Heterogeneity, returns to scale, and collective action

Alexander Karaivanov *Department of Economics, Simon Fraser University*

Abstract. I analyze the effects of resource inequality and valuation heterogeneity on the provision of public goods with increasing or decreasing returns to scale in production. The existing literature typically takes the agents' characteristics as given and known to the researcher. In contrast, this paper compares collective action provision across groups of agents with resources and valuations for the public good drawn from different known joint distributions. Specifically, I characterize the *expected* equilibrium public good level as function of various distributional properties and moments. A resource-valuation distribution that first-order stochastically dominates another distribution always results in higher expected public good provision level, independent of the production technology. With decreasing returns to scale in the public good production, higher resource inequality results in higher expected provision. With increasing returns the same result holds when the mean resource level is relatively low, but expected provision decreases in inequality when the mean resource level is high. A parallel result holds for agents' valuations. JEL classification: H41, D61

Hétérogénéité, rendements à l'échelle, et action collective. L'auteur analyse les effets de l'inégalité des ressources et de l'hétérogénéité dans l'évaluation de biens publics dont la production se fait avec des rendements croissants et décroissants à l'échelle. La littérature spécialisée suggère habituellement que les caractéristiques des agents sont données et connues du chercheur. A contrario, ce mémoire compare l'action collective de groupes d'agents dont les ressources et les évaluations du bien public sont tirées de distributions conjointes connues. Spécifiquement, le niveau d'équilibre anticipé du bien public est fonction de divers moments et propriétés de ces distributions. Une distribution conjointe ressources-évaluations qui en domine une autre (au sens stochastique de premier ordre) résulte toujours en un niveau de bien public anticipé plus élevé, quelle que soit la technologie de production. Quand les rendements à l'échelle sont décroissants, une inégalité de ressources plus grande résulte en un niveau de bien public anticipé plus élevé. Quand les rendements sont croissants, le même résultat s'ensuit si le niveau moyen de la ressource est relativement bas, mais le niveau de bien public anticipé décroît quand le niveau moyen de la ressource est élevé. Des résultats parallèles sont notés pour ce qui est des évaluations du bien public par les agents.

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Email: akaraiva@sfu.ca

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1. Introduction

A wide variety of economic, political, or social phenomena fall under the heading of 'collective action,' defined here as the process of voluntary provision of a public good. Examples range from strikes to revolutions, from raising funds for charity to organizing lobbies, from digging a village well to building the Eurotunnel. International public goods such as environmental protection, disease prevention, or even world peace are further examples, since the lack of global enforcement prevents centralized provision. Social capital, which has strong collective good characteristics by definition (Coleman 1988), has been advanced as one of the determinants of economic development (Putnam 1993; Knack and Keefer 1997). Looking at the charitable sector of the US economy, Andreoni (1988) reports that 85% of all households make donations[1] and over 50% of all tax returns include charitable giving. On aggregate, the charitable sector constitutes about 2% of US national income.

Given the importance and prevalence of collective action, a natural question is, how does the amount provided depend on the characteristics of the contributors? In particular, should we expect higher levels of collective action in more homogeneous or more heterogeneous groups? For instance, how does the recent expansion of NATO to less wealthy Eastern European countries affect the provision of collective defence? Does reduction of land inequality through a land reform affect agricultural productivity by changing the voluntary provision of collective goods such as irrigation? Such comparisons across contributor groups with different characteristics can have important policy implications. A leading example is the recent interest in the economic, social, and political effects of diversity or ethnic fractionalization. For example, Alesina and La Ferrara (2000, 2005) or La Ferrara (2004) review ample evidence for both positive and negative effects of heterogeneity.

The large existing literature on voluntary public good provision (see Cornes and Sandler 1996 for an excellent review) most often addresses these questions by taking the distribution of agent types as given or looks at certain specific types of redistribution transfers across or within the contributors and non-contributors. A typical example is the 'distribution neutrality theorem' (Warr 1983; Bergstrom and Cornes 1983, among others) stating that the total amount of voluntary contributions to a pure public good is independent of the distribution of wealth among contributors. The main question is thus, how does the contribution equilibrium look like taking as given agents' preferences and incomes.

In contrast, this paper looks at these distributional effects from an aggregate viewpoint, comparing *across groups* of agents with different degrees of heterogeneity in resources and/or valuations for the public good[2] that are not

1 The same number applies for Canada, too: according to Statistics Canada (2004), 85% of Canadians aged 15 and over donated a total of $8.9 billion to charitable or non-profit organizations in 2004.

2 For example, women vs. men (possibly differing in their valuations), high-income vs. low-income neighbourhoods, and retired vs. working people (differing in spare time).

necessarily the result of particular types of transfers or particular initial resource allocations. The question I study is, for given distributions of characteristics in the population, what is the provision level we can expect as a function of the properties of the distribution (e.g., means, variances, correlation)?

Naturally, my main objective is thus *positive*, as opposed to normative analysis; before one can make statements and design policies about optimal wealth (re-)distribution one needs to study first what the expected outcome without any interventions looks like. In addition, performing similar cross-group comparisons for welfare as I do for total provision requires making strong assumptions about the social welfare function. Therefore, for the most part, I abstract from analyzing the optimal (social-welfare maximizing or Pareto improving) wealth (re-)distributions, for example, as in Itaya, de Meza, and Myles (1997); Olszewski and Rosenthal (2004); Cornes and Sandler (2004); and Bardhan, Ghatak and Karaivanov (2007).

The theoretical model I use features a finite number of agents who differ in their endowments of some resource (e.g., wealth, time) that they can contribute towards a pure public good or consume privately, and in their valuations for the public good. The main assumption that differentiates this paper from most of the existing literature is that the agents' resource and valuation levels are known to them but not to the researcher. The researcher knows only that the agents' characteristics are drawn from some population distribution Φ. I analyze the contribution game equilibria and the *expected* public good provision level as a function of the properties of the joint distribution of resources and valuations. A major advantage of this approach is its predictive power under low information availability: knowing only the distribution of characteristics in the population, what inference can be made about expected total provision? This could be useful in various contexts: fund-raising by charities (how many donations should be expected from different neighbourhoods), public policy (private vs. government public good provision), or political economy (e.g., in the case of collective actions such as insurgencies, strikes).

In addition, the public good production function – that is, the relationship between the contributed amount and the provision level – differs significantly across various public goods: some public goods are characterized by increasing returns in production, while others are characterized by decreasing returns.[3] The majority of the literature[4] focuses on the constant returns case; that is, it is typically assumed that the public good provision level simply equals the sum of individual contributions. This leaves out many interesting (and more general) cases of collective action with convex or concave production functions, which are

3 Public goods with increasing returns are typically mass actions, the effect of which increases with the number of participants, at least up to some level, such as strikes, demonstrations, or revolutions (Marwell and Oliver 1993). In contrast, most 'material' collective goods exhibit decreasing returns in production.

4 Cornes (1993) is a notable exception. Also, Andreoni (1998) uses a step production function (a limit case of increasing returns) to explain 'leadership gifts' and 'seed grants.'

analyzed here. This is important, as I show that the effect of heterogeneity on expected provision critically depends on the production technology.

The main results are as follows. First, with both increasing and decreasing returns in the public good production, a resource-valuation distribution that first-order stochastically dominates another distribution is characterized with higher expected total contribution level. Second, distributions characterized by positive assortative matching between valuations and resources lead to strictly higher expected contribution compared with distributions with negative matching. With decreasing returns to scale in the public good production, holding agents' valuations constant, higher resource inequality always leads to (weakly) higher expected provision, as it is the high resource agents who contribute. Holding resources constant, the effect of valuation heterogeneity on expected provision is generally ambiguous and depends on the second-order properties of the contribution functions. With increasing returns in the public good production, the effects of heterogeneity in agents' valuations and resource levels depend crucially on the mean valuation and resource levels. At low mean resource levels, increasing the resource variance (weakly) increases expected total contributions, while at high mean resource levels higher resource variance has a negative effect on total contribution. A symmetric result holds for valuations. The intuition is that, if average resources or valuations are low, heterogeneity can alleviate the free-riding problem and the zero provision equilibrium by having the agents with high valuations for the public good contribute, regardless of the action of the others. In contrast, at higher mean resources or valuations, heterogeneity has a negative effect on total provision, since it leads to non-contribution by some agents who contribute under homogeneity.

A key result of the standard private provision model is that the extent of free-riding increases as the number of agents grows. Addressing this issue, Olson (1965) suggests that 'selective incentives' may be needed to ensure non-zero provision, while Andreoni (1988) calls for a revision of the traditional model, calling it incapable of accounting for the observed widespread collective action in large groups. I show that the expected total level of collective action is always increasing in the number of agents, regardless of the shape of the public good production function. Intuitively, when there are heterogeneous agents with characteristics randomly drawn from a given distribution, increasing the number of agents leads to a higher probability that combinations favouring higher total contributions will be drawn, since only a subset of all agents contribute and contributions cannot be negative. Furthermore, if we look at per capita provision, I argue that, while Andreoni's criticism is valid under decreasing returns to scale in the collective action (i.e., expected per capita provision decreases with group size), the exact opposite result can hold with increasing returns.[5]

5 There are also other models consistent with widespread collective action in large groups, for example, the impure public good model of Cornes and Sandler (1984), and the 'weaker link' and 'weakest link' models (e.g., see Cornes and Sandler 1996).

Finally, the paper relates closely to the recent, primarily empirical, literature analyzing the effects of wealth inequality on collective action provision. Evidence at both the micro level (Wade 1994; Alesina and La Ferrara 2000; Bardhan 2000; Dayton-Johnson 2000; Khwaja 2004; Benjamin, Brandt, and Giles 2006) and the macro level (Knack and Keefer 1997; Banerjee 2004; Banerjee, Iyer, and Somanathan 2005) strongly suggests that the propensity of individuals to join groups, participate in social activities, or cooperate in various collective action problems is negatively related to inequality. On the other hand, a classic argument by Olson (1965) states that inequality may be beneficial for collective action, since the greater the interest of any single agent, the greater his incentive to see that the collective good is provided, even if he would be the only contributor. Higher inequality thus may help to reduce free-riding and increase provision (see also Itaya, de Meza, and Myles 1997). I explore under what conditions the former or the latter effect may prevail.

The paper proceeds as follow. Section 2 describes the basic model of voluntary public good provision. Section 3 analyzes the case of decreasing returns to scale in the public good production deriving the first (level) and second-order (inequality) effects of changes in the joint distribution of resources and valuations on expected total contributions. Section 4 performs the same analysis for the case of increasing returns in the collective action and outlines the differences from the decreasing returns case. Section 5 concludes.

2. Model

Consider an economy of n agents, indexed by $i = 1, \dots, n$ and two goods – a private good, x and a pure public good, Z. Agent i has an endowment, $r_i > 0$, of the private good (hereafter *resource*, which could be time, income, etc.). The resource can be either privately consumed by the agent or contributed to the production of the public good.[6] Let x_i denote the agent's consumption of the private good and g_i his resource contribution to the public good.

Agents' preferences are given by $U^i(x_i, Z)$, where U^i is increasing in both goods, (weakly) concave in x_i, and differentiable. The public good, Z, is produced from the total contributed amount, $G \equiv \Sigma_{i=1}^n g_i$, using the production function P,[7] that is, $Z = P(G)$. I assume that P is differentiable and strictly increasing. The curvature of P is crucial for the following analysis, which is why no assumptions about it are made at this point.

6 As is standard in the private provision literature (e.g., Bergstrom, Blume, and Varian 1986), I assume that the agents cannot trade, lend, or borrow their resource endowments. This could be due to transaction costs or simply the nature of the good (e.g., spare time).

7 While I adopt a 'production function' interpretation for P, which provides better motivation for the case of increasing returns to scale, it is clear that there is an equivalent 'preference' interpretation (see Cornes and Sandler 1996).

The agent's problem is to allocate his resources, r_i, between private consumption, x_i, and a contribution to the public good, $g_i \in [0, r_i]$:

$$\begin{aligned} &\max_{g_i}\ U^i(r_i - g_i, P(g_i + G_{-i})), \\ &\text{s.t. } 0 \leq g_i \leq r_i, \end{aligned} \tag{1}$$

where G_{-i} is the total contribution of all other agents. I assume Nash behaviour; that is, all agents take the actions of all others as given when simultaneously determining their optimal g_i and x_i. To parametrize the preference heterogeneity in a tractable way (via a single parameter) and analyze its effect on expected total provision, assume the following.

Assumption A1. *The preferences U^i satisfy a single-crossing property: for any given $x, G \in \mathcal{R}_+$,*

$$\frac{U^i_Z(x, P(G))}{U^i_x(x, P(G))} = v_i \frac{U_Z(x, P(G))}{U_x(x, P(G))} \tag{A1}$$

That is, for any x, G, the agents' marginal rates of substitution between the private and the public good are ordered according to the ordering of the parameters $v_i > 0$. I call v_i agent i's *valuation* for the public good, since, when everything else is held constant, higher v_i reflects a higher preference for the public good. An example of a utility function satisfying property (A1) is $U^i(x_i, Z) = u(x_i) + v_i w(P(G))$ for some u and w.

As discussed in the introduction, assume that the agents' resources, r_i and valuations, v_i are random draws from a known joint distribution[8] $\Phi(r, v)$. The agents have perfect information about each others' valuations and resource levels, but the latter are unobservable to the researcher. The paper characterizes the Nash equilibrium of the contribution game for any given draws $\{r_i\}$ and $\{v_i\}$, $i = 1, \ldots, n$. Furthermore, I show that the expected level of collective action, Z, depends critically on the characteristics of the distribution $\Phi(r, v)$ and the shape of the production function P and characterize this dependence.

I start the analysis with the case of decreasing returns to scale in collective action (non-convex P), which is isomorphic to the standard voluntary provision model (e.g., Bergstrom, Blume, and Varian 1986; hereafter BBV), and hence provides the natural benchmark for analyzing the effects of the properties of the resources and valuations joint distribution on expected total contributions. I then analyze the case of increasing returns to scale in the collective action in which the simpler binary (corner solution) structure of the equilibrium contribution strategies allows slightly more complete characterization and contrast the results with the decreasing returns case.

8 For the most part, Φ is assumed continuous for simplicity, although most results go through for a discrete distribution.

3. Collective action with decreasing returns

Start with the case of decreasing returns to scale (DRS) in the public good production, or alternatively, constant returns in production and concave utility in the public good.

3.1. Equilibrium

The agent's problem (1), with $P(G) = G$ and U^i strictly increasing and quasi-concave in both arguments and with both goods normal (isomorphic to the problem with U^i concave in Z and P concave that I study here) has been widely analyzed in the voluntary provision literature (see BBV or Cornes and Sandler 1996 for a review). It is well known that a unique Nash equilibrium in the agent's contributions, g_i^* exists, so I omit repeating the proof here.

While the analysis that follows can be done using the traditional methods of BBV, I follow the more recent, and perhaps more elegant, 'replacement function' approach of Cornes and Hartley (2007a; hereafter CH). This allows me to characterize the effects of changes in the resources and valuations in a very easy and intuitive way, as it avoids the proliferation of dimensions when many heterogeneous players are involved.

As in CH, define $M_i \equiv r_i + G_{-i}$ to be agent i's 'full income.' Thus, we can rewrite the agent's problem (1) as

$$\begin{aligned} &\max_{x_i, G}\ U^i(x_i, P(G)), \\ &\text{s.t. } x_i + G \leq M_i \text{ and } x_i \leq r_i. \end{aligned} \tag{2}$$

That is, the agent chooses his optimal quantities of the private good, x_i and the public good, G ($=g_i + G_{-i}$) subject to the full income budget constraint and $x_i \leq r_i$ (the agent cannot 'undo' the contributions of others). As in CH, assume that the private and public goods are normal goods for all players and define $\xi_i(M_i)$ to be person i's demand for the public good. By the normality assumption, ξ_i is increasing and thus its inverse, $\xi_i^{-1}(G)$, is well defined and increasing on the range of ξ_i. Call

$$\rho_i(G, r_i) \equiv \max\left\{r_i - \xi_i^{-1}(G) + G, 0\right\} \tag{3}$$

player i's *replacement function.*[9] By CH's proposition 2.1, the function ρ_i has the following properties: (1) there exists a finite value (called by CH the 'standalone value'), $\tilde{G}_i$, at which $\rho_i(\tilde{G}_i, r_i) = \tilde{G}_i$, namely, the crossing of ρ_i when plotted as a function of G with the 45° line – see figure 1 (adapted from figure 2 in CH); (2) ρ_i is defined for all $G \geq \tilde{G}_i$; and (3) $\rho_i(G, r_i)$ is continuous, everywhere non-increasing in G, and strictly decreasing whenever $\rho_i > 0$.

9 See Cornes and Hartley (2007a) for further details and motivation.

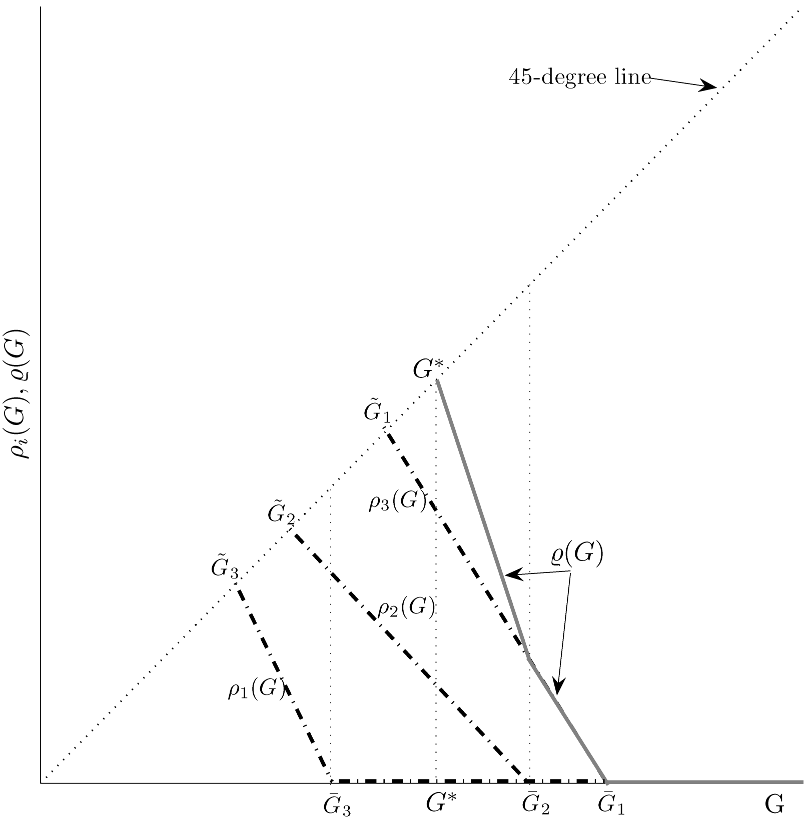

FIGURE 1 Individual and aggregate replacement functions

Define also the *aggregate replacement function*, $\varrho(G, \mathbf{r}) \equiv \Sigma_{i=1}^{n} \rho_i(G, r_i)$, where $\mathbf{r}$ denotes the vector $(r_1, r_2, \ldots, r_n)$. By proposition 2.2 in Cornes and Hartley (2007a), $\varrho(G, \mathbf{r})$ is defined for all $G \geq \max_i \tilde{G}_i$, and is continuous, everywhere non-increasing in G, and strictly decreasing when $\varrho > 0$.

A Nash equilibrium of the voluntary contribution game is a collection of individual contributions, $\{g_i^*\}_{i=1}^{n}$ such that $g_i^* = \rho_i(G^*, m_i)$ for $i = 1, 2, \ldots, n$ and where $G^* = \Sigma_{i=1}^{n} g_i^*$ (see CH). That is, the total equilibrium contribution G^* solves the equation $\varrho(G, \mathbf{r}) = G$; that is, given $\mathbf{r}$, it can be found as the crossing of the aggregate replacement function when plotted as a function of G with the 45-degree line. By the properties of ϱ (continuous and non-increasing), it is immediate to see that a unique such value and hence a unique Nash equilibrium exist. The elegance and simplicity of the Cornes and Hartley approach is that the characterization of the Nash equilibrium requires simply vertically adding up the functions ρ_i.

Assume also (as in assumption A4* in CH and standard for this literature) that for each agent there exists a finite level of the public good (a 'dropout value'), call it $\bar{G}_i$, above which player i will stop contributing; that is, $\rho_i(G, r_i) = 0$ for all $G \geq \bar{G}_i$. The dropout values determine which players contribute in equilibrium: any player with $\bar{G}_i < G^*$ will be a non-contributor in equilibrium, since $\rho_i = 0$ for such a player.

The positions of the replacement functions ρ_i (plotted against G) relative to each other are determined by the agent's resources and valuations for the public good. That is, for any $\{v_i, r_i\}$ drawn from the distribution Φ, the properties of Φ will affect these positions and therefore (after adding up) the overall equilibrium provision.

Therefore, to be able to characterize the distributional effects, we first need to study the effect of changes in the agent's resource and valuation levels r_i and v_i on his replacement function ρ_i (holding G_{-i} fixed). In addition to what was already assumed about U^i, assume also that U_Z is (weakly) increasing in x_i and U_x is (weakly) increasing in Z (i.e., the public and private good are weak complements). By assumption 1, the first-order condition for the public good in (2) is:[10]

$$U_x(M_i - G, P(G)) = v_i U_Z(M_i - G, P(G))P'(G), \tag{4}$$

from which one can derive the demand function for the public good $\xi_i(M_i)$ (which depends on v_i) and the replacement function $\rho_i(G, r_i, v_i)$, as defined in (3).

LEMMA 1. *The agent's replacement function, ρ_i, is (weakly) increasing in his valuation level, v_i, and his resource level, r_i, for any given G. The agent's standalone value, $\tilde{G}_i$, and dropout value, $\bar{G}_i$, are increasing in r_i and v_i for any given G.*

Proof. It suffices to show that, plotted against G, the replacement function shifts to the right as r_i and/or v_i increase. First, $\xi_i(M_i)$, which is the solution in G to (4), is (weakly) increasing in r_i and v_i. This is so by the properties of U: a higher r_i (weakly) decreases the l.h.s. but (weakly) increases the r.h.s., and so a larger G is needed to restore equality. Similarly, a higher v_i raises the r.h.s. without affecting the l.h.s.; so again, a higher G is needed to restore equality. By CH, whenever an agent is contributing, his replacement function $\rho_i(G, r_i)$ solves

$$\xi_i(r_i + G - \rho_i) - G = 0. \tag{5}$$

For given G, larger r_i and v_i raise the l.h.s. of (5) and, since it is a continuous decreasing function of ρ_i (see CH, 205), this implies that the function $\rho_i(G)$, whenever positive, shifts up in r_i and v_i. Hence, its crossing point with the horizontal axis, $\bar{G}_i$, shifts to the right. The result for $\tilde{G}_i$ follows immediately by its definition as the crossing point of $\rho_i(G)$ with the 45° line. ■

10 By the assumed normality and the properties of U and P, a positive solution always exists.

3.1.1. A Cobb-Douglas example

A specific example further clarifies how the functions ρ_i depend on r_i and v_i. Suppose we have a Cobb-Douglas (log-log) specification ($u(x) = \ln x$ and $P(G) = \ln G$); that is, $U^i(x_i, P(G)) = \ln x_i + v_i \ln G$. Then, from the agent's full income optimization problem, (2) we have

$$\xi_i(M_i) = \frac{v_i}{1+v_i} M_i \text{ and hence } \xi_i^{-1}(G) = \frac{1+v_i}{v_i} G.$$

Thus, $\rho_i(G, r_i) = \max\{r_i - (1/v_i)G, 0\}$ which is strictly increasing in r_i and v_i for any $G < \bar{G}_i$. The dropout and standalone values $\bar{G}_i = r_i v_i$ and $\tilde{G}_i = r_i v_i/(1+v_i)$ are also obviously increasing in both r_i and v_i. The Nash equilibrium total contribution, G^* solves $\Sigma_C[r_i - (1/v_i)G] = G$, where C is the set of contributors in equilibrium, i.e., all agents with $\bar{G}_i = r_i v_i > G^*$.

In general, the contribution Nash equilibrium with DRS is characterized as follows:[11]

PROPOSITION 1 (DRS equilibrium properties). *The equilibrium total contribution, G^* depends on the agents' resources, valuations, and replacement functions as follows:*

- i) *If we compare two groups A and B of size n with valuations and resources $\{v_i^A, r_i^A\}$ and $\{v_i^B, r_i^B\}, i = 1, \ldots, n$ such that $v_i^A \geq v_i^B$ and $r_i^A \geq r_i^B$ $\forall i$, then the total equilibrium contribution for group A is (weakly) larger than that for group B.*
- ii) *The set of contributors consists of the m agents, $m \in [1, n]$, with the m highest dropout values, $\bar{G}_i$.*
- iii) *Order the agents in decreasing order of their dropout values; that is, $\bar{G}_1 \geq \bar{G}_2 \geq \cdots \geq \bar{G}_n$. If agent m ($n \geq m \geq 1$) is the last contributing agent (i.e., agent $m+1$, if such exists, does not contribute), then total equilibrium contribution satisfies, $\bar{G}_m > G^* \geq \tilde{G}_1$. For $m = 1$, a precise upper bound for G^* is $\hat{G}$ defined as the solution to $n\rho_1(G, r_1) = G$.*

Proof. Part (i) follows directly from lemma 1 and the fact that $\rho_i^A(G) \geq \rho_i^B(G)$ $\forall i, G$. Thus, the aggregate replacement function ϱ^A is above ϱ^B and hence crosses the 45° line at a higher level of G (see figure 1). Part (ii) follows from the definition of ϱ as the vertical summation of the functions ρ_i and from the fact that G^* is the crossing point of ϱ with the 45° line (see CH). Finally, part (iii) follows from the fact that a positive amount ρ_k (if k is a contributor) is added to ϱ only for $G < \bar{G}_k$. Thus, if m agents are contributing, since the aggregate replacement function $\varrho(G)$ is decreasing, its intersection with the 45° line, G^* is (weakly) to the right of $\tilde{G}_1$($G^* = \tilde{G}_1$ only if $m = 1$) but also *strictly* to the left of $\bar{G}_m$ because

11 Most of these results follow easily from Cornes and Hartley (2007a), but they are listed here, since they are heavily used later on.

$\varrho(G)$ is strictly downward sloping for $G < \bar{G}_m$. The bound $\hat{G}$ for $m = 1$ (which is smaller than $\bar{G}_1$, since ρ_1 is downward sloping) is tight and is achieved when $\rho_i = \rho_j \, \forall i, j = 1, \ldots, n$. ■

Start characterizing at the aggregate distributional effects on expected provision by looking at the effect of group size, n. Specifically, if we were to draw at random n data points, (r_i, v_i), from some given joint distribution $\Phi(r, v)$, the proposition below characterizes the effect of group size on expected contribution level, $E_\Phi(G^*)$.

PROPOSITION 2 (group size). *The expected total contribution is (weakly) increasing in the number of agents, n.*

Proof. Take any draw $\{r_i, v_i\}_{i=1}^n$ from Φ. Suppose we draw one more agent with resource $\tilde{r}$ and valuation $\tilde{v}$. For any G this additional agent's replacement function adds either a strictly positive amount or zero to the aggregate replacement function, ϱ, so the latter either stays constant or shifts up and to the right. The aggregate replacement function can never shift down – agents cannot undo others' contributions. Thus, a larger number of agents always leaves unchanged or increases the total contribution. Since this result holds for any possible draws $\{r_i\}_{i=1}^n$ and $\{v_i\}_{i=1}^n$, the expected total contribution (weakly) increases in group size, n. ■

Proposition 2 restates in terms of expected provision the familiar result from the voluntary provision literature that adding an agent with positive endowment cannot decrease total contributions. The reason for the beneficial effect of group size here is simple: having more agents with characteristics randomly drawn from a given distribution increases the probability of generating ('good') draws with valuation and resource levels that correspond to high (positive) individual contributions, while at the same time negative contributions (for 'bad' draws) are impossible.

3.2. First-order distributional effects on collective action

This and the following section characterize the relationship between the properties of the joint distribution of valuations and resources, $\Phi(r, v)$, and the expected total equilibrium contribution. Specifically, I study what contribution levels and patterns should be expected when comparing across groups with different underlying distributions of resources and valuations.

Start by taking two population groups or size n with characteristics, $\{r_i, v_i\}$, independently drawn from two joint distributions, $F(r, v)$ and $H(r, v)$, with the same support and such that F first-order stochastically dominates H.[12] That

12 The original definition of first-order stochastic dominance (f.o.s.d.) by Rothschild and Stiglitz (1970) covers only the univariate case. Here, I extend the concept to multivariate random variables, as in Shaked and Shanthikumar (2006), using the (equivalent in the univariate case)

is, the probability of an agent having r_i, v_i with $r_i \leq \bar{r}$ and $v_i \leq \bar{v}$ for any fixed resource and valuation levels $\bar{r}$, $\bar{v}$ is always (weakly) lower under the distribution F than under H. In other words, on average, n randomly drawn agents from the first distribution would have higher resources and/or valuations compared with n agents drawn from the second distribution.

PROPOSITION 3 (first order stochastic dominance). *Let $F(r, v)$ and $H(r, v)$ be two probability distributions with the same support, such that F first-order stochastically dominates H. Then, the expected total contribution computed over draws $\{r_i, v_i\}_{i=1}^n$ from F is higher than the expected total contribution over draws $\{r_i, v_i\}_{i=1}^n$ from H.*

Proof. By Lemma 1, each agents' replacement function, ρ_i is (weakly) increasing in v_i and r_i. Thus, holding everything else constant, for any two points, $\mathbf{x}$ and $\mathbf{y}$ in the support of the distributions F and H such as $\mathbf{y} \geq \mathbf{x}$ (i.e., $y_1 \geq x_1$ and $y_2 \geq x_2$), we have $\rho_i(\mathbf{y}) \geq \rho_i(\mathbf{x})$. Furthermore, total contribution G^* determined by the intersection of the aggregate replacement function $\varrho = \Sigma_{i=1}^n \rho_i(G)$ with the 45° line is also (weakly) increasing in the valuations and resources. That is, by proposition 1, for any two collections $\{r_i, v_i\}_{i=1}^n$ and $\{r_i', v_i'\}_{i=1}^n$ such that $r_i' \geq r_i$ and $v_i' \geq v_i$, we have (abusing notation slightly) $G^*(\{r_i', v_i'\}) \geq G^*(\{r_i, v_i\})$. Further, by the definition of f.o.s.d. (see fn 12), the fact that F f.o.s.d. H implies that F puts higher probability on any such $\{r_i', v_i'\}$ than H (and lower probability on $\{r_i, v_i\}$). Therefore, by the properties of f.o.s.d. (e.g., Shaked and Shanthikumar 2006, chap. 5), we have $E_F(G^*) \geq E_H(G^*)$, where E_i denotes the expectation computed under distribution $i = F, H$. ■

Proposition 3 follows from the fact that agents' replacement functions and hence individual contributions are (weakly) increasing in their valuation and resource levels (lemma 1). Because of this, higher individual (and hence higher total) contributions obtain on average if the agents' characteristics are drawn from a distribution putting more mass at larger resources and valuations.

3.3. Second-order distributional effects: the role of heterogeneity

Propositions 2 and 3 outline the group size and first-order (level) effects of resources and valuations on expected collective action. However, as argued in the introduction, perhaps a more important distribution characteristic, especially in terms of policy debate, is the degree of heterogeneity (inequality) among agents. Proposition 1 and the Cobb-Douglas example imply that the set of contributing agents and total contributions are determined by agents' valuations and resources through their effects on $\tilde{G}$ and $\bar{G}$. For example, the highest $\bar{G}$ agent (agent 1, if ordered as in proposition 1(iii)) would be the sole contributor if $\tilde{G}_1 > \bar{G}_2$, that is,

statistical definition of *usual stochastic order*. Specifically, let X and Y be two random vectors in $\mathcal{R}^n$. If prob($X \in U$) $\leq$ prob($Y \in U$) for all upper sets $U \subseteq \mathcal{R}^n$ (a set $U \subseteq \mathcal{R}^n$ is called *upper* if $\mathbf{y} \in U$ whenever $\mathbf{y} \geq \mathbf{x}$ and $\mathbf{x} \in U$), we say that 'Y is larger than X in the usual stochastic order,' or 'Y f.o.s.d. X.'

when he supplies on his own his desired level of the public good. All other agents free-ride, as in the classic 'exploitation of the great by the small' result of Olson (1965), because their desired levels are lower.

These remarks suggest that, in general, the second-order effects of the resource and valuation distribution on expected total contributions can be quite complicated, since r and v interact in a non-linear way in determining the standalone and dropout values. Still, a tractable general analysis is possible, if we hold one of the characteristics constant while varying the other. An even more detailed analysis can be done for the IRS case in which the contribution equilibrium structure is simpler (see the next section).

PROPOSITION 4 (heterogeneity and collective action with DRS).

a) *Suppose all agents have the same valuations, $\bar{v}$, and n is large. Let $F(r)$ and $H(r)$ be two distributions with the same mean $\bar{r}$, from which $\{r_i\}_{i=1}^{n}$ are randomly drawn and such that $H(r)$ is a mean preserving spread of $F(r)$. Then, expected total contributions under H, $E_H(G^*)$, is (weakly) larger than expected provision under F.*

b) *Suppose all agents have the same resources, $\bar{r}$, and their replacement functions ρ_i are convex in v_i. Let $F(v)$ and $H(v)$ be two distributions with the same mean $\bar{v}$ from which $\{v_i\}_{i=1}^{n}$ are randomly drawn, and such that $H(v)$ is a mean preserving spread of $F(v)$. Then, expected total contributions under H, $E_H(G^*)$, is (weakly) larger than expected provision under F. If, instead, the replacement functions are concave in v_i, valuation heterogeneity can decrease expected total contribution.*

Proof. (a) Start with the case that both F and H are the degenerate distribution with all mass at $\bar{r}$ – call it $\bar{F}(\bar{r})$. Then all agents have the same replacement function, $\rho(G, \bar{r})$, and so all contribute in Nash equilibrium. Total contributions are the amount $\hat{G}(\bar{r}, n)$, which solves $n\rho(G, \bar{r}) = G$.

Now keep F equal to $\bar{F}(\bar{r})$, and let H be a mean preserving spread (m.p.s.) of $\bar{F}$. Order the agents in increasing order of their resources. Remember that (see lemma 1 and proposition 1), given that agents' valuations are the same, $\rho\ (G, r_i)$ shifts to the right the higher r_i and also that the high-resource agents contribute in equilibrium (they are those with the rightmost ρs for any given G). Expected equilibrium contribution under H can be written as

$$E_H(G^*) = \sum_{i=1}^{n-1} \text{prob}\ (C_i) \sum_{j=i}^{n} \rho(G, r_j) + \text{prob}\ (C_n) \sum_{i=1}^{n} \rho(G, r_i),$$

where C_i denotes the set of contributors $\{i, \ldots, n\}$ and prob(C_i) denotes the probability of all draws $\{r_i\}$ from H such that only the agents in the set C_i contribute. Take a sequence of distributions, $H_0, H_1, \ldots, H_k$, .. such that H_l is a m.p.s. of H_{l-1}, $\forall l \geq 1$ and $H_0 \equiv \bar{F}(\bar{r})$. By the definition of the function ρ, it is linear in r_i and so, for any draw such that all agents contribute ($C \equiv C_n$), expected equilibrium contributions (determined by the crossing of $\Sigma_{i=1}^{n}\rho\ (G, r_i)$ with the 45° line) is the same as $\hat{G}(\bar{r}, n)$ under $\bar{F}$, since $\hat{\varrho} \equiv \Sigma_{i=1}^{n}\rho(G, r_i)$ is independent of the distribution of r_i if we hold total resources constant. Thus, $E(G^*)$ remains the same whenever *all* agents contribute.

Next, look at the cases in which only *some* agents (less than n) contribute. Since the contributors are always the agents with the rightmost ρs, on average in those equilibria $\Sigma_{j=i}^{n}\rho(G, r_j)$ is larger than $\hat{\varrho}$. This is so because, when all agents contribute, $\hat{\varrho}$ is determined by the overall average resource, $\bar{r}$, while the average contributors' resource – and thus the position of the aggregate replacement function when fewer than n agents contribute – is higher, which results in larger G^*. In other words, in those cases the contributors' replacement functions are higher on average[13] and hence their sum ϱ crosses the 45° line at a point to the right of $\hat{G}(\bar{r}, n)$ on average. In addition, by the definition of m.p.s., for any constant $c < \bar{r}$ the prob($\bar{G}_1 < c$) grows as H becomes more unequal. Thus, the probability of all agents contributing in equilibrium (weakly) falls as resource inequality rises, and so more probability mass in total is shifted with cases with less than n contributors in equilibrium, each of which has higher total contribution on average compared with the case when all agents contribute. Thus, $E(G^*)$ increases as H becomes more unequal in the m.p.s. sense (see also the Cobb-Douglas example, below, for further intuition).

(b) The argument is similar to that in part (a); however, now we need to take care of the fact that, in general, ρ_i depends on v_i non-linearly and so $\Sigma_{i \in C_i}\rho_i(G, \bar{r})$ changes as the distribution of v_i changes, even if the set of contributors remains constant. By the properties of mean preserving spreads (e.g., Rothschild and Stiglitz 1970), we know that if ρ is convex in v, more inequality in the valuation distribution (keeping the mean constant) would *increase* the expected value of $\Sigma_{i=1}^{n}\rho_i(G, \bar{r})$ and hence the positive effect of heterogeneity on expected provision as in (a) would be reinforced. If, however, ρ is concave in v, then the decrease in $\Sigma_{i=1}^{n}\rho_i(G, \bar{r})$ as the valuations distribution becomes more unequal may be enough to offset the probability mass shift to the cases in which fewer agents contribute and expected total contributions could actually decline. ■

Proposition 4 states that, keeping the mean resource constant, increasing resource inequality always (weakly) increases expected total contributions to the public good. The same is true for valuation heterogeneity if the replacement functions are convex in v_i. Intuitively, under DRS, larger resource inequality increases the probability of equilibria in which not all agents contribute. As an agent becomes a non-contributor at the margin, total contributions cannot decrease (if they did, this would violate revealed preference for the agent's dropping out – remember ρ drops to zero continuously). Furthermore, in such equilibria total

13 Basically, this is saying that if one draws n numbers from a distribution with mean μ, then (for n large) the average of the draws is μ, while if one takes only the largest m draws ($m < n$), their average would be larger than μ.

contributions grow on average with inequality (if v is constant across agents or if ρ is convex), since the contributors are always the subset of agents with the highest resources and/or valuations (hence the rightmost ρ_i), which implies that the aggregate replacement function $\varrho = \Sigma_{i\in C}\rho_i$ (and therefore total contribution being its crossing with the 45° line) also grow on average as heterogeneity increases. Intuitively, the zero lower bound on the replacement function creates an asymmetric response to increases in resource inequality for poorer and richer agents. Once a poor agent does not contribute at the current total provision level, making him poorer on average by raising inequality still keeps his ρ at zero, while for the contributing richer agents with $\rho > 0$, higher inequality shifts their whole replacement function to the right and so raises their ρ (and thus the aggregate ϱ) for any given G that increases equilibrium provision.

3.3.1. A Cobb-Douglas example

The Cobb-Douglas utility example introduced earlier further clarifies the intuition behind proposition 4. Suppose, first, that all agents' valuations are the same, $U^i(x, P(G)) = \ln x_i + \bar{v} \ln G$. Then, as seen above, $\rho_i(G, r_i) = \rho(G, r_i) = \max\{0, r_i - (G/\bar{v})\}, \forall i$. For simplicity, assume also that r_i has a binomial distribution on the points $r_1 = \bar{r} - \sigma$ and $r_2 = \bar{r} + \sigma$, with probability 1/2 at each point (so the distribution mean is $\bar{r}$). Clearly, an increase in σ corresponds to moving to a more unequal distribution in a mean preserving spread sense.

Overall, there are $n + 1$ different outcomes that may happen, characterized by k draws at $\bar{r} - \sigma$ and $n - k$ draws at $\bar{r} + \sigma$, $k \in [0, n]$. These outcomes occur with fixed probabilities proportional to the binomial coefficients,

$$\pi_j = \binom{n}{j} \Big/ 2^n,$$

and so expected total contribution, $E(G^*)$, can be written as

$$E(G^*) = \sum_{j=0}^{n} \pi_j \hat{G}(n - j, \bar{r}, \sigma), \tag{6}$$

where $\hat{G}(m, \bar{r}, \sigma)$ is defined as the solution to $m\rho(G, \bar{r} + \sigma) + (n - m)\rho(G, \bar{r} - \sigma) = G$; that is, the equilibrium contribution in the case of m draws at $\bar{r} + \sigma$. Since ρ is *linear* in the agent's resource level, and since $\pi_j = \pi_{n-j}$, it is easy to see that as long as both $\rho(G, \bar{r} - \sigma)$ and $\rho(G, \bar{r} + \sigma)$ are positive, for any $j = 1, \ldots, n$, the jth and $n - j$th terms of the above sum add up to an amount independent of the resource inequality parameter,[14] σ. Consider, first, perfect equality, $\sigma = 0$. Then all agents contribute and total contribution is $\hat{G}(\bar{r})$ defined as the solution to $n\rho(G, \bar{r}) = G$. Now, begin increasing σ away from zero. Initially, nothing happens

14 If n is even, the $j = n/2$th term is independent of σ itself.

in terms of expected total contributions (all agents still contribute and, since ρ is linear in r_i, expected total provision stays the same). However, at some threshold $\hat{\sigma}$, the dropout value for the poorer agents (which decreases in σ from $\bar{G}(\bar{r})$), $\bar{G}(\bar{r}-\sigma)$ become such that $\bar{G}(\bar{r}-\sigma) = \hat{G}(n-1, \bar{r}, \sigma)$. For $\sigma > \hat{\sigma}$, in any draw with $n-1$ agents at $\bar{r}+\sigma$ there will be only $n-1$ contributors, since $\rho(G, \bar{r}-\sigma) = 0$, and so total contributions in this case solves $(n-1)\rho(G, \bar{r}+\sigma) = G$ and as such clearly *increases* in σ. Since all agents still contribute in the corresponding term with 1 contributor in (6), the sum $\pi_1\hat{G}(1, \bar{r}, \sigma) + \pi_{n-1}\hat{G}(n-1, \bar{r}, \sigma)$ also increases in σ for $\sigma > \hat{\sigma}$. All other cases still have all agents contributing, and so all other terms in (6) still add up pairwise to the same amount (independent of σ) as before. This implies that expected total contribution, $E(G^*)$, begins to grow in σ for $\sigma > \hat{\sigma}$. As σ increases further, there is another threshold value at which $\bar{G}(\bar{r}-\sigma) = \hat{G}(n-2, \bar{r}, \sigma)$ (note $\hat{G}(m, r, \sigma)$ is decreasing in m), after which expected total contribution in the case of $n-2$ agents at $\bar{r}+\sigma$ also starts to grow in σ and so on, until σ becomes so high that $\bar{r}-\sigma < \hat{G}(1, \bar{r}+\sigma)$ and n agents contribute (i.e., σ does not affect provision) only for the extreme cases of n draws at $\bar{r}-\sigma$ or at $\bar{r}+\sigma$. Now look at valuation heterogeneity as in proposition 4(b). Assume equal resources, $\bar{r}$ and suppose agents' valuations are binomially distributed on $v_1 = \bar{v}-\gamma$ and $v_2 = \bar{v}+\gamma$ with probability 1/2, where $\gamma \in [0, \bar{v})$. We then have $\rho_i(G, \bar{r}) = \max\{0, \bar{r} - (G/v_i)\}$, which, if positive, is concave in v_i. If we proceed as above, the sum of the jth and $n-j$th terms in (6), if both are positive, equals

$$n[\rho_1(\hat{G}, \bar{r}) + \rho_2(\hat{G}, \bar{r})] = 2\bar{r}n - 2\bar{v}n\hat{G}\left(\frac{1}{\bar{v}^2-\gamma^2}\right),$$

which is decreasing in γ. Thus, if γ is small enough that all agents contribute in any possible draw $\{v_i\}$, expected total contribution surely *decreases* in γ. However, for higher γ, as the low valuation agents drop out in some equilibria, total contribution solves

$$m\left(\bar{r} - \frac{G}{\bar{v}+\gamma}\right) = G$$

(if there are $m < n$ contributors), which is *increasing* in γ, and so the overall effect of inequality on expected provision is ambiguous and likely to be positive for high γ.

In general, it is evident from the analysis that, if both valuations and resources are heterogeneous, the impact of heterogeneity on expected total provision would depend on how the set of contributors changes as we vary the properties of $\Phi(r, v)$. The outcome would depend on how the ordering and spacing of the standalone values $\bar{G}(r_i, v_i)$ changes as heterogeneity increases. For instance, the intuition from proposition 4 suggests that, holding valuations constant, higher resource inequality is likely to increase total contribution via the asymmetric response effect in the replacement functions, but, if valuations vary as well, the shape of the function ρ in v also plays an important role. Further, when we compare across groups with the same valuations but different resource distributions, expected provision would be lower in groups with $\Phi(r, v)$ distributions that assign fewer resources to high valuation agents.

The following result addresses some of these issues by looking at the effects of the correlation between valuations and resources in the joint distribution Φ.

PROPOSITION 5 (correlation between resources and valuations). *For any given draw of resources and valuations $\{r_i\}$ and $\{v_i\}$, $i = 1, \ldots n$, total contributions are highest when $\{r_i\}$ and $\{v_i\}$ are perfectly positively correlated and lowest when $\{r_i\}$ and $\{v_i\}$ are perfectly negatively correlated.*

Proof. See the appendix.[15]

Perfectly negative correlation between valuations and resources hampers public good provision, while the opposite is true for positive correlation. The reason is the asymmetric effect in the agents' replacement function discussed after proposition 4. Since it holds for each draw, the proposition result holds on average as well, so if we hold the distributional means constant, expected provision will be largest if the agents' resources and valuations are perfectly positively correlated in the joint distribution Φ and smallest if they are perfectly negatively correlated.

4. Collective action with increasing returns

Suppose, now, that the public good production function, $P(G)$ is *convex* (at least for the studied resource range), which corresponds to *increasing returns to scale* (IRS) in the collective action. Public goods with IRS are typically mass actions, the effect of which increases with the number of participants and which exhibit positive interdependence (complementarity) among individual contributions; that is, each contribution makes the next one more worthwhile (Marwell and Oliver 1993). All other examples are discrete collective action problems (e.g., Andreoni 1998, among others), in which the quantity provided is zero up to some total contribution and then jumps discontinuously, for example, building a bridge.

For tractability, assume that agents' preferences are separable:

$$U^i(x_i, Z) = u(x_i) + v_i w(P(G)), \tag{7}$$

with u increasing and (weakly) concave.

To make the increasing returns assumption have full bite, assume that $u(0)$ is finite and the public good production function $P(G)$ is sufficiently convex that

15 The full details of this and all subsequent proofs are available in an on-line appendix linked to this article in the CJE archive: http://economics.ca/cje/en/archive.php.

$w(P(.))$ is convex. Then, without loss of generality, we can write the second term simply as $v_i P(G)$. In addition, assume that $P(G)$ is sufficiently convex that the overall function $U^i(r_i - g_i, P(G))$ is convex in g_i.[16] An example satisfying these properties is quasi-linear utility linear in x_i.

4.1. Equilibrium

Normalize $P(0) = 0$ and let $S^i \equiv [0, r_i]$ be the set of contribution strategies, g_i, of agent i. I am looking for Nash equilibria, that is, strategy profiles $s = (g_1, \ldots, g_n)$ such that, $\forall i = 1, \ldots, n$, $U^i(g_i, P(\Sigma\, g_{-i} + g_i)) \geq U^i(g_i', P(\Sigma\, g_{-i} + g_i'))$ for all $g_i' \in S^i$. By the assumed sufficient convexity of the production function, U^i is convex and attains maximum at a corner solution: either $g_i = r_i$ or $g_i = 0$. I can thus concentrate on the much simpler game in which the strategy set for each player is simply $S^i \equiv \{0, r_i\}$. That is, the agent either does not contribute or gives her total resource. Alternatively, if one feel uneasy about the zero corner solution for the private good, one can directly assume that only two possible contribution levels are possible: 0 and r_i – 'not participate' or 'participate' in the collective action and interpret r_i as the efficiency units (e.g., time or effort) contributed to the collective action.

The agent's optimal contribution is therefore (normalizing $u(0) = 0$):

$$g_i^* = \begin{cases} r_i & \text{if } v_i \geq \dfrac{u(r_i)}{P(r_i + G_{-i}) - P(G_{-i})} \\ 0 & \text{otherwise.} \end{cases} \tag{8}$$

The inequality for v_i in (8), equivalent to $U^i(r_i, P(G_{-i} + r_i)) > U^i(0, P(G_{-i}))$, defines a cut-off level above which an agent with resource r_i will switch from non-contributing to contributing, given the others' total contributions, G_{-i}.

LEMMA 2. *The agents' contributions are (weakly) increasing in their resources, r_i, and valuations, v_i, for any given G_{-i} and (weakly) increasing in G_{-i} for any given r_i, v_i.*

Proof. See the appendix.

By the properties of u and P, an agent is more likely to contribute or contributes more if her valuation of the public good, v_i is high and her available resource amount r_i is high. In addition, when P is convex, g_i^* also increases (i.e., can jump from 0 to r_i) in the contribution of all others, G_{-i}. This captures the strategic complementarity between individual contributions in contrast to the 'canonical' case with $P(G) = G$ and concave w, for example, as in BBV, where the individual contribution decreases in G_{-i}.

16 A sufficient condition is u'' to be small enough, that is, the convexity of P to offset the concavity of u.

I proceed to characterize the Nash equilibria of the IRS contribution game. I concentrate only on pure strategy Nash equilibria (PSNE). Because of the corner solution optimal contribution strategies, one cannot directly use the classical apparatus of BBV or the 'replacement function' approach[17] from section 3 to show the existence of Nash equilibrium. Therefore, I provide an existence proof.

PROPOSITION 6 (equilibrium existence). *For any positive valuations and resources, $\{v_i\}_{i=1}^n$ and $\{r_i\}_{i=1}^n$, there exists at least one pure strategy Nash equilibrium (PSNE) of the IRS contribution game.*

Proof. See the appendix.

Proposition 6 (see also the proof in the on-line appendix) shows the possibility that multiple Nash equilibria exist for the same agent characteristics. In particular, it is possible to have both 'all agents contribute' and 'no agent contributes' as PSNE for the same set of $\{r_i, v_i\}$ as in a 'coordination failure' game (e.g. Palfrey and Rosenthal 1984). This is perhaps easiest to see in the two-player case. For example, let $U^i = x_i + v_i P(G)$, $i = 1, 2$ with $v_1 = v_2 = v$ and $r_2 > r_1$. Suppose

$$\frac{r_2}{P(r_2 + r_1) - P(r_1)} < v < \frac{r_2}{P(r_2)}.$$

Then, using (8), it is easy to verify that both (0, 0) and (r_1, r_2) are Nash equilibria; if neither agent has a high enough valuation level to be the sole provider, he contributes only if the other player does.

4.2. Distributional effects on collective action

When there exist multiple equilibria, it is necessary to discuss equilibrium selection. In particular, to be able to compare expected provision levels for different resource and valuations distributions or for different group sizes, as I do below, it is crucial to adopt and hold constant a rule of how to select among the multiple equilibria. One possibility could be to assume that whenever 'no agent contributes' is a Nash equilibrium (potentially among others), it is played with probability one (coordination failure). Alternatively, one can assume that if 'all agents contribute' is a Nash equilibrium (potentially among others), it is played with probability one. From now on, whenever comparing expected provision across different characteristics distributions or different group sizes, I assume that the equilibrium selection criterion is always held constant.

4.2.1. Group size and first-order distributional effects

I first characterize the effect of group size and the first-order (level) effects of the resource and valuations distribution, holding constant the equilibrium selection rule adopted.

17 See, however, Cornes and Hartley (2007b) for an example with convex public good technology, which requires redefining the replacement function as a correspondence.

PROPOSITION 7 (group size and f.o.s.d.)

a) *The expected total contribution,* $E_\Phi(G^*)$ *is (weakly) increasing in the number of agents, n, holding constant the criterion to select among multiple equilibria.*

b) *Let* $F(r, v)$ *and* $H(r, v)$ *be two probability distributions with the same support, such that F first-order stochastically dominates H. If we hold constant the criterion to select among multiple equilibria, the expected total contribution computed over all possible draws* $\{r_i, v_i\}_{i=1}^n$ *from F is higher than the expected total contribution over all possible draws* $\{r_i, v_i\}_{i=1}^n$ *from H.*

Proof. See the appendix.

The basic intuition from the DRS case still holds, since what matters is the fact that contributions cannot be negative and that individual contributions are (weakly) increasing in both r and v. However, there are also some observations specific to the IRS case. First, note that the maximum contribution level is $\Sigma_{i=1}^n r_i$, which is also the first-best level if the sum of the agents' valuations is large enough. Thus, if we hold mean resources constant, larger group size makes it more likely that the first-best provision level will be achieved. The group size result in part (a) is also consistent with the observation that collective action events with increasing returns (e.g., revolutions, strikes) are likely to be more successful when the number of participants is high.

4.2.2. Second-order distributional effects: the role of heterogeneity

A detailed characterization of all possible equilibria for given agents' characteristics $\{r_i\}_{i=1}^n$ and $\{v_i\}_{i=1}^n$ and the effects of valuation or resource heterogeneity on expected public good provision is possible if we assume, in addition to assumption A1,

ASSUMPTION A2. *The functions P and u are such that, for any a, b with* $0 \le a < b$, *the function* $u(b - a)/(P(b) - P(a))$ *is decreasing in* $b \in (a, \infty)$, *given a, and decreasing in* $a \in [0, b)$, *given b.*

It is easy to verify that A2 holds for any convex function P and linear u (i.e., when U^i is quasi-linear).[18] More generally, A2 requires that P is sufficiently convex relative to u (alternatively, that u is not too concave relative to P).

Without loss of generality, index the agents in increasing order of their valuations, $v_1 \le v_2 \le \cdots \le v_n$. We have the following lemma.

LEMMA 3. *Under assumptions A1 and A2, all pure strategy Nash equilibria of the IRS contribution game can be characterized as follows: agents* $i = 1, \ldots, n - m$, $0 \le m \le n$ *with the lowest valuations do not contribute* ($g_i^* = 0$), *while the rest of the agents contribute* ($g_i^* = r_i$).

18 Basically, the property restates the fact that the steepness of $P(x)$ increases with x (sketching a graph helps).

Proof. See the appendix.

Lemma 2 implies that all Nash equilibria of the IRS contribution game share the property that the set of contributors consists of the agents with the m highest valuation levels. This can be seen as a manifestation of Olson's (1965) famous 'exploitation of the great by the small' result: the agents who value the provision of the public good free-ride less on those who value it more. The lemma also implies that total contribution is determined by the resource endowments of the highest valuations agents. For example, suppose we compare groups with different $\{r_i, v_i\}$ distributions. Then, the lemma tells us that a group in which the highest valuation agents have few resources would achieve lower aggregate contribution on average compared with another group for which the opposite is true. I explore this further later on.

4.2.3. Equal valuations

To focus solely on the effects of resource inequality on expected provision, start with the case of all agents having equal valuations, $v_i = \bar{v}, \forall i$. The proofs of proposition 6 and lemma 2 go through, since they use weak inequalities. I analyze the heterogeneous valuations case in the next section.

For tractability, order the agents in increasing order of their resources – that is, $r_1 \leq r_2 \leq \cdots \leq r_n = r_{\max}$, where $r_{\max}$ denotes the maximum resource level and let $R \equiv \Sigma_{i=1}^n r_i$.

LEMMA 4. *The pure strategy Nash equilibria (PSNE) of the IRS contribution game when all agents have equal valuations are as follows:*
(i) 'no agent contributes' ($g_i^* = 0, \forall i$) *is the unique PSNE if*

$$\bar{v} < \frac{u(r_{max})}{P(R) - P(R - r_{max})};$$

(ii) 'all agents contribute' ($g_i^* = r_i, \forall i$) *is the unique PSNE if* $\bar{v} \geq u(r_{max})/P(r_{max})$*; and*
(iii) both 'no agent contributes' and 'all agents contribute' are PSNE if

$$\bar{v} \in \left[\frac{u(r_{max})}{P(R) - P(R - r_{max})}, \frac{u(r_{max})}{P(r_{max})}\right).$$

Proof. See the appendix.

Note two important differences between lemmata 4 and 3. First, the intermediate case of m ($0 < m < n$) agents contributing is no longer possible if agents' valuations are identical. Second, with equal valuations, the resource endowment of the richest agent, $r_{\max}$, affects to a large extent the equilibrium contribution level, which is either $G^* = 0$ or $G^* = R$. Let $\Phi_r(r)$ denote the marginal distribution of

r, given, $\bar{v}$. For simplicity, assume that Φ_r is continuous[19] and has mean, $\bar{r}$, and a finite variance, σ. I am interested in how varying the mean and the variance of the resource distribution affects the expected equilibrium supply of the public good.

Because of the possibility of multiple equilibria in lemma 4(iii), I need to specify an equilibrium selection criterion to be able to compare expected total provision across different resource distributions. I consider two possible such criteria: (L) 'coordination failure'; whenever both $G = R$ and $G = 0$ are PSNE, pick the equilibrium in which *no agent* contributes and (H); whenever both equilibria are possible, pick the equilibrium in which *all agents* contribute. Clearly, any other selection criterion would produce expected contributions between those two levels.

Lemma 4 implies that, under equilibrium selection criterion L, $G^* = 0$ if $\bar{v} < u(r_{\max})/P(r_{\max})$ and $G^* = R$ otherwise. Define $\phi(r) \equiv u(r)/P(r)$ for $r \in (0, R]$. It is a continuous, monotonically decreasing function and hence invertible with an inverse ϕ^{-1}, a monotonically decreasing function of v. Thus, if we hold $\bar{v}$, $\bar{r}$, and n constant and use $E(R) = n\bar{r}$, the expected total contribution overdraws $\{r_i\}$ from Φ_r under criterion L is

$$E^L(G^*) = [1 - prob(r_{\max} < \phi^{-1}(\bar{v}))]n\bar{r}. \tag{9}$$

Similarly, call

$$\psi(r) \equiv \frac{u(r)}{P(R) - P(R-r)},$$

defined for $r \in (0, R]$. The convexity of P implies $\psi(r) \le \phi(r)$, with equality only for $r = R$. By the assumptions on u and P (see A2), ψ is a continuous increasing function in r; hence its inverse ψ^{-1} exists and is increasing in v on its domain. Then, by lemma 4, holding $\bar{v}$, $\bar{r}$ and n constant, the expected contribution overdraws $\{r_i\}$ from Φ_r under criterion H is

$$E^H(G^*) = [\text{prob}\,(r_{\max} \le \psi^{-1}(\bar{v}))]n\bar{r}. \tag{10}$$

Further, $\psi^{-1}(\bar{v})$ increases in $\bar{v}$ from 0 to R on the interval[20] $(\psi(0), \psi(R)]$, and $\phi^{-1}(\bar{v})$ decreases in $\bar{v}$ from R to 0 on $[\phi(R), \infty)$ with $\psi(R) = \phi(R)$. If $\bar{v} < \psi(0)$, we have $G^* = 0$ for any draw $\{r_i\}$ from Φ_r under both criteria, H and L, so focus on the case $\bar{v} \ge \psi(0)$.

19 This assumption is solely for tractability reasons; most results below hold for a discrete distribution as well.

20 By A2 $\psi(0)$ is finite. For example, if u is linear, we have by L'Hopital's rule that $\psi(0) = 1/P'(R)$. In general, a sufficient condition is $u'(0)$ to be finite; that is, u linear, $u(x) = \log(x + b)$ for some $b > 0$, and so on.

PROPOSITION 8 (resource inequality effects with equal valuations).

a) *Under criterion L, the effect of resource inequality, σ, on expected total contribution, $E^L(G^*)$, depends on the relative magnitudes of the mean resource, $\bar{r}$, and the valuation, $\bar{v}$ as follows: (i) if $\bar{v} < \phi(\bar{r})$ (low valuation), $E^L(G^*)$ (weakly) increases in the resource variance; (ii) if $\bar{v} \geq \phi(\bar{r})$ (high valuation), $E^L(G^*)$ (weakly) decreases in the resource variance.*
b) *Under criterion H, the effect of resource inequality, σ on expected total contribution, $E^H(G^*)$, depends on the relative magnitudes of the mean resource, $\bar{r}$, and the valuation, $\bar{v}$, as follows: (i) if $\bar{v} < \psi(\bar{r})$, $E^H(G^*)$ (weakly) increases in the resource variance; (ii) if $\psi(\bar{r}) \leq \bar{v}$, $E^H(G^*)$ (weakly) decreases in the resource variance.*
c) *Expected total and per capita contribution, holding any of the two equilibrium selection criteria fixed, are (weakly) increasing in the mean resource level, $\bar{r}$, and the valuation, $\bar{v}$.*

Proof. See the appendix.

The resource inequality effects in parts (a)–(b) depend crucially on the mean resource level and the agents' valuation. For both equilibrium selection criteria, even though the exact valuation cutoffs are different, the effect of resource inequality on expected provision (holding mean resource $\bar{r}$ constant) changes as $\bar{v}$ increases from a (weakly) positive effect to a (weakly) negative effect. Intuitively, when the agents' resource levels are low and similar (alternatively, fixing $\bar{r}$, when $\bar{v}$ is relatively low), the only possible equilibrium is that in which none of them contributes. In this case sufficiently high resource inequality is the only way to ensure non-zero provision, since then the richest agent(s) would contribute unconditionally, even if the others free-ride. Conversely, if the mean resource level is high relative to $\bar{v}$ (or, fixing $\bar{r}$, when valuations are relatively high), the unique equilibrium that would obtain if resources are equal is 'all contribute.' In this case, high resource inequality is likely to decrease total provision, since it destroys the 'all contribute' equilibrium if $r_{\max}$ is low enough for some draws. In other words, for high valuation, groups that are relatively homogeneous in resources exhibit higher collective action provisions compared with more unequal groups. The opposite is true for relatively low valuation.

Note, also (see the proposition proof), that there exists a minimum valuation level ($\phi(R)$ under L and $\psi(0)$ under H) necessary to generate positive total contribution (a threshold effect). This is due to the complementarity between agents' optimal contribution strategies.

Finally, from the proof of proposition 8, since $1 - prob(r_{\max} < \phi^{-1}(\bar{v}))$ is increasing in n,[21] it follows that, under criterion L, not only expected total contributions (see proposition 7) but also expected per capita contributions, $E^L(G^*)/n$

21 This holds, since, by the properties of order statistics, $1 - \text{prob}\,(r_{\max} < \phi^{-1}(\bar{v})) = 1 - \Phi_r(\phi^{-1}(\bar{v}))^n$ (see, e.g., David and Nagaraja 2003).

are increasing in the group size. Thus, the relative extent of free-riding measured by the ratio of the first-best contribution level (which, if $\Sigma\ v_i$ is large enough, equals $n\bar{r}$) to the expected Nash total contribution level diminishes as group size grows. Intuitively, with IRS, higher G increases the marginal utility of the public good for all agents, making them contribute more. This result stands in contrast to the 'standard' intuition that free-riding is stronger in larger groups (e.g., Olson 1965; Andreoni 1988). Olson's and Andreoni's conclusions thus rely on assuming non-increasing returns in the public good production and, as such, are inapplicable[22] to collective action problems with convex production such as those studied here.

4.2.4. Heterogeneous valuations

Now, to isolate the effect of heterogeneity in valuations on expected total contributions, take the resource levels of the agents as given. Without loss of generality, order the agents by their valuation, $v_1 \leq v_2 \leq \cdots \leq v_n$. Denote $R = \Sigma_{i=1}^n r_i$ as before. Recall from proposition 6 that, with heterogeneous valuations, it is possible to have equilibria with contribution levels, 0, R (as before), but also between 0 and R, as well as multiple equilibria. Because of the complications that arise in characterizing all cases and equilibrium types, one can derive analytically only an upper and a lower bound on expected provision, as opposed to solving for its exact value, as is possible with equal valuations. The upper bound, $E^{ub}(G^*)$, is obtained by assuming that total contribution equals R whenever $G = 0$ is *not* the unique equilibrium, given $\{v_i\}_{i=1}^n$. The lower bound, $E^{lb}(G^*)$ is computed by assuming that total contribution is 0 whenever $G = R$ is not the unique equilibrium. The bounds are precise and achieved with equality, for example, if $v_i = \bar{v}\ \forall i$ and using respectively criterion L or criterion H from section 4.2.3.

LEMMA 5

a) *Given $\{r_i\}_{i=1}^n$, a sufficient condition for 'no agent contributes' ($G^* = 0$) to be the unique PSNE of the IRS contribution game is*

$$v_{\max} < \frac{u(r_{\min})}{P(R) - P(R - r_{\min})}.$$

b) *Given $\{r_i\}_{i=1}^n$, a sufficient condition for 'all agents contribute' ($G^* = R$) to be the unique PSNE of the IRS contribution game is $v_{\min} \geq u(r^{\min})/P(r^{\min})$, where $r^{\min} = \min\{r_1, r_2, \ldots, r_n\}$.*

22 In fact, the earlier results imply that with increasing returns there can exist a minimum group size below which contribution is zero, so it is small, not large, groups that are likely to be plagued by zero provision or high extent of free-riding.

Proof. See the appendix.

To derive the bounds on expected contributions we thus need to compute prob($v_{\min} \geq \phi(r_{\min})$) and prob($v_{\max} \geq \psi(r_{\min})$), where the functions ϕ and ψ are defined as in section 4.2.3. The special case of equal resources is easily subsumed by setting $r_{\min} = \bar{r}$ in all expressions. As before, let $\Phi_v(v)$ denote the marginal distribution of v, and assume for simplicity that Φ_v is continuous and has mean $\bar{v}$ and a finite variance γ.

PROPOSITION 9 (valuation heterogeneity). *For any given resource levels,* $\{r_i\}_{i=1}^{n}$ *we have the following.*

a) *A lower bound on expected total equilibrium contribution is*

$$E^{lb}(G^*) = prob\ (v_{min} \geq \phi(r_{min}))R = [1 - \Phi_v(\phi(r_{min}))]^n R. \tag{11}$$

The lower bound, $E^{lb}(G^*)$ *is increasing in the total resource amount R and decreasing in the number of agents, n. For relatively low mean valuation levels,* $\bar{v}\phi(r_{min})$*, the lower bound is (weakly) increasing in valuation heterogeneity,* γ*. For relatively high mean valuation,* $\bar{v} \geq \phi(r_{min})$*,* $E^{lb}(G^*)$ *is (weakly) decreasing in* γ*.*

b) *An upper bound on the expected total contribution is*

$$E^{ub}(G^*) = \ prob(v_{max} \geq \psi(r_{min}))R = [1 - (\Phi_v(\psi(r_{min})))^n]R. \tag{12}$$

The upper bound, $E^{ub}(G^*)$ *is increasing in the total resource amount, R and the number of agents, n. For relatively low mean valuation,* $\bar{v} < \psi(r_{min})$*, the upper bound is (weakly) increasing in valuation heterogeneity,* γ*. For relatively high mean valuation,* $\bar{v} \geq \psi(r_{\min})$*,* $E^{ub}(G^*)$ *is (weakly) decreasing in* γ*.*

Proof. See the appendix.

Since $\phi(r) \geq \psi(r)$ for any r, with equality only at R, the upper bound above is always larger than the lower bound. Both bounds increase in the mean valuation level, $\bar{v}$, suggesting that on average expected total contribution increases as well. The results regarding valuation heterogeneity parallel those obtained for resource inequality in proposition 8. Both bounds, and hence, on average, expected total contributions, increase in the degree of valuation heterogeneity for low mean valuation levels, $\bar{v} < \psi(r_{\min})$, but decrease in γ at higher mean valuation levels, $\bar{v} \geq \phi(r_{\min})$. The intuition is the same as before. The reason why the lower and upper bounds diverge as n grows is that the number of intermediate equilibria increases when more agents participate in the contribution game.[23]

23 This was verified using numerical simulations in the working paper version of this paper. The results are available from the author upon request.

Now, take agents' valuations, $\{v_i\}_{i=1}^n$, as given and look at the effect of changes in the resource distribution on expected total contribution. Define a lower bound on expected provision as in proposition 9. By lemma 5 and the properties of the function ϕ defined earlier, a sufficient condition for 'all agents contribute' to be the unique equilibrium is $r_{\min} \geq \phi^{-1}(v_{\min})$, while a sufficient condition for 'no agent contributes' to be the unique equilibrium is $r_{\min} > \psi^{-1}(v_{\max})$. However, in contrast to proposition 9, since the function ψ depends on $R = \Sigma_{i=1}^n r_i$ in a non-linear way, it is hard to establish whether the latter condition is more likely to be satisfied for low or high mean resource, $\bar{r}$.[24]

PROPOSITION 10 (resource inequality effects with heterogeneous valuations). *For any given valuation levels, $\{v_i\}_{i=1}^n$ we have the following.*

a) *A lower bound on expected total equilibrium contribution is*

$$E^{lb}(G^*) = prob\big(r_{min} \geq \phi^{-1}(v_{min})\big)n\bar{r} = [1 - \Phi_r(\phi^{-1}(v_{min}))]^n n\bar{r}.$$

For relatively low mean resource level, $\bar{r} < \phi^{-1}(v_{min})$. the lower bound is (weakly) increasing in resource inequality σ. For high mean resource, $\bar{r} \geq \phi^{-1}(v_{min})$, $E^{lb}(G^*)$ is (weakly) decreasing in σ.

b) *An upper bound on the expected total contribution is given by*

$$E^{ub}(G^*) = prob\ \big(r_{min} \leq \psi^{-1}(v_{max})\big)n\bar{r} = \big[1 - \big(1 - \Phi_r(\psi^{-1}(v_{max})\big)^n\big]n\bar{r}.$$

If $\bar{r} \leq \psi^{-1}(v_{max})$, the upper bound, $E^{ub}(G^*)$ is weakly decreasing in resource inequality σ, while, if $\bar{r} > \psi^{-1}(v_{max})$, the upper bound is weakly increasing in σ.

Proof. Omitted: analogous to proposition 9.

Proposition 10 confirms the findings from the equal valuations case: for low mean resource levels, high inequality likely increases total contribution (by raising the lower bound); for high mean resource levels, high inequality likely reduces expected contribution.

As in the DRS section, I finish the analysis of the effects of heterogeneity on expected provision by looking at the impact of the correlation between valuations and resources in the joint distribution Φ.

PROPOSITION 11 (correlation between resources and valuations). *For any given draw of resources and valuations $\{r_i\}$, $\{v_i\}_{i=1}^n$ from $\Phi(r, v)$, holding the criterion to select among multiple equilibria constant, total contribution is highest when $\{r_i\}$ and*

24 For example, if $u(x) = x$ and $P(x) = x^2$, we have $\psi^{-1}(v) = 2R - (1/v)$, and so $\bar{r} \leq \psi^{-1}(v_{\max})$ is likely to hold for $\bar{r}$ sufficiently high relative to $1/v_{\max}$ consistent with part (a).

$\{v_i\}$ are perfectly positively correlated and lowest when $\{r_i\}$ and $\{v_i\}$ are perfectly negatively correlated.

Proof. See the appendix.

The intuition is that it is more probable that an agent will contribute if he has both a high valuation and a high resource level (see lemma 2). Furthermore, since the highest valuation agents contribute all their resources under IRS, the higher these resources are, the higher the total contributed amount will be. Under perfectly negative correlation, the highest valuation agents have low resource levels, which reduces provision. Finally, since it holds for each draw, the proposition result holds on average as well. Thus, holding the distributional means and equilibrium selection rule constant, expected total contribution would be maximized if the agents' resources and valuations are perfectly positively correlated.

4.3. Discussion

A direct comparison of the results in sections 3 and 4 emphasizes the importance of the shape of the collective action production function for the expected total contribution.

1. In the decreasing returns case, higher resource inequality always (weakly) increases expected total contributions, while this is true only for low mean resources or valuations in the increasing returns case. The reason for this important difference is that, with DRS, if all agents have equal resources, they all contribute no matter what the average resource or valuation levels are and total contributions are at its minimum possible level. Higher heterogeneity then increases provision by the asymmetry in the replacement functions effect explained above. With IRS, if average resources are low and all agents are equal, 'no agent contributes' is the unique equilibrium, and so heterogeneity may increase provision by making some agents contribute. However, if mean resources are high and agents are equal, all agents contribute their resources and total contribution is at its maximum possible level, R. Starting from that situation, inequality decreases provision by making some agents (those with low resources or valuations) contribute zero, while the remaining contributors cannot increase their contributions further, in contrast to the DRS case.
2. In the decreasing returns case we generically have a unique Nash equilibrium with $g_i^* < r_i \; \forall i$, while multiple equilibria are possible with increasing returns.
3. In contrast to IRS, total contribution under decreasing returns is always positive: zero provision never occurs. This is because agents' marginal utility from the initial units of the public good is always higher than their marginal cost by the Inada conditions on P.
4. In both cases the set of contributors consists of the high-valuation/high-resource agents. The number of contributors in equilibrium depends crucially on the agents' resources and valuations and how they affect the agent's replacement functions or optimal contributions.

The intuition underlying these results is based mainly on the fact that with DRS the marginal utility of consuming the public good decreases as more of it is provided, while the opposite is true with IRS. Thus, with DRS, if the public good is already being supplied at a sufficiently high level by some agents, no other agents would be willing to contribute, in contrast to the increasing returns case in which the agents' marginal utility from the public good is increasing in total contributions, G.

The results of propositions 8–10 are consistent with the empirical findings of Chan et al. (1999), who perform a laboratory experiment to study the effects of income and preference heterogeneity on public good provision in a similar model of non-decreasing returns to scale in the public good production (they assume $U^i = x_i + v_i G + x_i G$). In a non-cooperative contribution setting with $n = 3$, they find that larger heterogeneity (especially in both agents' characteristics simultaneously) leads to higher total contributions. As seen, this is a prediction of my model for relatively low average valuation or resource levels. Unfortunately, Chan et al. did their experiments with constant means, so their results do not explore the full empirical content of the theory in this paper.

5. Conclusions

I studied the problem of collective action in an economy with agents heterogeneous in their preferences and resource endowments. Unlike most of the previous literature, the main assumption here is that the researcher does not know the actual realizations of the agents' characteristics but knows only that they are drawn from some given distribution. Thus, I do not analyze equilibria corresponding to particular allocations, transfers, or redistributions but instead the *expected,* or average, provision level as a function of various distributional moments, the size of the group, and the production function for the public good (increasing or decreasing returns to scale).

The main conclusion is that agent heterogeneity has different effects on total provision, depending crucially on the mean levels of valuation and resources, as well as the correlation between the two. On the one hand, inequality can alleviate the free-riding problem, since it introduces the possibility for agents with high valuation for the public good or high endowments to contribute, regardless of the action of the others. This effect dominates in the decreasing-returns case and also in the increasing-returns case for relatively low mean resource or valuation levels. In contrast, for higher mean resources or valuations, heterogeneity can have a negative effect on total provision with IRS, since it may lead to non-contribution by some agents who would contribute under homogeneity. Group size has a positive effect on the expected quantity of the public good provided, although the relative extent of free-riding measured by the ratio of the first best to the Nash equilibrium contribution level may increase or decrease in the number of agents, depending on the public good production function.

Several empirically (or experimentally) testable implications follow from the results. First, resource inequality may imply both higher and lower levels of collective action compared with a more equal distribution depending on the returns to scale in public good production. In the IRS case inequality is generally beneficial for total provision levels at low mean resource levels, while at high mean resource levels it can be detrimental. In the DRS case resource inequality always (weakly) raises expected provision. Second, when we compare two groups with the same mean resource and valuation levels, the group whose most resource endowed members are also the most interested in the collective action will achieve higher provision level compared with a group where the opposite is the case. Third, collective action characterized by increasing returns (strikes, revolutions, etc.) is more likely to be provided at a larger scale by larger groups and might not occur at all if the group size is too low. Finally, if it is possible to elicit people's valuation for a public good, we should expect that under decreasing returns in production, the contributors are the people with highest valuations.

An important implication of the increasing returns case results is that redistribution policies[25] (e.g., affecting the resource variance σ) might have different effects on public good provision depending on the agents' mean resource or valuation, their variances, or the correlation between resources and valuations. For example, reducing the resource variance/inequality (holding the mean constant) would have opposite effects on expected total contributions in groups with high vs. low mean resources, or in groups with high vs. low average valuations. Thus, implementing blanket policies across population groups differing in their distributional characteristics might lead to unintended effects, as the relationship between inequality and collective action can be ambiguous.[26]

The analysis above also shows that, for a given number of contributors, depending on agents' resources and valuations, there can be equilibria with high or low total contribution levels. The highest equilibrium contribution occurs when the contributors have both high valuations and resources that are positively correlated. This observation implies that, if it is possible to select among all potential contributors those with most favourable characteristics, one can achieve higher total contribution levels per given number of people. Indeed, selection of potential contributors is a widespread practice among foundations, charities, universities, and other organizations that raise voluntary contributions or are involved in collective action in general.[27]

25 By this I mean one-shot redistributions (e.g., land reform), after which the agents play the contribution game with their new resources.

26 Bardhan, Ghatak, and Karaivanov (2007) reach the same conclusion in their study of the resource distributions that maximize total contribution or social surplus. See also Olszewski and Rosenthal (2004), who use a political economy approach to analyze Pareto-improving redistributions in the voluntary provision model.

27 For example, universities keep records of their alumni and target potential big donors; insurgency leaders carefully select their associates based on their belief in the cause and resources.

Various extensions are possible. It would be interesting to relax the complete information assumption among the agents and analyze the distributional effects on expected provision in an asymmetric information setting.[28] It may be also useful to consider different assumptions about the agents' preferences and the public good technology, for instance, as in Cornes and Hartley (2007b). Another important issue is contributors' selection, where a more detailed analysis should provide further insights on the effects of heterogeneity on collective action.

Appendix

This appendix contains the proofs of propositions 5–11 and lemmata 2–5 from 'Heterogeneity, Returns to Scale, and Collective Action.' The equation numbers refer to the main text.

Proof of proposition 5. Relabel the drawn resources and valuations so that $v^1 \leq v^2 \leq \cdots \leq v^n$ and $r^1 \leq r^2 \leq \cdots \leq r^n$ (the superscript j thus refers to the $j-th$ smallest r or v level, not the agent). Suppose that $\{r_i\}$ and $\{v_i\}$ are not positively assortatively matched and look at the following two agents: agent A with (v^n, r^i) and agent B with (v^j, r^n) for some $i, j \neq n$. I will show that total contribution (weakly) increases if we switch r^i with r^n, i.e. match v^n with r^n for agent A.

By lemma 1 and proposition 1, agent A always contributes after the switch (since she has the highest $\bar{G}$). There are several cases to consider. If both agents were not contributing before the switch, the fact that A contributes ex-post implies that the aggregate replacement function would shift up (the other agents' replacement functions remain constant) and hence G^* increases. If only one of A and B was contributing before then, since ρ is increasing in r, v, after the switch A would have a replacement function above any of A's or B's before the switch and total contribution would go up again. The only remaining case is when both A and B were contributing ex-ante. If both of them still contribute ex-post, total contribution remains unchanged by the distribution neutrality theorem.

Thus, suppose B does not contribute ex-post and suppose (even though A does) total provision drops after the switch. I will show that this leads to contradiction. Denote by G_e^* the ex-ante (before the switch) contribution when both A and B contribute. By definition, G_e^* solves

$$r^n - \xi_j^{-1}(G) + G + r^i - \xi_n^{-1}(G) + G + \eta(G) = G \tag{A1}$$

28 In the working paper version of this paper I show examples of Bayesian Nash contribution equilibria with IRS technology in which it is possible that complete information among the agents is harmful (i.e., prevents achieving the first-best provision) or beneficial (i.e., increases total provision compared with under incomplete information).

where η (G) denotes the sum of all other agents replacement functions and ξ_k^{-1} applies to the agent with valuation v^k. Ex-post, after the switch only A contributes and total contribution, G_p^* solves:

$$r^n - \xi_n^{-1}(G) + \eta(G) = 0. \tag{A2}$$

Since B does not contribute ex-post, it must be that, $r^i - \xi_j^{-1}(G_p^*) + G_p^* < 0$ (that is, $\rho^B = 0$). If $G_e^* > G_p^*$ (total provision decreases after the switch), then the properties of ξ^{-1} imply that $r^i - \xi_j^{-1}(G_e^*) + G_e^* < 0$ as well. But then, rearranging (A1), evaluated at G_e^* we obtain:

$$r^n - \xi_n^{-1}\big(G_e^*\big) + \eta(G_e^*) = -\big(r^i - \xi_j^{-1}(G_e^*) + G_e^*\big) > 0,$$

which contradicts (A2) since its l.h.s. is decreasing in G and so must be negative for any $G > G_p^*$ such as G_e^*.

In the same way one can show that total contribution cannot decrease when (given the match r^n, v^n) we also match v^{n-1} with r^{n-1} and so on. Thus, maximum contribution is achieved under positive assortative matching. The reverse argument applies for perfectly negative correlation between resources and valuations. ■

Proof of lemma 2. If the agent contributes, r_i increases g_i^* directly. Further, $u(r_i)/P(r_i + G_{-i}) - P(G_{-i})$ is decreasing in r_i since its first derivative being negative is equivalent to

$$\frac{u'(r_i)r_i}{u(r_i)} \frac{P(r_i + G_{-i}) - P(G_{-i}))}{r_i} < P'(r_i + G_{-i})$$

which is true, since $P(r_i + G_{-i}) - P(G_{-i}))/r_i < P'(r_i + G_{-i})$ by the convexity of P and $\frac{u'(r_i)r_i}{u(r_i)} \leq 1$ by the concavity of u and $u(0) = 0$. Thus, higher r_i also makes it more likely that the agent contributes. Clearly, higher v_i has the same effect. The result for G_{-i} follows directly from the convexity of P. ■

Proof of proposition 6. The proof is by construction, that is, for any positive $\{v_i\}_{i=1}^n$ and $\{r_i\}_{i=1}^n$, I exhibit an action profile that constitutes a Nash equilibrium. Define:

$$F_1^i(r_i) \equiv \frac{u(r_i)}{P(r_i)},\ F_2^i(r_1, \ldots, r_n) \equiv \frac{u(r_i)}{P(R) - P(R - r_i)}$$
$$\text{and } F_3^i(r_i, G_{-i}) \equiv \frac{u(r_i)}{P(r_i + G_{-i}) - P(G_{-i})}$$

where $R = \sum_{i=1}^n r_i$.

Consider the possible configurations of the valuations $\{v_i\}_{i=1}^n$ relative to F_1^i and F_2^i. I will exhibit a Nash equilibrium for any such configuration which will complete the proof. Suppose first that $v_i \geq F_2^i, \forall i$. Using $G_{-i} = R - r_i$, by (7) 'all agents contribute' is a Nash equilibrium. Similarly, if $v_i < F_1^i$ for all i then 'no agent contributes' is a Nash equilibrium. More generally, suppose $v_i \geq F_1^i$ for $i \in S_1$, $v_i < F_2^i$ for $i \in S_2$ and $v_i \in [F_2^i, F_1^i)$ for $i \in S_3$ where $S_1 \cup S_2 \cup S_3 = \{1, 2, \ldots, n\}$ and the sets S_j, $j = 1, 2, 3$ have empty pairwise intersections.

Denote by C the strategy to contribute and by N the strategy not to contribute. I show how to construct an action profile which constitutes a Nash equilibrium. First, assign C to all agents in S_1 and N to all agents in S_2. These actions are optimal for them, independent of what the agents in S_3 (if such exist) do. To see that, notice

$$F_1^i \geq F_3^i \geq F_2^i \quad \text{for all } i,\ r_i,\ G_{-i} \geq 0,$$

which follows from the convexity of P. Thus, we have $v_i \geq F_1^i \geq F_3^i(r_i, G_{-i})$ for all $i \in S_1$, $G_{-i} > 0$ (so C is optimal for $\forall i \in S_1$) and $v_i < F_2^i \leq F_3^i(r_i, G_{-i})$ for all $i \in S_2$ and $\forall G_{-i} > 0$ (so N is optimal for them).

If $S_3 = \varnothing$, we are done. Otherwise, denote $R^* \equiv \sum_{i \in S_1} r_i$ and assign action C to all agents in the set $S_3{}' = \{i \in S_3 | \ v_i \geq u(r_i)/P(R^* + r_i) - P(R^*)\}$. If $S_3{}' = \varnothing$, assign N to all agents in S_3 which is clearly best response for them. Note that C is best response for each agent in $S_3{}'$ regardless of what the rest of the agents in S_3 do. This follows by the fact that F_3^i is decreasing in G_{-i} (by the convexity of P). Now, if $S_3 \backslash S_3{}'$ is empty, we are done, otherwise denote $R^{**} = \sum_{i \in S_1 \cup S_3'} r_i$ and assign action C to all agents in the set $S_3{}'' = \{i \in S_3 \backslash S_3{}' | \ v_i \geq \frac{u(r_i)}{P(R^{**}+r_i) - P(R^{**})}\}$. If S_3'' is empty, assign N to all agents in $S_3 \backslash\ S'_3$ and so on. Continue in this way until all agents are assigned either C or N. Since the number of agents is finite, this assignment process cannot continue forever. Finally, note that it is possible to have multiple equilibria – for example, if $F_1^i > v_i \geq F_2^i\ \forall i$, both 'all agents contribute' and 'no agents contributes' are Nash equilibria. ■

Proof of proposition 7.

(a) Take any draw $\{r_i, v_i\}_{i=1}^n$ from Φ and any Nash equilibrium possible at these values. Suppose we draw one more agent with some resource $\tilde{r}$ and valuation $\tilde{v}$. Suppose first that this $n+1$-st agent finds it optimal to contribute. Note that, for all agents that were contributing before, contributing is still best response as their marginal utility from the public good is now higher (because of the convexity of P). Clearly total contribution in such an equilibrium cannot be lower compared to before. Alternatively, if the $n+1$-st agent has $\tilde{v}, \tilde{r}$ such that he does not wish to contribute, the other agents'u choices and trade-offs are unchanged, so total contribution stays constant (but cannot decrease). Finally, since this result holds for any possible draws $\{r_i, v_i\}_{i=1}^n$, it holds on average as well; that is, expected provision (weakly) increases in group size, n.

(b) Lemma 2 implies that each agent's contribution is (weakly) increasing in v_i and r_i. The proof is thus the same as in proposition 3 and hence omitted. ■

Proof of lemma 3. The proof is by contradiction. Suppose in equilibrium agent i contributes ($g_i^* = r_i$), i.e. $v_i \geq u(r_i)/P(r_i + G_{-ij}) - P(G_{-ij})$ but agent $j > i$ does not ($g_j^* = 0$), i.e. $v_j < u(r_j)/P(r_i + G_{-ij} + r_j) - P(G_{-ij} + r_i)$. Then, $G_{-j} = G_{-ij} + r_i = G_{-i} + r_i$. Denoting $x = r_i + r_j + G_{-ij}$, $y = r_i + G_{-ij}$ and $z = G_{-ij}$ we have, by A2, that $u(y - z)/P(y) - P(z) > u(x - z)/P(x) - P(z) > u(x - y)/P(x) - P(y)$ and so the r.h.s. of the inequality for v_i above is larger than the r.h.s. of the inequality for v_j, implying $v_i > v_j$ – a contradiction. Therefore, it is not possible to have a Nash equilibrium in which an agent with lower valuation contributes while an agent with a higher valuation does not. Hence, there exists a cutoff level, depending on $\{r_i\}$ and $\{v_i\}$, such that all agents with valuations higher than this level contribute, while all agents with valuations below it do not contribute. If this cutoff level is higher than the highest valuation, $v_{\max}$ then no one contributes in equilibrium. If it is lower than the lowest valuation then all agents contribute in equilibrium. ■

Proof of lemma 4. Note first that the intermediate case of $m < n$ agents contributing is no longer possible if all valuations are equal. To see that, note that by A2,

$$\frac{u(r_m)}{P(\sum_{j=m}^{n} r_j) - P(\sum_{j=m+1}^{n} r_j)} \leq \frac{u(r_{m+1})}{P(\sum_{j=m+1}^{n} r_j) - P(\sum_{j=m+2}^{n} r_j)}, \forall m$$

i.e., if $m + 1$ contributes, then m does too as it cannot be that $\bar{v}$ is larger than the r.h.s. but smaller than the l.h.s. of the above inequality. Thus, if (the richest) agent contributes, all agents contribute. To have 'no agents contribute' be the unique equilibrium, we therefore must have $\bar{v} < u(r_{\max})/P(R) - P(R - r_{\max})$. Similarly, 'all agents contribute' is the unique PSNE if agent n would contribute even if $G_{-n} = 0$, that is, if $\bar{v} \geq u(r_{\max})/P(r_{\max})$. Finally, both these equilibria are possible (as in a coordination game) if[29] $\bar{v} \in [u(r_{\max})/P(R) - P(R - r_{\max}), u(r_{\max})/P(r_{\max}))$ – if agent n contributes, all others do since $\bar{v} \geq u(r_i)/P(R) - P(R - r_i)\ \forall i$, (the function $u(r)/P(R) - P(R - r)$ is increasing), rendering n's strategy optimal, while if agent n does not contribute, all others do not contribute either since $\bar{v} < u(r_i)/P(r_i)\ \forall i$, (the function $u(r)/P(r)$ is decreasing), rendering n 's strategy optimal. ■

Proof of proposition 8

(a) The proof uses the fact that the maximum of n independent draws from a distribution F (known as the n-th *order statistic*) has a cdf, denoted $F_{[n]}(x)$,

29 Note that, by the convexity of P, $u(r_{\max})/P(R) - P(R - r_{\max}) < u(r_{\max})/P(r_{\max})$.

which equals $(F(x))^n$ – see for example David and Nagaraja (2003) for proofs and discussion. By the properties of ϕ outlined above, if $\bar{v} < \phi(R)$[30] then $r_{\max} \leq R < \phi^{-1}(\bar{v})$ and so, by lemma 3, $E^L(G^*) = 0$ which is constant in σ. Suppose $\bar{v} \geq \phi(R)$, i.e., the probability of $r_{\max} < \phi^{-1}(\bar{v})$ is less than 1. There are two cases. First, if mean resource, $\bar{r}$ is relatively low compared with $\bar{v}$ or, equivalently, valuation is relatively low compared with $\bar{r}$ (so that $\bar{r} < \phi^{-1}(\bar{v}) \iff \bar{v} < \phi(\bar{r})$) then low resource variance ($\sigma = 0$ or σ close to zero if the support of ϕ is bounded) still yields $E(G) = 0$ but raising the resource variance increases the probability that $r_{\max}$ exceeds $\phi^{-1}(\bar{v})$ and so $E^L(G^*)$ eventually becomes positive and increases in σ as more and more mass shifts above $\phi^{-1}(\bar{v})$. Conversely, if the mean resource is relatively high (or $\bar{v}$ is high) so that $\bar{r} > \phi^{-1}(\bar{v}) \iff \bar{v} > \phi(\bar{r})$, then small variance (e.g. σ zero or close to zero) ensures that all probability mass in Φ_r is above the cutoff $\phi^{-1}(\bar{v})$ and thus $E^L(G^*) = n\bar{r}$, while increasing σ eventually leads to probability mass shifting below $\phi^{-1}(\bar{v})$ and thus expected provision decreases in resource inequality.

(b) By the properties of ψ, if $\bar{v}$ is large ($\bar{v} \geq \psi(R)$) then $prob(r_{\max} \leq \psi^{-1}(\bar{v})) = 1$ and so $E^H(G^*) = n\bar{r}$ which does not depend on σ. For lower valuations the results obtain analogously to those in part (a).

(c) Since $\phi^{-1}(\bar{v})$ is decreasing in $\bar{v}$, it is clear that $prob(r_{\max} \leq \phi^{-1}(\bar{v})) = [\Phi_r(\phi^{-1}(\bar{v}))]^n$ is (weakly if the probability is 1) decreasing in $\bar{v}$, that is, expected total, $E^L(G^*) = (1 - [\Phi_r(\phi^{-1}(\bar{v}))]^n)\bar{r}n$ and per capita contributions under criterion L is (weakly) increasing in $\bar{v}$ and $\bar{r}$. Similarly, under criterion H, since $\psi^{-1}(\bar{v})$ is increasing in $\bar{v}$ we have $prob(r_{\max} \leq \psi^{-1}(\bar{v})) = [\Phi_r(\psi^{-1}(\bar{v}))]^n$ is (weakly) increasing in $\bar{v}$, so $E^H(G^*)$ and $E^H(G^*)/n$ are also (weakly) increasing in $\bar{v}$ and $\bar{r}$. ■

Proof of lemma 5

(a) The stated condition ensures that $v_i < (u(r_i)/P(R) - P(R - r_i))\ \forall i$ since $\psi(r_i)$ is increasing in r_i given R. Thus, even if all others would contribute, no agent would find it optimal to contribute if the sufficient condition holds, so 'no agent contributes' is the unique possible PSNE.

(b) The stated condition ensures that $v_i \geq (u(r_i)/P(r_i))\ \forall i$, since $\phi(r_i)$ is decreasing in r_i given R. Thus, each agent would contribute even if no one else is contributing, which implies that 'all agents contributing' is the unique possible equilibrium. ■

Proof of proposition 9. The proof is analogous to that of proposition 8 using that the minimum of n independent draws from a distribution F has a cdf

30 R is a random variable itself, so a (very strong) sufficient condition that $G^* = 0$ with certainty is $v < \phi(\min_{\{r_i\}^\sim\Phi_r} \sum r_i)$. For n large, by the Law of Large Numbers, $R = n\bar{r}$ and so $\phi(R)$ is independent of the draw $\{r_i\}$.

$1-(1-F(x))^n$ (see David and Nagaraja 2003). The comparative statics with respect to $\bar{v}$, R and n follow immediately from the expressions in (10) and (11). Consider first the effect of the valuation heterogeneity, γ on $E^{lb}(G^*)$ in part (a). When $\bar{v}$ is low, $\bar{v} < \phi(r_{\min})$, low valuation heterogeneity (e.g. $\gamma = 0$) implies $E^{lb}(G^*) = 0$ and only if γ is high enough can $v_{\min} \geq \phi(r_{\min})$ hold for some draws $\{v_i\}$. Thus, higher variance (weakly) increases the lower bound of expected total contribution away from zero. Conversely, if mean valuation is relatively high, $\bar{v} > \phi(r_{\min})$, then no valuation heterogeneity ($\gamma = 0$) implies $E^{lb}(G^*) = R$, while increasing the valuation variance decreases the probability in equation (10) away from 1, hence (weakly) reducing $E^{lb}(G^*)$ away from R. The results in part (b) follow analogously. ■

Proof of proposition 11. Proceed as in proposition 5, using the same notation. Suppose $\{r_i\}$ and $\{v_i\}$ are not positively assortatively matched and look at the following two agents: agent A with (v^n, r^i) and agent B with (v^j, r^n) for some i, j. I will show that total contribution (weakly) increases if we switch r^i with r^n. There are three possibilities to consider. First, suppose that before the switch both B and A were not contributing, i.e., (by Lemma 3) total contribution was zero. Since $r^n \geq r^i$, by lemma 3, agent A may choose to contribute after the switch, thus raising total contribution (some other agents may choose to contribute too by the complementarity between contributions). Second, suppose before the switch B was not contributing but A was. Then A would continue to contribute (with a higher amount since $r^n \geq r^i$) after the switch and, in addition, other agents may start contributing too. Finally, suppose that, before the switch, both A and B were contributing. I will show that both contribute after the switch as well. First, A clearly contributes after the switch since $r^n \geq r^i$. For B, notice that if $v^j \geq u(r^n)/P(G^O + r^n + r^i) - P(G^O + r^i)$ (where G^O is the total contribution of all others), then by A2 and the convexity of P, $u(r^n)/P(G^O + r^n + r^i) - P(G^O + r^i) \geq u(r^i)/P(G^O + r^n + r^i) - P(G^O + r^n)$, i.e., B contributing after the switch is best response given the others' actions. In the same way one can show that total contribution cannot decrease when we match v^{n-1} with r^{n-1} and so on. Thus, maximum contribution is achieved under positive assortative matching. The second part is proved analogously. ■

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