Potts (e.g., 2005) argues that natural language meaning is multi-dimensional. For example, *John read the damn book* asserts that John read a particular book (its at-issue entailments), and simultaneously conveys that there is something about the situation that the speaker disapproves of.

Potts claims that any given formative can contribute either to at-issue entailments, or else to some other dimension of meaning, but not to both. This is intended to cover at least supplements (*John, who is smart, is here*) and expressives (*John read the damn book*). The claim that lexical items are dimensionally pure in this way has been challenged by a number of people, including Bach 2006, McCready 2009, Kubota and Uegaki 2009, and Gutzmann 2009, among others.

(1) John arrested the shyster.

For instance, Bach (following suggestions of Horn) argues that the epithet in (1) simultaneously contributes to the referential content of the noun phrase at the same time that it contributes information about the speaker’s opinion of that referent.

We will propose a formal logic that seeks to characterize the possible interactions between multi-dimensional elements and the at-issue material that surrounds them.

On the system of Kubota and Uegaki, the contribution of each element is divided into two separate parts: the at-issue entailments (written below a horizontal line), and the expressive content (above the line). Because lexical items have access to the at-issue dimension and the expressive dimension, it is possible for them to make contributions at both levels simultaneously.

But the mechanism behind the two-level notation used by Kubota and Uegaki was originally designed to handle scope-taking expressions such as quantificational DPs. Therefore, on their account, since expressives have access to the same formal resources as quantifiers, they have as much freedom to interact with at-issue material as quantifiers do. For instance, it would be easy to have a lexical item “negex” that, say, reversed the polarity of a subordinate expressives, so that *John negex read the damn book* meant something like ‘John read the book, and that was a good thing’. As far as we know, such operators do not exist.

We propose the following refinement of Potts’ descriptive generalization:

(2) The not-at-issue component of a multi-dimensional element does not interact with at-issue material.

This generalization is not guaranteed by Kubota and Uegaki’s system.

We will show that LG (for “Lambek/Grishin”, Moortgat 2009) offers a principled account of (2). LG is a categorial grammar with the usual syntactic types built from / and \, and in addition has dual connectives \(\odot\) and \(\ominus\). The usual connectives and the dual connectives interact only via structural inference rules proposed by Grishin 1983, most relevantly including the following two (compiled here with residuation rules, and assuming a display-calculus presentation of LG):

\[
\frac{p \odot (q \otimes r) \rightarrow s}{(p \otimes q) \otimes r \rightarrow s} \quad \frac{p \odot (q \otimes r) \rightarrow s}{q \otimes (p \otimes r) \rightarrow s}
\]

At-issue composition proceeds as usual with / and \(. The syntactic category of a not-at-issue element is given using the special dual categories, so that shyster might have category \((S \odot S) \otimes N\). Interaction between local and non-local elements is regulated entirely by the Grishin interaction postulates. In particular, the Grishin postulates allow the expressive part \((S \odot S)\) to climb in only one direction: upward to some enclosing clause.
John (arrested (the N)) ⊢ (S ⊗ S) ⊕ S
(S ⊗ S) ⊗ (John (arrested (the N))) ⊢ S
John (S ⊗ S) ⊗ (arrested (the N)) ⊢ S
John (arrested (S ⊗ S) ⊗ (the N)) ⊢ S
John (arrested (the (S ⊗ S) ⊗ N)) ⊢ S
John (arrested (the shyster)) ⊢ S

Reading the derivation proof from bottom to top, *shyster* receives the syntactic category *(S ⊗ S) ⊗ N*. The nominal part, *N*, contributes a property that, in combination with the definite determiner, picks out an appropriate individual. The expressive component, *S ⊗ S*, detaches and climbs upward by means of the Grishin interaction postulates, making its semantic contribution (that the speaker disapproves of individuals who satisfy the property given by *N*) only when it escapes to the right side of the turnstyle, which is where the non-local part of the language lives.

As in Kubota and Uegaki’s system, the dual categories of LG provide an account of general quantificational scope-taking. For instance, the syntactic category of a quantificational DP is *(S ⊗ S) ⊗ DP*. The crucial difference is that LG has an additional semantic level of representation (technically, a CPS transform). It is only at this level that we can give quantifiers lexical values that allow them to violate (2). Our hypothesis, then, is that what is special about multi-dimensional components is that their contribution can be encoded already in terms of the pre-CPS semantic representation (as we will explain, the pre-CPS semantics is encoded in the λµ̃μ̃ calculus of Curien and Herbelin 2000). Thus non-at-issue meaning stays entirely within the proof-theoretical derivation, as governed by the Grishin postulates, without access to the full power of the lexical semantics.

We give a full formal fragment in the paper, providing derivations of epithets, supplements, expressives, and the compositional expressives of Geurts 2007 and of Schwager and McCready 2009.

From a theoretical point of view, the situation is analogous to the treatment of indexicals, though with a reversal of direction in the flow of semantic information. That is, with indexicals, information flows from the context to the indexical: the enclosing context determines the value of an indexical, no matter how deeply embedded; but with not-at-issue implications, information flows from the deeply-embedded element to the context.

Indexicals differ from pronouns in that the values of indexicals (typically!) depend only on the context of utterance, and do not interact with intervening material. In Kaplan’s classic account, this difference is modeled by providing two levels of semantic interpretation: character and content. A sentences type denotes a character. Characters combine with a context of utterance to produce a content, in which indexicals are replaced with their context-determined values. The content then contains normal pronouns that interact with quantifiers and other semantic elements in the normal way. (Refined systems also handle untypical ‘monstrous’ indexicals.)

LG was originally designed with two semantic levels for purely technical reasons. We show how exploiting this independently-motivated feature provides exactly the right theoretical power to account in a principled and fully compositional way for one of the signature properties of multi-dimensional meaning. Just as a two-level conception of semantics can give an insightful account of the difference between indexicals and other pronouns, a two-level conception of non-local semantic scope-taking can provide new insights into the different kinds of multi-dimensional meaning.