Can a raise in your wage make you worse off?
A public goods perspective

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Abstract

We show that a seemingly paradoxical result is possible—an increase in one’s wage can reduce one’s welfare. Such outcome can occur in an economy populated by agents who value a private good bought using labor income and a public good produced by voluntary time contributions. A raise in the wage (in general, opportunity cost of time) makes each agent substitute away from contributing to the public good, failing to internalize the negative externality imposed on others. The result is a decrease in public good provision. Under quite general conditions, the implied cumulative negative effect on agents’ welfare can more than offset the positive effect of the wage raise from increased private good consumption and lead to an equilibrium in which all agents are worse off. Our result is particularly relevant for developing economy settings as it holds for relatively low initial wage levels. We discuss the applicability of our findings to a number of important problems in development, such as market integration, cooperation in common pool resource conservation and social capital.

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1. Introduction

The traditional development economics literature has devoted surprisingly little attention to public goods. Until recently, the quality of life was assessed mainly in terms of private consumption. In the past few years, however, both in academia and on the policy front (for example, see the United Nation’s Human Development Index), there has been much more
emphasis on other measures of the quality of human life apart from income per capita. As Besley and Ghatak (2003) point out, “two groups of people with similar private consumption will have very different qualities of life if there are significant differences in their access to safe drinking water or medical care.” They highlight the importance of public goods as a necessity for a working theory of development focusing on provision by voluntary contributions, the government, mixed private–government institutions or non-governmental organizations (NGOs).  

Our paper identifies an inherent tension present in the interaction between local public goods provision and improvements in private market opportunities brought about by the economic development process. It is conventional economic wisdom than an increase in one’s labor wage holding other prices constant should improve one’s welfare. While this is certainly true in a situation with no externalities, this paper describes a seemingly paradoxical general scenario where a wage raise can be actually welfare reducing for everyone in the economy.

Specifically, we show that in a setting where agents value a private good bought using labor income and a pure public good produced by contributing part of their time endowment, it is possible that an exogenous raise in the labor wage (or, in general, any factor that affects the opportunity cost of contributing) can make all agents worse off in the resulting equilibrium. The intuition is the following. As the wage rises, agents devote more time to working in the wage sector substituting away from contributing time to the public good production. When deciding on their time allocation, they fail to internalize the negative externality imposed on everyone else or, in other words, act atomistically. Given the nature of public goods, the result of all agents behaving in such a manner creates a compounded negative welfare effect which can more than offset the positive effect of the wage increase on private good consumption and thus decrease their welfare in comparison to the initial (lower wage) equilibrium.

Throughout the paper, we have worded our analysis in terms of studying the effect of a rise in a private “wage” aiming for expositional and analytical simplicity. However, all that matters for our results is the fact that the opportunity cost of time spent in the public good production is rising. Thus, improvements in people’s outside options, migration or any other reason that leads to a voluntary choice to reduce the time devoted to the public good sector could all lead to the same outcome. It is the individual’s optimal decision to change her time allocation in response to an exogenous factor that is crucial for our result. The actual reason for this change in time allocation could be rising wages or opportunity cost in general, availability of private market job opportunities that require time spent away from public good production, or any similar factor. We acknowledge, however, that improvements in outside options can also have an additional negative effect on cooperation, which remains outside the scope of our model, namely better exit options can undermine the ability of agents to punish deviators (free-riders). Nevertheless, the “opportunity cost” effect that we study is likely present and significant in almost all practical situations.

Recent field studies from various developing countries provide motivating evidence for our story. An example that we believe fits our story well is cooperation in common property resource (CPR) preservation. A CPR is basically a common plot of land (fishing pond, forest, dyke, etc.), which is jointly maintained by people in a village or community and whose resources are shared by these people. To avoid the well-known “tragedy of the commons” problem of over-

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1 See also Dreze and Sen (1999), Hart et al. (1997) and Besley and Ghatak (2001).

2 We view our analysis as best suited for the case of labor contributions to public goods and provide examples accordingly. For discussion on the case of monetary contributions (e.g. as in Bergstrom et al., 1986), see Section 3.
exploitation and degradation of the resource various mechanism of cooperation emerge among the agents. These cooperative actions have strong public good (collective action) aspects and they typically require a significant amount of labor input for both actual conservation measures as well as enforcement. Jodha (1992) and Baland and Platteau (1996) provide an excellent and very extensive discussion (see also Ostrom, 1990; Bandiera et al., 2005). Specific collective action problems discussed in the literature include: management of irrigation and drainage systems (see Bardhan, 2000), conservation of forest resources\(^3\) and grazing pastures, steep-slope anti-erosion control, guarding coastal fishing grounds against encroachment by mechanized boats, etc.

Many authors have identified a negative effect on cooperation in CPR conservation resulting from increasing market integration (e.g. Baland and Platteau, 1996; Shanmugaratnam, 1996). Economic growth opens or improves the options for work outside the community raising the opportunity cost of supplying labor in CPR maintenance. This adjustment can be both on the intensive margin (working more hours per year far from the village) or on the extensive margin (increased geographic and occupational mobility). In addition, market integration can lead to a reduction of the value of CPR to people (i.e. again, effectively increasing the relative value of other private options) or erosion of traditional authority and social cohesion in general further harming cooperation (Jodha, 1992).

Indeed, numerous empirical studies have found a negative “opportunity cost” effect in CPR collective action problems such as the one stipulated in our theoretical model. Weinberger and Jutting (2000) study the collective action problem of community participation in risk management in Chad villages and find a negative and significant effect of income on participation, which they attribute to an opportunity cost of time effect. Adger (1998) looks at collective action in coastal defence and water management in Vietnam and concludes that “...private economic activity is leading to higher opportunity costs of time and less cooperation in collective action.” Shanmugaratnam et al. (1992) report that in Mali the emergence of absentee herd owners with high outside economic opportunities appears to be a major stumbling block on the way towards pastoral institution-building for sustainable rangeland management. Fariss (2004) studies the collective management of pastoral commons in Peru and finds significant evidence of an “opportunity cost” effect—tourism involvement exerts a significant negative effect on household herding labor, especially in the dry season. Bellon (2001) studies the problem of maintaining crop diversity which requires coordinated labor-intensive effort of all farmers in a village in the Oaxaca region of Mexico and concludes that expanded participation in labor markets increases the opportunity cost of time for farmers and their families to maintain crop diversity which leads to negative consequences. Similar findings are reported in Pender and Scherr (2002) for natural resource conservation in Honduras and Steele (1999) for fisheries in Sri Lanka. Finally, to the extent that better outside option not only undermine the possibility to punish deviators but also lead to a decrease in actual time spent in CPR maintenance, the studies of Bardhan (2000) on public irrigation systems in Tamil Nadu, India; Fujita et al. (2001) on irrigator’s associations in the Philippines; and of Janvry et al. (2001) on ejido communities in Mexico in the 1990s could provide further motivation to our story as they find better outside options to have a negative effect on contributing to the cooperative effort.

A close parallel to the above discussion exists in the social capital literature. As pointed out by Coleman (1990), social capital has significant public good features. Social capital or its outcomes have been found to be positively correlated with growth and development (Putnam, 1993; Knack

\(^3\) Edmonds (2002) estimates that one third of the world population relies on firewood for energy. See also Reddy and Chakravarty (1999) on the welfare impact of disabling common access to forests in northern India.
and Keefer, 1997; Udry, 1994). Recently, a strand of literature has appeared lamenting the “erosion of social capital” brought about by market penetration, commercialization, etc. (e.g. Putnam, 2000; Wiig, 2003). A significant factor in this process is said to be the increase in opportunity cost of time or outside options as well as the spreading of “urban” or “modern” individualistic values.

Social capital, unlike other forms of capital, is embedded in the relationship between people and not the people themselves or the means of production. Thus, it is clear that actually spending time with others in the community is crucial in maintaining social capital. The “opportunity cost” effect thus will be particularly strong and it cannot be avoided by substituting time contributions by any other form (e.g. monetary). It also leads to less frequent interactions among people and “reduces the scope for interlinked long-term relationships and the size of the social units within which informal cooperation mechanisms take place” (Baland and Platteau, 1996). Coleman (1990, p. 316) provides a classic example related to our analysis. A family’s decision to move away from a community because of a job opportunity may be entirely profitable from the individual point of view of that family. However, because social capital is embedded in interpersonal relations, others may experience extensive losses due to the severance of their ties with members of that family, a severance over which they had no control. Such losses may entail the weakening of norms and sanctions that aid enforcement or help parents and schools in socializing children. The total cost that each family experiences as a consequence may well outweigh the benefits that come from those few decisions it has control over.

Historical development studies also discuss similar phenomena. For example, in their seminal work on the rise and fall of the manorial system in Western Europe, North and Thomas (1971) document evidence that before the 14th century protection was provided by Lords who required in exchange that serfs work in their fields. The authors argue that serfdom in Western Europe was very different from slavery in North America and Eastern Europe and, in contrast, essentially represented an efficient contractual system (which they call the “customs of the manor”) rather than an exploitative arrangement where Lords ‘owned’ labor. After the Black Death and the resulting sharp decline in population, wage rates increased and eventually the system disappeared. Thus, a rise in the wage rate diverted peasants from the Lord’s fields, which in turn brought about the decline of a presumably efficient contractual system featuring strong public good characteristics.

A relevant question to ask in view of the above evidence is: “Why should we care?” If people voluntarily choose to reduce their participation in CPR preservation or creating and maintaining social capital, does it not mean, through a revealed preference argument, that they must be better off as a result? This is certainly true for a private good setting by a standard “change in relative prices” textbook argument but we show that this logic can fail in our collective action or public

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4 In general, as Baland and Platteau (in press) and Dasgupta and Kanbur (2005) suggest, labour inputs are not the only possible form of contributions. If contributions can be made in cash, the expectation is that agents with a higher opportunity cost of time would prefer this form of contribution and, under some conditions, it is possible that public good provision does not decline as opportunity cost goes up. We discuss the implication of allowing monetary contribution in Section 3.

5 “The absence of an effective central political authority made the provision of protection purely a local matter. The constant threat of piracy and brigandage, and the less frequent but always possible incursions of Vikings, Huns, or Moslems, made local defense a matter of prime importance. The general lack of order compelled dependence on specialized individuals possessing superior military skills and equipment, and their presence was welcomed by peasants unskilled in warfare and therefore otherwise helpless. This affords the classic case of a public good, since protection of one peasant family involved protection of his neighbors as well” (North and Thomas, 1971).
good setting due to the presence of externalities. In essence, this paper provides a theoretical, efficiency-based argument of why we may indeed worry about decline in CPR protection, erosion of social capital or related issues. The fact that a rise in the opportunity cost of contribution leads to substituting time away from public good provision is not new—what we demonstrate is that the welfare of all agents may decline as a result.

We characterize the exact conditions under which the immiserising effect occur and show that it is more likely to take place for a range of relatively low initial wages which makes our results particularly relevant for less developed economies. Specifically, we show that a local negative welfare effect would always occur around the wage (opportunity cost of time) level at which the agent starts devoting time to the private market sector (in our interpretation this is time spent away from the community). As the wage rises further, the welfare effects tend to turn positive, i.e. the immiserisation could be viewed as a transitional phenomenon in environments with low initial wages. The relevance to development is also strengthened by the fact that institutional problems preventing non-private provision of local public goods of the type we study are also more likely to be prevalent in such environments. As has been argued by Durlauf and Fafchamps (2005): “In poor countries, there are many situations in which the state could, theoretically, intervene to provide a public good, but where it is unable to do so because its tax base and its capacity to organize are limited. Collective action can serve as a substitute for the state.”

In its general flavor, our immiserising result bears similarity to the findings of Bergstrom (1989) and Dasgupta (2001) who also analyze public good settings with voluntary provision and derive “seemingly paradoxical” results in which apparently beneficial changes in the economic environment make agents worse off. In Bergstrom (1989), one of two agents contributing to a public good is given a subsidy on contributions (financed by lump-sum tax on both agents), which could make her worse off as a result because the other agent free-rides more in the new equilibrium. In Dasgupta (2001), a lump-sum transfer to the wife in a household financed by a proportional tax on the husband could make her worse off as the husband cuts back on contributing to childcare (viewed as a public good). Thus, in both papers, the emphasis is on potential perverse effects of redistributionary policies and the results rely on introducing asymmetric treatment of the agents. In contrast, our results hold in a multi-agent economy and do not require any heterogeneity, asymmetric treatment or redistribution policy considerations. We show that the basic force needed to generate an immiserisation effect is just the existence of externalities and the agents’ atomistic behavior in a Nash setting coupled by an exogenous change in the opportunity cost of contributing that can be possibly attributed to economic growth or market integration. Thus, it is the change in the relative cost of contributing coupled with the presence of externalities that could bring about the immiserisation not the transfer policy itself. Finally, unlike ours, their settings feature monetary contributions to the public goods which makes our results complementary as both types of public goods are likely to be important in reality.

The rest of the paper is organized as follows. In Section 2, we present the basic theoretical model, show existence and uniqueness of equilibrium, and derive our main theoretical results. Section 3 shows how the basic model can be extended to include leisure and briefly covers the

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6 Ghosh and Kanbur (2003) is another recent work exploring a similar idea.
7 This is basically the “exploitation of the rich by the poor” effect discussed by Olson (1965).
8 If both agents receive the subsidy or transfer in Bergstrom’s or Dasgupta’s models, they will not be worse off. In the Bergstrom (1989) model, one can actually show that (at least) for a quasi-linear utility function linear in the private good if both agents’ contributions are subsidized and the subsidy is financed by a lump-sum tax both agents can be better off as a result.
consequences of introducing taxation of labor income. We also discuss a version of the model where agents make monetary instead of labor contributions. In Section 4, we outline some possible areas where our result has theoretical and empirical significance, conclude and propose some potential future research directions.

2. The model

The basic structure of the model follows the well-developed literature on voluntary public good provision (Bergstrom et al., 1986; Cornes and Sandler, 1996 among others). Consider an economy consisting of \( n \) identical agents with preferences:

\[
U(c, G) = u(c) + G
\]

where \( G \) is the amount of a pure public good consumed, \( c \) is consumption of a private good, and \( u \) is strictly increasing and concave. Each agent is endowed with one unit of time that she can allocate between working for a market wage, \( w \), or working for the production of the public good. This exclusivity assumption is important for the results to follow, i.e. our analysis is not applicable to public goods created at the market workplace. If agent \( i \) spends \( y_i \) units of time contributing towards the public good, then its total provision is given by \( G = G(Y) \) where \( Y = \sum_{i=1}^{n} \) is the total amount of time allotted to the public good production. We assume that the production function, \( G \), is increasing and strictly concave.

The consumption good can be purchased with labor income only. For simplicity, we restrict our attention to a static model with no savings or uncertainty. Thus, given that \( u \) is strictly increasing, the agent’s budget constraint is (normalizing the price of the private good to 1):

\[
c_i = (1-y_i)w
\]

The optimization problem of an agent in our economy is:

\[
\max_{y_i \leq 1} G(y_i + Y_{-i}) + u((1-y_i))w
\]

where \( Y_{-i} \) is the total time contribution of all other agents, which is taken as given by agent \( i \).

For the most part of the paper, we interpret \( w \) as a (market) wage but notice that we can easily accommodate the case in which the agent can earn some income, \( y_i w_p \), by working in the public good sector in addition to his income from the market sector, \((1-y_i)w_m\) where \( w_p < w_m \). In such case, the agent’s total income (and hence consumption) will be \( w_m - y_i(w_m - w_p) \). Hence, the marginal cost (in consumption units) of spending time in the public sector is given by the difference between the market and public sector wages, \( w_m - w_p \). Since this marginal cost is all that matters for the agent’s optimal time allocation decision, we can actually interpret \( w \) as the difference between the market and public sector wages and normalize \( w_p = 0 \) for simplicity. Below, we will be looking at comparative statics with respect to changes in \( w \) and these can be also interpreted as changes in the magnitude of the differential between the market wage and the public good sector wage. In this respect, our main result that an increase in \( w \) may lead to

\[9\] Later in the paper, we show via an example that additive separability is not necessary for our results. It was assumed to ensure analytical tractability.

\[10\] For example, public goods like seminars created at academic departments.

\[11\] Note that assuming that preferences are linear as opposed to concave in \( G \) is without loss of generality since \( G \) is strictly concave itself.
worsening of the welfare of the agents in the economy can occur in general because of any reason that causes a widening of the gap between the wages in the two sectors of the economy, and not only because of an absolute increase in the private sector wage.

2.1. Characterization of the equilibrium time allocation

We assume that the agents in the model economy behave non-cooperatively so we will be looking for a Nash equilibrium in their time contributions towards the public good production. Since there is no heterogeneity across the agents, we concentrate on symmetric equilibria in which \( y_i = y \) and \( c_i = c \) for all \( i = 1, \ldots, n \). Under some relatively mild assumptions stated below, we show existence and uniqueness of such equilibrium.

**Assumption A1.** The public good production function, \( G(Y) \), is twice continuously differentiable with \( G'(Y) > 0, G''(Y) < 0 \) for all \( Y \geq 0 \) and \( \lim_{Y \to 0} G'(Y) = \infty \). The utility function \( u(c) \) is strictly increasing, twice continuously differentiable and (weakly) concave.

The above assumption does not put any uncommon restrictions on the functional forms used in the analysis below—it basically imposes concavity and some widely used Inada conditions. The first-order condition of (1) is:

\[
G'(Y) \geq u'(c_i)w
\]

with equality when \( y_i < 1 \). In a symmetric equilibrium, we must have: \( y_i \frac{Y}{n} \) for all \( i \); hence, the above can be written as:

\[
G'(Y) \geq u' \left( w \left( 1 - \frac{Y}{n} \right) \right) w
\]

with equality when \( Y < n \). Notice also that under Assumption A1 it is not possible to have \( Y = 0 \) in equilibrium since the right hand side of (2) is a finite number, \( u'(w)w \), while the left hand side goes to infinity.

**Proposition 1.** Under Assumption A1, there exists a unique symmetric Nash equilibrium for the time allocations, \( \{y_i \}_{i=1}^n \), in the model economy.

**Proof.** See Appendix A. □

Given the result of Proposition 1, let us denote the equilibrium total contribution \( Y^* \) by \( g(w) \) as we will be interested on how total provision, \( G(Y^*) \), and agent’s indirect utility vary due to changes in the wage. We concentrate on the case \( g(w) < n \) since, if the wage is so low so that \( g(w) = n \), the equilibrium public good provision level and the agent’s indirect utility are locally invariant in \( w \). This implies that (2) will hold as equality and we can write it as:

\[
G'(g(w)) - u' \left( w \left( 1 - \frac{g(w)}{n} \right) \right) w = 0
\]

We proceed by characterizing the behavior of the total equilibrium public good provision, \( G(g(w)) \), as a function of the wage. Since \( G \) is increasing, it is enough to look at the behavior

\[\text{12} \] Other equilibria, which we rule out, might be also possible, for example, when all agents are at an interior equilibrium and \( u \) is linear around the equilibrium (see (2) below).
of \( g(w) \) with respect to changes in \( w \). Differentiating (3) with respect to \( w \) and re-arranging we have:

\[
g'(w) = \frac{u'(c) + u''(c) \left(1 - \frac{g(w)}{n}\right)w}{G''(g(w)) + \frac{u'(c)w^2}{n}} = \frac{u'(c) \left[1 + \frac{u''(c)c}{u'(c)}\right]}{G''(g(w)) + \frac{u'(c)w^2}{n}}
\]

The denominator is always negative due to the concavity of \( G \) and \( u \), so the sign of \( g' \) depends on the sign of the numerator. Since \( u'(c) > 0 \), we have that \( g(w) \) is a decreasing function of \( w \) whenever \( 1 + \frac{u''(c)c}{u'(c)} > 0 \) or, in other words, whenever the agent’s relative risk aversion\(^{13} \) \( \rho(c) = \frac{-u''(c)c}{u'(c)} \) is less than 1 (i.e. the agents are not too risk averse). If, conversely, \( \rho(c) > 1 \), we have that \( g(w) \) is always increasing in the wage level. These results are summarized in the following:

**Proposition 2.** If the coefficient of relative risk aversion of \( u \) is less than 1 at the equilibrium time allocation, the total amount of the public good provided, \( G \), is decreasing in the labor wage, \( w \). Conversely, if the agents’ relative risk aversion is higher than 1, the total amount of the public good provided is increasing in the wage level.

**Proof.** See above. □

There are two effects of the wage increase working in opposite directions: a substitution effect which results in reduced time contribution to the public good and an income effect reducing labor hours. In general, the overall effect of an increase in the wage on public good provision is unclear and depends on the relative strength of these two effects. If the agent’s utility has a high curvature (\( \rho(c) > 1 \)), the substitution effect of the wage increase which results in reduced time contribution to the public good is relatively small and is swamped by the income effect, raising \( G \). Notice that this implies that wage labor must be decreasing in the wage, i.e. a “backward bending” labor supply curve. This scenario appears to be less empirically plausible—the estimates of labor supply elasticities in the literature are positive. On the other hand, in the lower curvature (\( \rho(c) < 1 \)) case, the deleterious (for the public good provision) substitution effect is stronger compared to the income effect resulting in a decrease in the total amount of the public good provided. Notice that, in general, all that we need for the proposition result is simply that private market labor supply not be backward bending.

We now turn to the welfare properties of the equilibrium described above. The equilibrium utility level obtained by an agent in the model economy at wage \( w \) is:

\[
U^*(w) = G(g(w)) + u\left(1 - \frac{g(w)}{n}\right)w
\]

Let us characterize how this indirect utility varies with the wage. We show that, for some range of \( w \) and when labor supply to the private market is not backward bending, an exogenous increase in the wage can actually make the agents worse off. Notice that this is a local result, i.e. it is not true for all starting values of the wage level. As we will see, it is

\(^{13} \)There is no risk in our model so \( \rho(c) \) is taken just as a characteristic of the utility function.
likely to hold for relatively low values of \( w \), thus qualitatively matching (but not exclusively) the reality of a growing developing economy. We have, using (3):

\[
\frac{dU^*(w)}{dw} = u'(c) \left[ 1 - \frac{g(w)}{n} + \frac{(n-1)}{n} wg'(w) \right]
\]

(4)

The first thing to notice is that the above expression is always positive when \( g(w) \) is increasing in \( w \). Thus, using the result of Proposition 2, when value of \( \rho(c) \) at equilibrium is larger than one, a local increase in the wage level will always lead to “backward bending” labor supply, higher public good contribution and hence higher welfare.

A much more interesting possibility occurs when labor supply is increasing in the wage, \(^{14} g'(w) < 0 \). In this case, we show that there always exists an interval for the wage level at which a local increase in the wage is immiserising for the agent. This result is demonstrated to hold for lower levels of \( w \), namely such around the critical value at which the agent starts to allocate a positive amount of time to the private market sector. Depending on the functional forms for \( u \) and \( G \), this interval for the wage can have a zero or strictly positive lower bound as we demonstrate in the examples in the next section. The wage increase is actually shown to have a positive effect for high enough values of \( w \).

**Proposition 3.** *If labor supply is increasing in the wage \((g'(w) < 0)\) and \( u'(0) \) is finite,\(^{15} \) then, for any \( n > 1 \), there exist an interval for the wage level in which a local increase in \( w \) reduces the agent’s welfare.*

**Proof.** When labor supply is increasing in the wage, i.e. \( g' < 0 \), the derivative negative when:

\[
1 - \frac{g(w)}{n} + \frac{(n-1)}{n} wg'(w) < 0
\]

(5)

Given the boundedness of \( u'(0) \), the first-order condition, (2), and Assumption A1 imply that there exists a positive wage level \( w_1 > 0 \) such that \( g(w) = n \) for \( w \in [0, w_1] \), i.e. the time feasibility constraint will be binding and hence agent’s welfare is independent of the wage. By continuity, in some interval \([w_l, w_m]\), the above derivative is negative as long as \( g' < 0 \) there. □

Proposition 3, stating that a wage increase can be welfare reducing, is the main result of the paper. The intuition comes from the fact that, in equilibrium, each agent does not take into account the positive externality created by supplying time towards the production of the pure public good. Thus, for some range of the wage level, a raise in the wage makes the agent spend more time working in the private good sector. When making this decision the agent tries to free ride on others assuming that they will keep their contribution at the same level as before, i.e. fails in internalizing the harm he causes everyone else by reducing his contribution towards the public good. Thus, each agent’s ‘greed’ to exploit the higher private wages leads to a compounded dramatic reduction in the supply of the public good.

Remember from our discussion above that the reduction in welfare can happen only when the substitution effect of the wage raise on the private good sector labor supply dominates the income effect, i.e. when the agent’s labor supply is increasing in the wage. In this case,

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\(^{14}\) This includes the case of linear utility examined in more detail later in the paper.

\(^{15}\) The finiteness of \( u'(0) \) is only a sufficient condition to guarantee the result. As the first example in Section 2.3 below shows, our main result that welfare can decrease in the wage can hold also for utility functions with unbounded \( u'(0) \).
there are two effects on welfare working in opposite directions: a positive effect through the increase in private good consumption and a negative effect through the reduction in the equilibrium public good provision. When the latter effect dominates the former, we obtain the proposition result. Note also that, in the assumed non-cooperative framework, the agents would be unwilling to go back to their previous higher contributions towards the public good as this is unsupportable in equilibrium since each agent would like to free ride on the others.

The proposition result implies also that the negative effect on utility is likely to hold at low wage levels and provides a natural interpretation of what “low” means. As long as labor supply is not backward bending, when the opportunity cost of time for the agent raises from zero to infinity, she optimally re-allocates her labor endowment from a situation where she contributes all her time to the public good to a situation where time contributed to the public good goes to zero. The above results show that, under these conditions, the welfare of the agent will always (weakly) decrease for lower levels of \( w \) (in a neighborhood around the threshold, \( w_t \), defined in the proposition proof). At higher levels of the wage, the increase in private good consumption is likely to offset the negative effect of the decreased level of public good provision. These findings, in our opinion, make the immiserisation result particularly relevant for developing economies. In particular, any of the factors discussed in the introduction that could raise the opportunity cost of contributing is more likely to have deleterious impact on welfare if the initial equilibrium is one with high public good participation.

Unfortunately, it turns out to be hard to characterize analytically the exact range of \( w \) for which Proposition 3 holds. To deal with this, we follow two complementary approaches. First, we provide numerical and graphical examples that illustrate the possibility of an immiserising wage increase in a wide range of settings and for various commonly used types of preferences. Second, we provide a much sharper characterization of the conditions under which wage increases are immiserising for the case of preferences linear in private consumption.

2.2. The linear utility case

It turns out that in the linear utility case, i.e. \( u(c) = c \), it is possible to characterize more precisely the conditions under which Proposition 3 holds as well as the behavior of agent’s welfare as a function of the wage. We show that, for any given \( n \) and under some mild assumptions on \( G \), there always exist threshold wage levels \( w_l(n) \) and \( w_h(n) \) such that the agent’s utility is constant for low wage levels, \( w < w_l(n) \), decreases in \( w \) for \( w \in [w_l(n), w_h(n)] \) and increases in \( w \) for \( w > w_h(n) \). Formally, we show the following:

**Proposition 4.** Suppose Assumption A1 holds and \( u \) is linear:

(a) Proposition 3 applies, so there exist real positive numbers, \( w_m(n) \geq w_l(n) > 0 \) such that agent's indirect utility \( U^*(w) \) is decreasing in \( w \) for \( w \in [w_l(n), w_m(n)] \) and constant in \( w \) for \( w \in [0, w_l(n)] \).

(b) If, in addition, we have that Assumption A2:

\[
\lim_{Y \to 0} \frac{G'(0)}{G''(0)} = t \geq -1,
\]

then there exists a real positive number, \( w_h(n) \), such that \( w_h(n) \geq w_m(n) \) and such that the agent’s equilibrium utility is increasing in \( w \) for \( w > w_h(n) \).

(c) If, in addition to Assumptions A1 and A2, it is also true that Assumption A3:

\[
\frac{G''(Y)G'(Y)}{(G'(Y))^2} \geq 1,
\]

then we have that \( w_m(n) = w_h(n) \), i.e. utility is initially constant, then decreases, and finally increases in the wage, \( w \).
Proof. See Appendix A. □

The sufficient conditions in Assumptions A1–A3 are not too restrictive. For example, it is easily checked that they are satisfied by any function of the CRRA form \( G(Y) = \frac{1}{\alpha} Y^\alpha \) for \( \alpha \in (0, 1) \) and \( G(Y) = \ln Y \). Notice also that, as before, our main result that there exists a range for the wage level in which agent utility is decreasing in \( w \) requires only the very standard assumption (A1). The proposition implies that at low wages all labor is spent on public good production but when wages rise agents start working in private jobs (e.g. leave the village).

![Fig. 1. Effects of a wage increase for various preferences.](image-url)
This relates to the examples of some of the effects of market integration discussed in the empirical literature survey in Introduction.

2.3. Examples and robustness

Take for instance $G(Y) = \sqrt[3]{Y}$ and $u(c) = \sqrt[3]{c}$. It is easy to show that $g(w) = \frac{1}{w^{1/3}}$, which is decreasing in $w$. With some simple algebra, it can be shown that $U^*(w)$ is also decreasing in $w$ whenever $w < \frac{\alpha g(n)}{n}$, which is the equivalent of condition (5) in this case. Notice that this implies that we cannot obtain the result of Proposition 3 for just two agents for any value of the wage.

To be more specific, let us set $n=4$ and compare the equilibrium utility that an agent gets at $w=1/3$ with the level obtained at $w=1/2$. We have $g(1/3)=12/7$, $g(1/2)=4/3$ so

$$U^*(1/3) = 1.7457 > 1.7321 = U^*(1/2)$$

Let us now look at the same example but under linear utility, $u(c)=c$. We obtain $g(w) = \min\{n, \frac{1}{w}\}$, i.e. total provision of the public good is once again decreasing in the wage. Clearly, $G(Y) = \sqrt[3]{Y}$ satisfies all the assumptions in Proposition 4. In this particular example, it is actually very easy to solve for the cutoff points $w_l$ and $w_h$:

$$w_l(n) = \frac{1}{2\sqrt{n}} \text{ and } w_h(n) = \frac{\sqrt{2n-1}}{2\sqrt{n}}$$

Thus, $U^*(w)$ is constant in $w$ for $w < \frac{1}{2\sqrt{n}}$ decreasing in $w$ for $w \equiv \left(\frac{1}{2\sqrt{n}}, \frac{\sqrt{2n-1}}{2\sqrt{n}}\right)$ and increasing in $w$ for $w > \frac{\sqrt{2n-1}}{2\sqrt{n}}$.

Fig. 1 studies the effect of a wage increase on agents’ welfare, consumption and total public good provision. Everywhere we have set $n=10$ and $G(Y)=Y$. The figure shows that our immiserisation result carries through for various preference specifications commonly used in the literature. Apart from the CRRA and linear preferences discussed above, the possibility for an immiserising effect of a wage increase\(^{16}\) is demonstrated to hold also for the cases of CARA (exponential) preferences, i.e. $u(c)=e^{-c}$ and HARA preferences, i.e. $u(c) = (1-\gamma)c^{\gamma-1}$. In these latter cases, the coefficient of relative risk aversion, $\rho(c) = -u''(c)/u'(c)$ varies with consumption and hence with the wage. This is why the public good provision level in the CARA utility case (i.e. increasing relative risk aversion initially goes down (while $\rho<1$) but eventually starts to increase in the wage level. The HARA family allows for both increasing and decreasing relative risk aversion\(^{17}\) depending on the sign of the parameter $\gamma$. Notice also that the separability in the public and private good, which we have assumed to simplify the analysis, is not a necessary condition for the validity of our findings. This is shown in the middle panel of Fig. 1 where we compute the case of non-separable utility given by $U(c,G)=e^{\alpha G}+G^2$ for $\alpha=0.9$.

Observe that in all cases the decrease in utility when wages increase occurs for the low wage levels. This makes the model well suited for developing country contexts—initially as wages increase, say due to increased productivity in the market sector, agents’ welfare may decrease but then it eventually increases. The perverse effect of the wage increase is thus more likely to be seen as a transitional phenomenon in environments with not too high initial wages.

\(^{16}\) The size of this effect is not trivial. For example, Fig. 1 shows that, in the linear utility case, the utility differential resulting from an increase in the wage from 0.2 to 0.7 is equivalent to the welfare loss that would be caused by a 40% decrease in private consumption, holding the public good level constant.

\(^{17}\) The economic literature seems to be split on the question whether relative risk aversion is decreasing or increasing in wealth. Arrow (1965) argues from a theory standpoint that it should increase in wealth, while a lot of the empirical literature (e.g. Ogaki and Zhang, 2002) holds the opposite view.
2.4. The role of population size

In this section, we analyze the effect of population size, $n$, on the contribution equilibrium as well as on the welfare effects that we identify. Let us first look at the effect of $n$ on equilibrium contributions, $Y$, holding the wage level fixed. Suppose the first-order condition, (2), holds at equality and $u$ is not linear. Then, by differentiating both sides with respect to $n$, we obtain:

$$\frac{\partial Y}{\partial n} = \frac{w^2 u'' Y}{G'' + \frac{w}{n} u''} > 0$$

by our assumptions on $G$ and $u$. Thus, for a given wage level, total contributions always increase in population size. In the linear case, we know that for low wage levels (below $w_l(n)$ as defined in
Proposition 4) we have that $Y=n$, so again it is increasing in population size. However, for higher wage levels, the first-order condition for optimum is independent of $n$ and hence $Y$ is the same as long as $w > w_1(n)$ holds when we vary population size. Given that $w_1(n)$ is decreasing in $n$,\footnote{From above, $w_1(n)$ is the solution to $G'(n) = w$; so $\frac{d}{dn} w_1 = G''(n) < 0$.} it is easy to see that in this case $Y$ will be also (weakly) increasing in $n$ holding $w$ constant.

From our results, it is also evident that population size, $n$, plays an important role in determining the range of values for the wage, $w$ in which welfare is decreasing, as well as in determining the strength of the welfare effects, i.e. are they more or less pronounced. In the general case, it is hard to characterize analytically these two effects. Thus, we provide formal results for the case of linear utility supplemented by numerical results for the non-linear case.

Suppose Assumptions A1–A3 hold and $u$ is linear. We already know that the lower bound of the “immiserising range”, $w_0(n)$, is decreasing in $n$. It turns out that, if\footnote{This condition is sufficient but not necessary.} $g(w_0) < 1$ for some given $n$, then $w_0(n)$ increases in $n$ as population size grows above this initial value, i.e. the range in which we observe a negative welfare effect expands (see Appendix A for a proof).

Finally, Fig. 2 analyzes numerically the effect of increasing the number of agents, $n$, on the strength the immiserisation effect in the CRRA case. We see that, as $n$ goes up, the effect becomes more pronounced, i.e. the range of wage levels for which it occurs expands and the relative decrease in welfare can become quite large. The intuition is that free riding increases as the number of contributors goes up which reinforces the negative effect on agents’ utility due to the decreased contributions to the public good.

3. Extensions

3.1. Monetary contributions

In our benchmark theoretical model, we have assumed that agents contribute time towards the public good production and alternative modes of contribution (e.g. monetary, in kind) are ruled out. Our analysis is thus best suited for situations in which interpersonal relationships are important and/or where labor is not easily substituted by money. The examples that we have used to motivate our analysis (cooperation in CPR maintenance, social capital) were chosen as likely to have these properties. In general, however, other forms of contributions may exist, separately or together with labor contributions. In this section, we investigate to what extent our results bear relevance for the case of monetary contributions studied in most of the standard references on voluntary public good provision (Bergstrom et al., 1986; Cornes and Sandler, 1996).

3.1.1. A simple model with monetary contributions only

Let us re-interpret our setting as follows. Assume all agents are endowed with the same amount of wealth normalized to 1 which they can spend either contributing to a pure public good, $G$, or to purchase a private consumption good, $x$, at a price of $p$. Agent $i$’s optimization problem is then\footnote{We use the notation from Bergstrom et al. (1986) who however fix $p=1$.}:

$$\max_{g_i, x_i} v(g_i + G - i) + u(x_i)$$

s.t. $px_i + g_i = 1$
where \( g_i \) is agent \( i \)'s (monetary) contribution towards the public good, \( G_{-i} \) is the total contribution of everyone else and \( x_i \) is the agent’s private consumption. The above problem is equivalent to:

\[
\max_{g_i} v(g_i + G_{-i}) + u\left(\frac{1}{p}(1-g_i)\right)
\]

which is exactly the same as our original problem, (1), with \( w = 1/p \). Thus, what we have said so far about the possibility of a wage increase to bring about a reduction in welfare in a symmetric contribution equilibrium holds for the current setting as well. In particular, under the conditions stated in Propositions 2–4, we can show that an exogenous decrease in the private consumption good price, \( p_x \), can make all agents worse off. Notice, however, that, in order for this to happen, we must have that the fall in the price leads to an increase in private good expenditure, \( p_x \), i.e. the private good must have a price elasticity of demand higher than 1. We see that the general flavor of the results is preserved when one looks at monetary contributions but the interpretation changes.

### 3.1.2. Labor or monetary contributions

One might ask why would the agents not hire someone else to work in the public good sector which would be feasible if the hired person is offered the current market wage. This hiring decision and contributing to the agent’s remuneration, however, would be likely plagued by the same free-riding problem as the original voluntary time contribution and hence would fail for the same reason. A single agent would also never pay \( w \) to someone to work in the public good sector as she could have done this herself at the same opportunity cost but chose not to in the new equilibrium.

Of course, the above argument relies heavily on the homogeneity across agents in our setting. Realistically, agents can be heterogeneous in their wages or wealth in general—this could weaken or even reverse the effect we identify but it will be still present. In general, if agents are heterogeneous, richer agents contribute more to the public good in a monetary contribution setting (Bergstrom et al., 1986). As pointed out by Dasgupta and Kanbur (2005), if agents are allowed to choose between labor and monetary contributions, this result also holds for various functional forms of the public good production function (e.g. quasi-linear in money and labor). However, they also acknowledge (p. 24) that the opportunity cost (substitution) effect that we analyze may lead to reduced contributions in general. In this respect, we view our results as complementary to theirs.

### 3.2. Allowing for leisure time

On a first glance, it may appear that the main result of the paper stated in Propositions 3 and 4 depends crucially on the fact that the agent is “forced” to split his time between private and public good production, and thus, if leisure time is introduced into the model, the result may no longer be true. We show that this is not the case, i.e. even with leisure time included in the agent’s preferences, there could exist an interval (potentially smaller than before) for the wage level in which agent’s utility is decreasing in \( w \). For expositional simplicity, we present only the linear utility case although the result is once again valid for \( u \) not too concave.

21 Note that contributing time or money has the same opportunity cost for all agents in our model.
Formally, suppose that agent’s utility is:

\[ U + G(y + Y - 1) + c + \gamma(l) \]

where \( l \) is leisure, \( y \) is the time contributed to the public good and \( c = (1 - y - l)w \) is private good consumption. The leisure utility function \( \gamma \) is assumed to be increasing and strictly concave and satisfy Inada boundary conditions. The agent’s optimization problem then becomes:

\[
\max_{l, y} G(y + Y - 1) + (1 - y - l)w + \gamma(l)
\]

with first-order conditions for an interior solution:

\[ G'(Y) = w \]
\[ \gamma' l = w \]

Proceeding as before, we get that, in the equilibrium, \( y^* = \frac{g(w)}{n} \) and \( l^* = f(w) \) where \( f(w) = \gamma^{-1}(w) \).

We then obtain:

\[ U^*(w) = G(g(w)) + \left(1 - f(w) - \frac{g(w)}{n}\right)w + \gamma(f(w)) \]

thus, using the first-order conditions:

\[
\frac{dU^*}{dw} = 1 - \frac{g(w)}{n} + \frac{n-1}{n} g'(w)w - f(w)
\]

Since \( g' < 0 \), by continuity, the above expression will be negative for a small enough interval with a lower bound \( w_l \) determined as the solution to \( 1 = \frac{g(w)}{n} + f(w) \). This equation always has a solution when \( G \) and \( \gamma \) are increasing, strictly concave and have a limit of infinity when their respective arguments go to zero. As before, for any \( w < w_l \), agents’ welfare will not depend on \( w \) as the time constraint will be binding and agents will not work in the private good sector.

3.3. Taxation of labor income

The result that higher wages can be detrimental for agents’ welfare implies that a tax imposed on labor income when the wage is in the interval where \( U^* \) is decreasing can be welfare increasing even if its proceeds are thrown away. Of course, the optimal thing to do is to use the tax to restore the first best which can be achieved in the standard way, for instance by imposing a proportional tax, \( \tau = \frac{n-1}{n} \) on labor income and refunding the tax revenue to the agents as a lump sum transfer such that each individual gets a transfer \( T = \tau w(1 - \frac{g(w)}{n}) \) in equilibrium. In such case, the agents’ optimality condition becomes:

\[ G'(Y) = u' \left( w \left(1 - \frac{g(w)}{n}\right) \right) w(1 - \tau) = u'(c) \frac{w}{n} \]

which is exactly the first-order condition corresponding to the first best.\(^{22}\)

\(^{22}\) This is simply a theoretical result—as usual, it may be impossible in reality to assess the right magnitude of this corrective tax so it may be unimplementable. Notice also that, as is common in such settings, the tax rate needed increases in \( n \) and is quite high (50% even in the two-agent case), which makes its use quite unrealistic.
4. Discussion and conclusion

This paper analyses some inherent economic tensions in the interaction between local public goods provision and the improvement in private market opportunities brought about by economic development. We have shown, under some relatively mild conditions, the possibility for a perverse effect whereby an apparently beneficial change (namely an increase in the market wage rate) can actually have welfare worsening consequences. Our findings relate to some of the literature on the effects of privatization in less developed economies. Some authors have argued (e.g. Soros, 2002) that, although the process of privatization has brought increased income in many areas by opening up markets and increasing production efficiency, it has adversely affected local public goods provision which is often characterized by private contributions. This paper indicates one possible channel through which this outcome might take place. Our analysis also contributes to the literature on market integration and its effects on village level activity (Baland and Platteau, 1996).

An interesting question is the potential empirical relevance of our theoretical analysis, in particular the main result that exogenous increases in the wage level may be welfare reducing. Remember that a necessary condition for the latter to occur is that rising wages be accompanied by a reduction in the level of public good provision. This is a potential testable implication of the model and we have already discussed in the introduction some real world evidence consistent with our results. Unfortunately, it is hard to test directly the welfare predictions of our theory lacking a direct welfare measure, although indirect measures of well-being such as life expectancy and various quality of life indices may offer some possibilities in this respect.

Our result that a decrease in welfare when wage levels rise is only possible when local public good provision is falling at the same time has some potential policy implications. We have seen above (e.g. Fig. 1) that this may only be a transitional phenomenon accompanying development and wage growth, but nevertheless it should deserve the policymakers’ attention. More specifically, our analysis suggests that the development process driven by opening up markets and/or technological adoption should be accompanied by measures ensuring that sufficient attention is paid to sustaining or improving the provision of local public goods.

In this paper, we have only looked at a single facet of “market integration”, namely the fact that it can lead to changes in agents’ opportunity cost of time. In general, the market can have many additional effects on CPR conservation or social capital that we do not address here but which deserve full attention. For example, by opening up new income-earning possibilities, market integration “…increases the likelihood that it will not any more be optimal for individual producers to conserve local natural resources…they may instead be induced to deplete them to a shut-down point” (Baland and Platteau, 1996). It can also lead to erosion of traditional authority by opening new avenues for social and economic mobility and propagating the view that anyone endowed with the proper skills can succeed. Jodha (1992) for instance finds that the decline in CPR is less pronounced in villages with a lower degree of commercialization, presumably because there is less erosion of social sanctions and informal arrangements protecting CPRs. The same argument clearly applies to social capital. Relatedly, Zufferey (1986) finds that a common outcome of the diversification and mobility brought about by development is the erosion of kinship ties. To the extent that these ties are a basic building block of social networks and discipline, this process can further erode the provision of public goods of the kind analyzed here.
We conclude by suggesting some potential areas to which our work can be extended. First, our model features only private provision of a pure public good which is the standard approach in the theoretical public goods literature (Bergstrom et al., 1986; Cornes and Sandler, 1996). Recently, there has been a lot of work where the provision of public goods is modeled in a richer framework. For example, Perrson and Tabellini (2003) consider scenarios where the public good is provided under different political regimes and analyze the corresponding effects on the provision level. It would be interesting to consider such a richer formulation of the process of public good provision and to investigate its effect on the results of this paper. We have also intentionally abstracted from the issue of government provision. Following the work of Besley and Ghatak (2001), it should be interesting to see whether a public–private combination of the public goods provision might influence the results pointed out in this paper. Second, to simplify things, we have concentrated on a homogeneous agents model. One might think of incorporating heterogeneity and study the distributional consequences of an increase in wage rate similarly to Dasgupta and Kanbur (2005). One simple way of doing this might be to consider agents differing in terms of their time endowments (e.g. sick vs. healthy) or non-labor income endowments. In a private public good provision framework, the issue of how inequality effects the provision level and welfare has been recently investigated in the works of Bardhan (2000), Bardhan and Dayton Johnson (2002) and Bardhan et al. (in press).

Finally, our theoretical setting can be extended to build a more complete model emphasizing the role of public goods for economic development by endogenizing the raise in w. In particular, our findings could have predictions for a “poverty trap” type of scenario in the following way. We have seen above that it is possible to have rising wage rates concomitantly with decreasing public goods provision. In a model where the development process explicitly incorporates voluntary provided public goods, one can think of a dynamic story where wage rates never rise enough to bring about increased welfare (remember that in our model we have a cut-off wage above which utility increases).

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Appendix A

Proof of Proposition 1. Consider first the case of finite \( u'(0) \). Notice that the total time contributed towards the public good production cannot exceed \( n \); thus, when the wage is low, namely \( w < \frac{G'(n)}{u'(0)} \), all agents’ time will be spent in the public good production and (2) will hold with inequality. In this case, the unique symmetric equilibrium is \( y^* = 1 \) for all \( i \). If the wage is higher than \( \frac{G'(n)}{u'(0)} \), the equilibrium total contribution, \( Y^* \equiv g(w) \) solves the f.o.c. as equality. Notice that by Assumption A1 such \( Y^* \) will always exist and be unique, since the l.h.s. of (2) is decreasing in \( Y \), the r.h.s. is increasing in \( Y \) and the r.h.s. is larger than the l.h.s. when both are evaluated at \( Y = n \).

Now suppose that \( \lim_{c \to 0} u'(c) = \infty \). The Inada conditions on \( G \) in Assumption A1 and the strict concavity of the two functions ensure the existence of a unique \( Y^* \equiv g(w) \) solving (2) taken as...
equality, given that its left hand side is decreasing in \( Y \) while the right hand side is increasing in \( Y \) with a limit of infinity\(^{23} \) as \( Y \rightarrow n \). The symmetric Nash equilibrium is then \( y_i^* = \frac{g(w)}{n} \) for all \( i \). \( \square \)

**Proof of Proposition 4.** Notice first that \( g'(w) \) is always negative in the linear utility case.

(a) \( u'(0) \) is finite for linear \( u \) so Proposition 3 applies indeed.

(b) We will show that the derivative \( dU^* \) becomes positive as \( w \) goes to infinity, which implies that there exists a finite number \( w_h(n) \) such that \( \frac{dU^*}{dw} > 0 \) for \( w > w_h \). Indeed, under linear utility, we have \( g(w) = G'^{-1}(w) \) from the agent’s f.o.c., so as \( w \rightarrow \infty \) we have, by Assumption A1 and using that \( G' \) is a continuous monotonically decreasing function, that \( g(w) \rightarrow 0 \). We also have \( g'(w) = 1/G''(g(w)) \) so that \( wg'(w) = G'(g(w)) \), thus

\[
\frac{dU^*}{dw} = 1 - \frac{g(w)}{n} + \frac{(n-1)}{n} \cdot wg'(w) \rightarrow 1 + \frac{n-1}{n} \cdot t > 0
\]

by Assumption A2. Notice that the restriction on \( t \) is sufficient as well as necessary.\(^{24} \) If \( t < -1 \), one can always find a large enough \( n \) such that, in the limit for \( w \rightarrow \infty \), \( \frac{dU^*}{dw} < 0 \). Once again, by a continuity argument, the derivative must be positive for any large enough \( w \).

(c) Under Assumption A3, we can actually show that \( \frac{dU^*}{dw} \) is strictly increasing in \( w \) and thus it crosses the horizontal axis at a unique point \( w^* \equiv w_m(n) = w_h(n) \). The derivative of \( U^* \) will then be negative for \( w \in [w_l, w_h) \) and positive for \( w \geq w_h \). To see that, differentiate \( U^* \) twice with respect to \( w \):

\[
\frac{d^2 U^*}{dw^2} = -\frac{g'(w)}{n} + \frac{(n-1)}{n} \left[ g'(w) + wg''(w) \right] = -\frac{g'(w)}{n} + \frac{(n-1)}{n} \cdot g'(w) \left[ 1 - \frac{G'G''}{(G'')^2} \right] > 0
\]

since, for linear utility:

\[
\frac{wg''(w)}{g'(w)} = G'(g(w))G''(g(w)) \frac{1}{dw} G''(g(w)) = -\frac{G'(g(w))G''(g(w))}{[G''(g(w))]^2}
\]

where we have also used that \( g'(w) = \frac{1}{G'(g(w))} < 0 \) by Assumption A1. \( \square \)

Effect of \( n \) on \( w_h(n) \) in the linear utility case.

Fix \( n \) and define \( \hat{w} \equiv w_m(n) \) as the wage level solving \( \frac{\partial U^*}{\partial w} \), i.e.:

\[
1 - \frac{g(\hat{w})}{n} + \frac{n-1}{n} \cdot \hat{wg}'(\hat{w}) = 0
\]

Suppose that \( g(\hat{w}) < 1 \). Then, by plugging in \( \hat{wg}'(\hat{w}) = \frac{g(\hat{w})-n}{n-1} \), it is easy to verify that

\[
1 - \frac{g(\hat{w})}{n+1} + \frac{n}{n+1} \cdot \hat{wg}'(\hat{w}) = 1 - \frac{g(\hat{w})}{n+1} + \frac{n}{n+1} \cdot \frac{g(\hat{w})-n}{n-1} < 0
\]

This implies that the derivative \( \frac{\partial U^*}{\partial w} \) is negative for \( w = \hat{w} \) at \( n+1 \). By continuity and by the fact that the derivative is negative (see Proposition 4) for wage values \( w < w_h(n+1) \), this implies that we must have \( w_h(n+1) > w_h(n) \). Combined with the fact that \( w_l(n) \) is always decreasing in \( n \), this

\(^{23} \) The only exception is when \( w = 0 \) in which case we obtain \( g(w) = n \) and the right hand side is finite.

\(^{24} \) We thank an anonymous referee for this observation.
implies that under these conditions the range of \( w \) in which negative welfare effect is observed grows with population size.

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