While we omit detailed lexical entries here[5], note that the fact that modalities have scope over the quantifiers and that $\Box$ is embedded under $\Diamond$ in the analyses of (1) and (2) results from a choice in lexical entries rather than from the compositional formalism; other lexical entries determine different scope possibilities for the modal operators and quantifiers.

We also give an account of MS using local accommodation, with a more complex type interpreting s. We need two additional parameters for the continuations: one of type t which is the restriction of the current modal structure (and the scope of the previous one) and one of type $t \to t \to t$ which expresses how the scope and the restriction have to combine (typically $\lambda b_1 b_2.b_1 \wedge \Diamond(b_1 \Rightarrow b_2)$). For *A wolf might enter. It would growl. It would eat you first* we get: $\Diamond\exists x.((\mathbf{wolf}\, x) \wedge (\mathbf{enter}\, x) \wedge \Box(((\mathbf{wolf}\, x) \wedge (\mathbf{enter}\, x)) \Rightarrow ((\mathbf{growl}\,(sel((x :: \mathtt{nil}) + \mathtt{nil}))) \wedge \Box(((\mathbf{wolf}\, x) \wedge (\mathbf{enter}\, x)) \Rightarrow ((\mathbf{eat\ you}\,(sel((x :: \mathtt{nil}) + \mathtt{nil}))))))))$.

Our continuation based framework is lexically based, flexible and powerful. It can model Veltman's [9] semantics for epistemic modalities by complicating the environment type. It can also analyze DRT's treatment of anaphorically linked yet independent attitudes. For example, you may want to marry an Italian and you may hope that she (or he) is rich. But you don't want to hope that she is rich; you hope that she is rich. A more famous example is the Hob-Nob sentence. A natural move to TY2 in our type-theoretic setting allows us to treat these examples with quantification over concepts. In TY2 with the appropriate lexical entries and a coercion story about the move from objectual quantification to concept quantification (x below is a variable of type $w \to e$), the Hob Nob sentence yields something logically equivalent to:

(4) $\exists x \forall w' \in \mathcal{B}_{h,w}(((\mathrm{witch}(x(w'), w') \wedge \exists! u(h's\ \mathrm{mare}(u, w') \wedge \mathrm{blighted}(x(w'), u), w')) \wedge \forall w'' \in \mathcal{B}_{n,w}(\exists! v(h's\ \mathrm{cow}(v, w'') \wedge\ \mathrm{killed}(x(w'), u, w'')))$

[1] P. de Groote. Towards a Montagovian Account of Dynamics. In *Proceedings of Semantics and Linguistic Theory* XVI, 2006. [2] A. Brasoeavnu. Structured Anaphora to Quantifier Domains A Unified Account of Quantificational and Modal Subordination and Exceptional Wide Scope. *Information and Computation.* To appear. [3] A. Frank and H. Kamp. On Context Dependence in Modal Constructions. In *Proceedings of SALT* VII, 1997. [4] H. Kamp. A Theory of Truth and Semantic Representation. In J. Groenendijk, T. Janssen, and M. Stokhof, eds, Formal Methods in the Study of Language. Foris, 1981. [5] H. Kamp and U. Reyle. *From Discourse to Logic.* Kluwer Academic Publishers, 1993. [6] C. Roberts. Modal Subordination and Pronominal Anaphora in Discourse. *Linguistics and Philosophy*, 12, 1989. [7] M. Stone and D. Hardt. Dynamic Discourse Referents for Tense and Modals. In *Proceedings of IWCS* 2, 1997. [8] R. van Rooij. A Modal Analysis of Presupposition and Modal Subordination. *Journal of Semantics*, 2005. [9] F. Veltman,. Defaults in Update Semantics. *Journal of Philosophical Logic* **25**, 1996.

[4]To simplify, all examples are interpreted with empty environments (`nil`), trivial continuations ($\lambda i_1 i_2.\top$) and conjunction of the two components ($\lambda b_1 b_2.b_1 \wedge b_2$) yielding the type t.

[5]For instance $[\![might]\!] = \lambda vs.\lambda i_1 i_2 k_1 k_2 f.f\ (\Diamond(v\, s\, i_1\, i_2\, k_1\, k_2 \Pi_1))(k_2\, i_1\, i_2))$.