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Pareto Improving Lotteries and Voluntary Public Goods Provision

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Abstract

This paper characterizes the utility possibility frontier resulting in a model of private voluntary provision of a public good. It is shown that ex-ante lotteries over resource distributions among the agents can be Pareto improving. A corollary is that an equal distribution of resources among the agents, or any distribution where all agents contribute in equilibrium, is always Pareto dominated by a lottery between two unequal distributions.

KEYWORDS: public goods, private provision, lotteries, efficiency

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1 Introduction

Most of the literature on private provision of public goods has been concerned with the question of maximizing the total amount of the public good provided. The reason is the well-known result that the positive externalities and free-riding associated with public goods result in underprovision, thus redistributive policies that raise total provision should be welfare improving. The famous 'neutrality theorem' (Warr, 1983; Bergstrom, Blume and Varian, 1986) states that the distribution of resources does not matter for the total provision of the public good if the set of contributors does not change with redistribution. This implies that it is only possible to raise total contribution if redistribution is done through taking resources away from non-contributors and giving them to contributors (Cornes and Sandler, 1996). While such redistributions lead to higher provision, their effect on social welfare is not clear. Since they typically do not achieve the first best, it is not obvious whether increasing total provision is equivalent to increasing social surplus or individual welfare of all agents (e.g. see Bardhan et al., 2006). As Cornes and Sandler (2000) point out, from a policy perspective, redistribution policies that improve everyone's well-being have more desirable normative properties than policies that just augment public good provision.

I show that, in a general, non-transferable utility setting, even when a Pareto improving redistribution per se may be impossible, it is typically the case that Pareto improving ex-ante *lotteries* between several resource distributions exist. This result holds under quite general conditions, essentially whenever the initial distribution is not too unequal. The reason is that there is an inherent non-convexity in the utility possibility frontier associated with the non-cooperative voluntary provision model studied here. Thus, as has been also pointed out in various other contexts, (e.g. Friedman and Savage, 1948; Guesnerie, 1975, Hansen, 1985, Townsend, 1993, Becker, Murphy and Werning, 2005 among others) such non-convexities in utility possibility sets, (indirect) utility functions or production sets can be "convexified away" using ex-ante lotteries over allocations to the effect of raising ex-ante expected utility and generating Pareto improvements. What this paper proves is that such lotteries will be generically optimal in the standard voluntary provision model. In addition to achieving higher welfare, the public good provision level at the post-lottery unequal distributions is also shown to be higher relative to what it would be at the pre-lottery resource distribution.

The current paper draws and builds upon the existing results of Itaya, De Meza and Myles (1997) and Cornes and Sandler (2000). Itaya, De Meza and Myles (1997) show that with transferable utility the social welfare function exhibits a non-convexity when agents are at the margin between contributing and non-contributing, thus, under certain initial conditions, it is possible to raise social welfare through inequality-increasing redistributive policies. Their results rely on the fact that, in a neighborhood of the non-convexity, the utility loss experienced by non-contributors is more than offset by the gain to contributors. In their transferable utility setting this permits a Pareto improvement. However, in the more general case of non-transferable utility (as studied here), it is not clear whether such redistribution policies should (or could) be pursued as they hurt one group of agents to benefit another. Therefore, it is important to know whether there exist Pareto improving redistributive policies, i.e. such that benefit all agents at the same time. In a recent contribution, Cornes and Sandler (2000) address this question and show that local Pareto improvements[1] (i.e. starting from a particular resource distribution) are indeed possible under certain conditions that require high initial number of non-contributors and strong aggregate contribution response to resource transfers received by the contributors. In particular, no Pareto improving redistribution is possible in a two-agent framework.

In a set of lecture notes, Bergstrom (2002) conjectures informally[2] the *possibility* of a non-convexity in the utility possibility frontier in a model with public goods. This paper proves that this non-convexity is not a mere possibility but rather an inevitable property of the frontier, i.e. the utility possibility set associated with the canonical voluntary public good provision model (Bergstrom et al., 1986) is *necessarily* non-convex, with or without utility transferability. This implies that Pareto improving lotteries over resource distributions would always exist around the non-convex portion of the frontier. The exact conditions and the set of initial wealth distributions that give rise to the non-convexity are fully characterized below. In particular, I show that any initial allocation that results in all agents contributing in equilibrium[3] will be *always* Pareto dominated in expected utility sense by offering all agents an ex-ante lottery over two (or more) unequal distributions. This is an interesting implication in light of the "neutrality" property discussed above which asserts than any direct redistributions that do not change the set of contributors have no welfare effects. Notice also that, unlike in Cornes and Sandler

[1]See also Ihori (1996) and Cornes and Hartley (2004) who demonstrate that relaxing the "canonical" assumption of identical unit costs across contributors could also lead to Pareto improvement.

[2]I thank an anonymous referee for bringing this to my attention. A sample figure is drawn but no justification is given of why or when this scenario may arise. See http://www.econ.ucsb.edu/%7Etedb/PubFin/pfweb.pdf, page 29.

[3]A particular case is the equal resource distribution.

(2000), this result does not require the existence of non-contributors in the initial equilibrium[4] and holds even in a two-agent setting.

The existing literature concentrates on showing how one can achieve a local Pareto improvement given the right initial conditions but does not explicitly analyze whether the resulting outcome lies itself on the Pareto frontier[5] or how large transfers are needed to achieve that. A further contribution of this paper is that it characterizes explicitly the entire Pareto frontier resulting from varying the initial resource distributions and hence provides a more complete picture of the nature and availability of redistributive policies that can be used to attain efficient allocations.

The remainder of the paper is organized as follows. The next section describes the model and proves the main result. Section 3 presents some graphical and numerical examples illustrating the main findings. Conclusions and discussion are provided in section 4.

2 Pareto Improving Lotteries Over Resource Distributions

Consider the "canonical" model of non-cooperative voluntary public good provision (Bergstrom, Blume and Varian, 1986). There are two goods - a private consumption good, x and a pure public good, G. The public good is provided through voluntary contribution and has a production function $G = \sum_{i=1}^{N} g_i$ where g_i is the resource contribution of agent i. There are $N = 2$ agents[6] with identical preferences represented by $u(x_i, G)$ and heterogeneous resource endowments (which could be wealth, time, etc.), $y_i \in \mathbf{R}_+$. The function u is strictly increasing in both arguments, concave, twice continuously differentiable, and satisfies $u_{12} \geq 0$, as well as Inada boundary conditions on both arguments. Agent $i's$ budget constraint is:

$$x_i + g_i = y_i$$

[4] It is worth pointing out, however, that the post-lottery unequal distribution must necessarily involve a positive number of non-contributors. The contributor/non-contributor distinction is thus still important (as in Cornes and Sandler, 2000) but in a different way. I thank an anonymous referee for this observation.

[5] Throughout the paper I refer only to the Pareto frontier resulting under voluntary Nash equilibrium provision. See Cornes and Sandler (1996, ch.7) for a general discussion of alternative contribution mechanisms and their efficiency.

[6] The $N = 2$ assumption is not crucial for the results since (as explained in more detail below) in the general case of $N > 2$ one can always arrange a sequence of 2-agent lotteries to generate a Pareto improvement.

We are looking for a Nash equilibrium in the individual contributions, g_i. In this framework it is well known (e.g. see Cornes and Sandler, 1996) that: (a) a unique Nash equilibrium exists; (b) if both agents contribute, they will have equal indirect utility, u^*, and (c) any resource redistribution that does not change their status of contributors will have no effect on both total provision and social surplus. Therefore, as Cornes and Sandler (2000) point out, it is only meaningful to talk about Pareto improving redistributions when, as a result, there exist non-contributing agents. Suppose that $y_i \leq y_j$, $i, j = 1, 2$, $i \neq j$, i.e. if resources are unequally enough distributed, agent i (the poorer agent) will be a non-contributor in equilibrium[7]. Since this paper is concerned with redistribution, let us fix the total resource amount, $Y = y_i + y_j$.

The first order condition of agent i's optimization problem is[8]:

$$-u_1(y_i - g_i, G) + u_2(y_i - g_i, G) \leq 0 \tag{1}$$

The above implies that, if $u_1(y_i, g_j) > u_2(y_i, g_j)$ in the contribution Nash equilibrium, agent i will not be contributing, i.e. $g_i = 0$. To see when such an equilibrium would occur, look at the contributor's (agent j) first order condition[9]:

$$u_1(y_j - g_j, G) = u_2(y_j - g_j, G)$$

At $g_i = 0$ the above can be written as:

$$u_1(y_j - g_j, g_j) = u_2(y_j - g_j, g_j) \tag{2}$$

which can be solved[10] for j's equilibrium contribution $g_j^* = \phi(y_j)$. Assuming that both x and G are normal goods, we know that $0 < \phi'(y_j) < 1$ everywhere (see Cornes and Sandler, 2000, p. 176).

Going back to (1) we see that if y_i and y_j satisfy:

$$u_1(y_i, \phi(y_j)) \geq u_2(y_i, \phi(y_j)) \tag{3}$$

then agent i will not be contributing in equilibrium. Given the assumptions on u, condition (3) holds for any y_i low enough relative to y_j. Denote by $\eta(y_j)$ the value of y_i that satisfies (3) at equality (such a value always exists given the Inada conditions), i.e. the resource level at which agent i is just

[7]See Bergstrom, Blume and Varian, 1986, theorem 5 for more details.

[8]The f.o.c.'s are necessary and sufficient for a maximum given the Inada assumptions and the concavity of u.

[9]Due to the assumed Inada conditions, an agent will never choose to contribute her total resource endowment.

[10]Under our assumptions on u the solution will be unique.

on the margin between non-contributing and contributing given y_j.The above analysis implies that the following two types of equilibria are possible:

- both agents contribute if their resource levels are close enough, i.e. $y_i > \eta(y_j)$;
- only the 'richer' agent contributes if $y_i \leq \eta(y_j)$.

Call y^* the resource level such that $y^* + \eta(y^*) = Y$. Such y^* always exists - it is simply the wealth level of the richer agent at which she contributes and the poorer agent is on the margin[11]. Putting together the definitions of $\eta(y)$, $\phi(y)$ and (2) implies that:

$$\eta(y^*) = y^* - \phi(y^*) \tag{4}$$

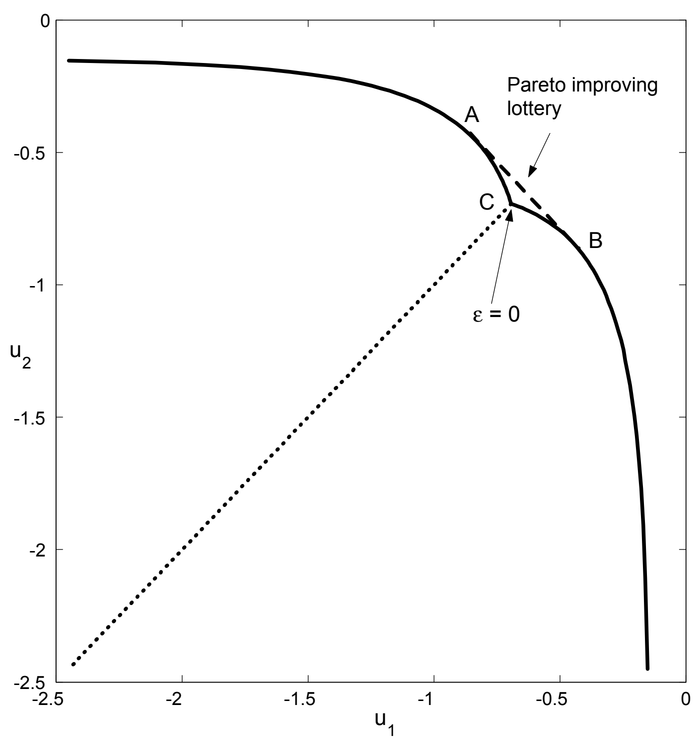

Figure 1: The Pareto Frontier (two agents)

We show next that there always exists a non-convexity in the Pareto frontier determined by the equilibrium indirect utilities of the two agents, (u^i, u^j) around the point where one of them is on the margin of contributing. The non-convexity implies that for any initial resource distribution $(y_i, y_j) = (\eta(y^*) -$

[11] One can find y^* for example by starting at an equal distribution and transfering resources from agent i to agent j until agent i is just at the margin of becoming non-contributor.

$\varepsilon, y^*+\varepsilon)$ for small enough $\varepsilon > 0$, ex-ante lotteries over two unequal wealth distributions can be Pareto improving as they provide a means to convexify the frontier. In addition, notice that for any resource distribution with $y_i > \eta(y^*)$, both agents contribute, thus $u^i = u^j = u^*$, i.e. all such distributions map into a single point on the Pareto frontier, namely the crossing with the 45-degree line (point C on fig. 1). Clearly then, Pareto improving lotteries are also possible for any initial contributors-only distributions (y_i, y_j) with $y_i \in [\eta(y^*), Y/2]$.

Proposition

There exists an $\bar{\varepsilon} > 0$ such that, $\forall \varepsilon \in [\eta(y^*) - Y/2, \bar{\varepsilon})$ and for any initial resource distribution $(y_i, y_j) = (\eta(y^*) - \varepsilon, y^* + \varepsilon)$, there always exists a lottery over two symmetric unequal distributions $(y_1, y_2)^A = (\eta(y^*) - \bar{\varepsilon}, y^* + \bar{\varepsilon})$ and $(y_1, y_2)^B = (y^* + \bar{\varepsilon}, \eta(y^*) - \bar{\varepsilon})$ which is welfare improving for both agents in expected utility sense[12].

Proof:

To prove the proposition it is enough to show that the Pareto frontier defined by the points with coordinates (u^i, u^j) where u^i is the indirect utility obtained in equilibrium by agent i, is non-convex around the point at which the 45-degree line in the (u^i, u^j) space crosses the frontier (point C with coordinates (u^*, u^*) on fig. 1). Since the frontier is symmetric around this point, to show the non-convexity it is enough to prove that the frontier's slope in the upper-side neighborhood of C is larger than 1 in absolute value, which would imply that the slope in the lower-side neighborhood is less than 1 and hence the frontier is not concave around point C. Notice also that all initial contributors-only distributions $(y_i, y_j) = (\eta(y^*) - \varepsilon, y^* + \varepsilon)$ for $\varepsilon \in [\eta(y^*) - Y/2, 0)$ map into point C, achieved also for $\varepsilon = 0$. Thus, without loss of generality, we concentrate on the case $\varepsilon \geqq 0$ as every property of the frontier at $\varepsilon = 0$ extends also to the case $\varepsilon \in [\eta(y^*) - Y/2, 0)$.

Given our analysis above, the non-contributor's indirect utility when we redistribute $\varepsilon \geq 0$ resources from agent i to agent j in the neighborhood of point C is:

$$u^{NC} = u^i = u(\eta(y^*) - \varepsilon, \phi(y^* + \varepsilon))$$

Notice that the non-contributor's indirect utility is decreasing in ε:

$$\frac{\partial u^{NC}}{\partial \varepsilon} = -u_1(.) + u_2(.)\phi'(.) < 0 \tag{5}$$

[12]Note that making this statement operational requires the implicit assumption that agents have von Neuman-Morgenstern utilities over lotteries which is a slight departure from the canonical voluntary provision model (e.g. Bergstrom et al., 1986).

since $\phi' < 1$ and $u_1(\eta(y^*) - \varepsilon, \phi(y^* + \varepsilon)) > u_2(\eta(y^*) - \varepsilon, \phi(y^* + \varepsilon))$ by (1). Similarly, the contributor's indirect utility is:

$$u^C = u^j = u(y^* + \varepsilon - \phi(y^* + \varepsilon), \phi(y^* + \varepsilon))$$

which, using (2) is increasing in ε:

$$\frac{\partial u^C}{\partial \varepsilon} = u_1(.)[1 - \phi'(.)] + u_2(.)\phi'(.) = u_1(.) > 0 \tag{6}$$

The results in (5) and (6) imply that the Pareto frontier is negatively sloping everywhere.

Let us now check the magnitude of the slope of the frontier in the neighborhood of point C. We will show that, as $\varepsilon \to 0$ from above, the limit of the frontier's slope is larger than 1 in absolute value. Using (2) and (4), we have that:

$$\lim_{\varepsilon \to 0} \left| \frac{\frac{\partial u^C}{\partial \varepsilon}}{\frac{\partial u^{NC}}{\partial \varepsilon}} \right| = \frac{u_1(\eta(y^*), \phi(y^*))}{u_1(\eta(y^*), \phi(y^*)) - u_2(\eta(y^*), \phi(y^*))\phi'(y^*)} = \frac{1}{1 - \phi'(y^*)} > 1$$

since $\phi' < 1$. Therefore, by continuity, there exists an $\bar{\varepsilon} > 0$ such that for any $\varepsilon \in [0, \bar{\varepsilon})$, a lottery between the resource distributions $(\eta(y^*) - \bar{\varepsilon}, y^* + \bar{\varepsilon})$ and $(y^* + \bar{\varepsilon}, \eta(y^*) - \bar{\varepsilon})$ is welfare improving for both agents.■

We have shown above that the utility possibility frontier is downward sloping everywhere. Thus, independent of the initial distribution and the number of contributors, a Pareto improving redistribution through transfers from one agent to another (as in Cornes and Sandler, 2000) is impossible. However, the above proposition shows that for any initial distribution characterized with not too much inequality (i.e. such that maps to section ACB of the Pareto frontier) there exist Pareto improving *lotteries* over two unequal resource distributions. Notice that this result is always true for all distributions where all agents are contributors[13]. The proposition result allows us to fully characterize the shape of the Pareto frontier. In particular, the inequality level, $\bar{\varepsilon}$ corresponding to points A and B can be found by setting the slope to -1.

The above results also have implications for the public good provision level that would be observed if such lotteries are used. Specifically, we can apply the result of Bergstrom, Blume and Varian (Theorem 5(v), p.38) which states that any transfer from a contributor to non-contributor would decrease

[13] Thus case corresponds to point C on the frontier.

total equilibrium provision. Since, by construction, each pre-lottery wealth distribution at which lotteries would be optimally used (i.e. not too unequal) can be obtained from the post-lottery distribution via such type of equalizing transfer, this implies that the post-lottery public good provision level would be also increased as an outcome of the lottery.

The characterization of the utility possibility frontier performed above can be also used to judge about how often or in what situations the initial wealth distribution could allow for Pareto improving lotteries. Notice that such lotteries are not possible only on the strictly concave portions of the frontier, corresponding to situations where the wealth distribution between the agents is very unequal and necessarily one of them does not contribute. In particular, the above result implies that for any wealth distribution where each agent contributes a positive amount[14], a possibility for Pareto improvement via lottery must exist. This suggests that such lotteries will be effective (i.e. agents would voluntarily choose to participate in them) in situations featuring non-too-unequal resource distributions but not in situations where the initial resource distribution is heavily biased towards one agent. For instance, in an application to local public goods, relatively homogeneous wealth distributions are likely to be expected due to endogenous selection in the residency choice.

The proposition result can be easily extended to the case of more than two players - it is enough to notice that any multilateral redistribution can be decomposed into a series of bilateral ones. Due to the non-convexity around the point where agents switch from contributing to non-contributing, there is always scope for Pareto improving lotteries for all not-too-unequal initial resource distributions that map into points on the frontier in a neighborhood of the equal utility point C. Notice that (see fig. 2) with more than two players there are more possibilities for Pareto improving lotteries than in the two-player case, namely with $N > 2$ such lotteries can be constructed between any of the points in the “inward bulge” of the frontier in N-dimensional space. By contrast with the analysis of Cornes and Sandler (2000), it is clear that this also holds for any initial distribution that leads to everyone contributing in equilibrium. We show a three-player example in the next section.

3 Examples

Consider first the two-player case. Let the utility function be Cobb-Douglas, $u(x, G) = \frac{1}{2}\ln x + \frac{1}{2}\ln G$. Following the same steps as in the previous section, we see that if $y_i \leq \frac{1}{2}y_j$ only agent j will contribute. Setting $Y = 1.5$, we have

[14] And also some distributions where just one agent contributes that are not too unequal.

$y^* = 1$, $\eta(y^*) = 1/2$ and $\bar{\varepsilon} \approx 0.12$. Notice that, when both agents contribute, total contribution and their private good consumptions are equal to $\frac{y_1+y_2}{3}$, while if only agent j contributes, we have $G = \frac{y_j}{2}$.

Figure 1 depicts the frontier traced by the agents' indirect utilities as we vary ε, i.e. their equilibrium welfare at resource levels $y_i = 1/2 - \varepsilon$ and $y_j = 1 + \varepsilon$. As we saw in the proposition proof, what is crucial for our results is that the rate of increase of the contributor's utility for small ε exceeds the rate of decline of the non-contributor's utility which creates a non-convexity in the Pareto frontier at the point $\varepsilon = 0$ (i.e. the level of inequality where both agents contribute). Thus, a lottery between points A and B on the graph yields higher expected utility for each agent compared to their (certain) utility in any distribution with a lower degree of inequality that maps into section ACB of the frontier.

For example, both agents are better off under a "fifty-fifty" lottery between the unequal distributions associated with points A and B compared to the case of equality (point C). At point C each agent contributes 1/4, total contribution is $G^C = 1/2$, and both agents enjoy utility of

$$u^C = \ln 1/2 \approx -0.693$$

while the expected utility obtained by each agent from the equal chance lottery between the resource allocations $(0.38, 1.12)$ and $(1.12, 0.38)$ at which only the richer agent contributes is:

$$u^{lottery} = \frac{1}{2}[u(0.38, 0.56) + u(0.56, 0.56)] \approx -0.677 > u^C$$

Therefore a Pareto improvement is achieved through an ex-ante lottery over resources although no (deterministic) local Pareto improving redistribution exists (compare with Cornes and Sandler, 2000).

Finally, figure 2 depicts the non-convex portion of the utility possibility frontier in the three-player case obtained through a numerical simulation. Notice that there exist numerous possible Pareto improving lotteries for initial distributions with various degrees of inequality around point C where all agents contribute and hence obtain equal utility. In principle, both three-way or two-way such lotteries exist, however, note that every lottery among three resource distributions can be decomposed into two lotteries between two distributions each, so our previous analysis carries through.

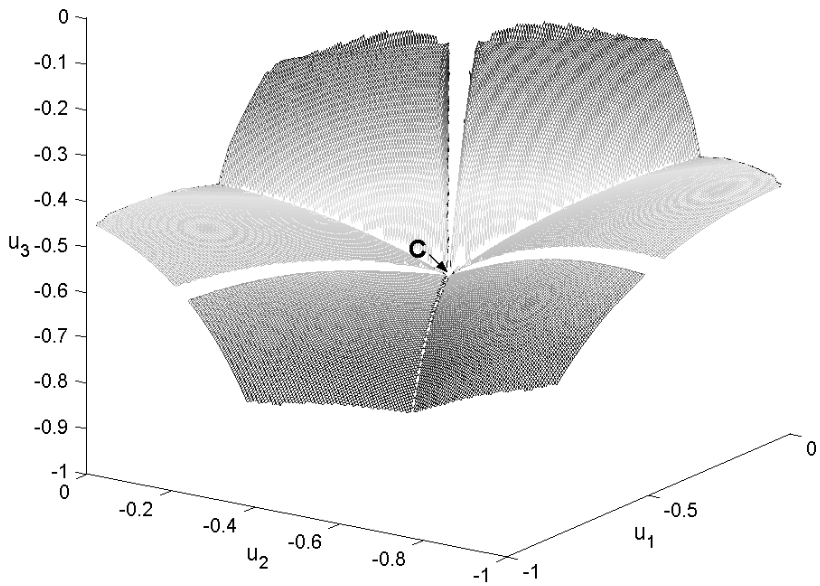

Figure 2: The Pareto Frontier (three agents)

4 Discussion and Conclusions

In this paper we have characterized non-convexities in the utility possibility frontier arising in a standard model of voluntary provision of public goods. It was shown that ex-ante lotteries over two or more unequal resource distributions can be Pareto improving under various initial conditions characterized by not-too-extreme inequality among the players' endowments. In particular, perfect equality between the agents is always Pareto dominated in expected utility terms by an equal chance lottery over two unequal allocations. This is a natural example when progressive redistribution policies may be suboptimal. In fact, somewhat contrary to standard economic intuition, the ex-post variance in outcomes resulting from the lottery can be welfare improving.

A potential caveat with respect to implementation is that such lotteries may be hard to enforce. To achieve the ex-ante welfare improvement the ex-post "loser" of the lottery is required to give up some of his resources which may be difficult to arrange. The cost of writing or enforcing such contracts may well outweigh the welfare gains from the lottery. Still, as per standard arguments, this issue is less likely to arise in a repeated game situation, e.g. between a husband and wife, etc.

The non-convexity of the utility possibility frontier around the point at which agents are on the margin between contributing and not-contributing (pointed out first by Itaya et. al., 1997) has been instrumental for our analysis. Such non-convexities, and hence the call for Pareto improving lotteries, arise also in various other settings that involve discrete choices or indivisibilities[15]. For example, Rosen (2002, p.13) calls this process "manufacturing inequality" and provides an excellent example of welfare improving gambles in a housing purchase problem.

Lotteries in our model act as randomization devices (gambles) assigning a high endowment to the "winner" and a low endowment to the "loser" with certain probabilities. The potential of lotteries as welfare improving mechanisms relates our results to those of Morgan (2000) and Amegashie and Myers (2003) who study the question of financing public goods by means of public lotteries (in the usual sense of the word) in lieu of voluntary contributions. The authors demonstrate that the usage of lotteries or raffles to collect funds for public good provision increases the provision level and can even achieve the first best. The main idea behind the result is that competition for the lottery prize creates a negative externality which can "neutralize" the positive spillover of the public good ignored by each contributor. Notice, however, that the role of lotteries is very different between Morgan's paper and ours. Participating in the lottery (i.e. buying tickets) is *the means* of contributing to the public good in Morgan (2000) *given* the wealth distribution among the players, while in our model the wealth distribution determining the final equilibrium contributions is the *outcome* of a lottery.

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