A Case for Bundling Public Goods Contributions

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Abstract

We extend the model of voluntary contributions to multiple public goods by allowing for bundling of the public goods. Specifically, we study the case where agents contribute into a common pool which is then allocated toward the financing of two pure public goods. We explore the welfare implications of allowing for such bundling vis-à-vis a separate contributions scheme. We show that for high income inequality or for identical preferences among agents bundling leads to higher joint welfare. Interestingly, a welfare improvement can in some cases occur despite a *decrease* in total contributions. On the contrary, when agents are heterogenous, for low income inequality bundling can lead to lower total contributions and may decrease welfare compared to a separate contribution scheme. Our findings have implications for the design of charitable institutions and international aid agencies.

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1. Introduction

This paper analyzes the following question. In a situation where agents contribute voluntarily to a single private good and multiple pure public goods, can there be a welfare improvement if contributions to the public goods are bundled? More precisely, can people be better off if, rather than contributing to the multiple public goods separately, they contributed money into a common pool that is then divided among the public goods according to a preannounced sharing rule? We find that when agents have identical preferences they can be made (weakly) better off by such a joint contribution scheme regardless of the resource heterogeneity among them. On the contrary, when there is heterogeneity among the agents in their preferences toward the public goods, then there are plausible scenarios where a joint contribution scheme might increase or decrease social welfare. Moreover, the efficacy of the joint scheme depends on the extent of resource heterogeneity across agents.

There exist numerous real world examples of multiple public goods that are voluntarily supplied at the same time. National governments simultaneously contribute to many international public goods, many individuals contribute to several charitable causes, local sports clubs, etc. at a time. For instance, consider disaster aid, for example, the 2005 tsunami disaster in Asia. There are, broadly speaking, three possible uses for aid—rescue, rehabilitation, and relief, all of which are important but not always "optimally" provided. Rehabilitation and relief have strong public good aspects and are financed to a large extent (especially in poorer countries) with voluntary donations that are often mediated by international nonprofit organizations, such as the United Nations (UN) or the Red Cross.

Our model also has implications for the design of organizations based on voluntary giving. For example, the United Way of America, a leading charity organization, collected 4.4 billion dollars in the year 2002–2003 from various donors. These funds were then allocated for various projects aimed at "... helping children and youth succeed, strengthening and supporting families, promoting self-sufficiency, building vital and safe neighborhoods, and supporting vulnerable and aging populations."[1] A pertinent question to our analysis is, Should the donors be allowed to make project-specific contributions instead? As another example, university alumni can make contingent donations to specific areas, say for the college football team or to a particular academic unit. A further, though less immediate, application can be the area of campaign contributions. Voters or firms could contribute money to parties or politicians who then allocate it toward supporting various issues of public good character within a given industry. Alternatively, as required by the recent McCain–Feingold Bill, one could put a ban on the soft money received by parties while allowing direct contributions to candidates. Our framework

[1]See http://national.unitedway.org/aboutuw/ciagenda.cfm.

could be used to shed light on the welfare implications of these alternative schemes.

Our theoretical setting is an economy with two agents and two public goods. The agents have certain resource (e.g., wealth) endowments that they can use for contributing to the public goods or private consumption. We first study the case of separate provision, where agents simultaneously decide to contribute separately to the two public goods. Next we analyze the equilibrium under joint contribution where each agent makes a total contribution toward the two public goods given an allocation rule in which this total contribution will be divided between the two public goods. We show that the welfare results for the two schemes depend on the two sources of heterogeneity in our model. For identical preferences or for heterogeneous preferences and high wealth inequality among the agents, the joint contribution scheme performs better in terms of total welfare than the separate contribution scheme. However, joint provision does not Pareto dominate separate provision. In contrast, with low wealth inequality and preference heterogeneity, it is possible that separate provision leads to higher welfare.

Addressing the pertinent free-riding problems associated with public good provision, this paper proposes a simple mechanism that can raise social welfare by requiring total voluntary contributions to be distributed between the two public goods in a way closer to what would be the first best allocation. Under separate provision, and especially when wealth inequality is high, the richer agent's preferences determine to a large extent the provision level of the two public goods (simply because he is the one contributing the most toward them). This may be socially suboptimal in a utilitarian framework where poorer agents have large enough weight in the social welfare function.

In general, there are several channels through which bundling contributions can lead to higher social surplus. First, in the generic case where the sharing rule affects the contribution amounts,[2] bundling can elicit higher total contributions which under some conditions transforms into higher welfare. Second, if the ex ante sharing rule is chosen to maximize the (ex post) social surplus resulting in equilibrium under the joint scheme, then it plays a redistributive role (i.e., changes the mix of public goods provided) that can offset some of the social inefficiency in the separate scheme caused by the differences between the players' valuations and the differences in their abilities to contribute.[3] Finally, joint provision can elicit contribution from some agents who would not contribute to a given public good under a separate provision scheme. This creates positive externalities for all other agents and can help increase joint welfare.

[2]As opposed to the homothetic preferences case when contributions are independent of the sharing rule (see Section 2).

[3]In a related paper, Cornes and Itaya (2004) show that the public good mix arising in a Nash equilibrium is suboptimal relative to the first best. The joint contribution scheme can raise social welfare by realigning provision toward the social optimum.

Interestingly, in our second best environment, a policy such as bundling, may in some cases (see Section 4) increase social welfare despite *reducing* total contributions to the public goods. This prompts a cautionary point about the "standard" presumption that, since public goods are normally underprovided due to free riding, a raise in total contributions must automatically increase welfare. While this intuition certainly holds in the single public good model commonly used in the literature, we show how it could fail when there are more public goods.

The majority of the existing literature on privately provided public goods, initiated by the seminal contributions of Bergstrom, Blume, and Varian (1986), Cornes and Sandler (1985) and Andreoni (1988) among others, have primarily concentrated on the single public good model. The question of private provision of multiple pure public goods has been briefly addressed in the past by Bergstrom, Blume, and Varian (1986) and analyzed recently in more detail by Cornes and Itaya (2004). These papers, however, concentrate on the issue of "distribution neutrality" (see also Warr 1983) in such a setting, i.e., under what conditions total provision is independent of the agents' resource endowment distribution.

Some of the questions that we address in this paper relate to the literature on "united charities," that is, centralized fund-raising agencies that provide multiple public goods (Bilodeau 1992, Bilodeau and Slivinski 1997). Bilodeau (1992) looks at a situation where individuals are allowed to contribute simultaneously to individual charities (providing a single public good) and to a united charity and shows that "contributing only to the united fund" can be a sub-game perfect equilibrium if the fund plays after all contributions are made and is able to offset direct contributions to individual charities. Bilodeau and Slivinski (1997) also study nonprofit organizations that may provide bundles of public goods but focus on the public good supply side, namely the rivalry between such organizations for donations, showing that, under some conditions, they will tend to specialize in the public goods they provide since diversification diminishes the equilibrium level of contributions collected. These papers are mostly interested in total or individual contribution levels and the possible commitment issues arising due to the fact that charities' own preferences toward the public goods may differ from contributors' ones. As such they do not provide a formal analysis of the individual and social welfare implications of joint versus separate provision schemes as well as the explicit role of heterogeneity that we focus upon.

More generally, our results relate to the literature on private goods bundling (Adams and Yellen 1976, McAfee et al. 1989) or excludable public goods bundling such as TV or access to electronic libraries (Fang and Norman 2004). Fang and Norman analyze the constrained optimal mechanism of public good provision in a model where agents' valuations are private information and show that it involves bundling. The intuition is that bundling alleviates the free riding problem in large economies by decreasing the extent of use exclusions. In contrast, our results apply to pure (nonexcludable) public goods where agents' preferences are public knowledge. In a related paper, Heal (2001) looks at bundling of public with private goods (e.g., by real estate developers) and shows that this can increase welfare by ensuring efficient level of public good provision (see also Fraser 2005), while Flores (1999) demonstrates how bundling of voting proposals about public good provision can be used to ensure passage or defeat.

2. Model

We consider private provision of multiple public goods as in Bergstrom, Blume, and Varian (1986) and Cornes and Itaya (2004). There are two agents indexed by $i = 1, 2$. Each agent divides her wealth $M_i > 0$ between private consumption, c_i, and contributions toward two pure public goods, G and H.[4] Let $M = M_1 + M_2$. Preferences are given by $u^i(G, H, c_i)$. Following the literature, the agents are assumed to act simultaneously and noncooperatively, and we use Nash equilibrium as the solution concept.

We compare public good provision and welfare under two different public good provision schemes: a "separate provision scheme" (hereafter SP) where the two agents are free to contribute any amount they like (including zero) to each of the two public goods, and a "joint provision scheme" (hereafter JP) where each agent makes a nonnegative contribution to a common fund, the total proceeds from which are then distributed between the two public goods using a preannounced fixed sharing rule.[5]

2.1. Separate Provision (SP)

Under separate provision, each agent decides on his contributions $g_i \geq 0$ and $h_i \geq 0$ toward the two public goods. The total supplied amount of the first public good is then given by $G = g_1 + g_2$ and the total supplied amount of the second public good is given by $H = h_1 + h_2$. Let $z_i = g_i + h_i$ denotes total contribution by agent i. The maximization problem of agent i is

$$\begin{aligned} &\max_{g_i, h_i, c_i} u^i(g_i + G_{-i}, h_i + H_{-i}, c_i) \\ &\text{s.t. } c_i + g_i + h_i = M_i, \\ &g_i, h_i, c_i \geq 0, \end{aligned} \tag{SP}$$

where, as usual, G_{-i} and H_{-i} denote total contributions by all agents different from i. Existence of a Nash equilibrium in the above setting is easily shown as in Bergstrom et al. (1986).

[4]Throughout the paper we assume that the prices of the private as well as the public goods are unity. This assumption is without any loss of generality.

[5]We assume that it is possible to credibly commit to any preannounced sharing rule. See Section 4.3 for a further discussion on this point.

2.2. Joint Provision (JP)

Under the joint provision scheme the two agents do not contribute separately to each public good as in the previous case, but instead decide on a total contribution amount $z_i \geq 0$. These total contributions are divided by a third party,[6] hereafter referred to as the "public good provider," using the following sharing rule which is announced to the agents before they make their contribution decisions. The share of the total contribution, $Z = z_1 + z_2$, which goes to the public good G is given by $\lambda \in [0, 1]$ and the share of total contribution that goes to H is given by $1 - \lambda$. In the following analysis we assume that the provider chooses λ to maximize the equilibrium social surplus (defined as the sum of utilities[7]) obtained when agents take λ as given. We also discuss the case when λ could be chosen to maximize total provision Z. Given λ, agent i's optimization problem is

$$\max_{z_i, c_i} u^i(\lambda(z_i + Z_{-i}), (1-\lambda)(z_i + Z_{-i}), c_i)$$

$$\text{s.t. } z_i + c_i = M_i,$$

$$c_i, z_i \geq 0. \tag{JP}$$

2.3. Equilibrium Conditions

The above formulation allows us to consider both homogeneous and heterogeneous preferences. We find that the two cases have qualitatively different implications. We make the following assumptions regarding agents' preferences.

ASSUMPTION A1: *For each agent i=1,2, the function* u^i (G, H, c) *is: (i) strictly concave and twice continuously differentiable with* u_1, u_2, $u_3 > 0$ *on* $\Re^3_{++}$, *(ii) it satisfies the Inada conditions:* $\lim_{G\to 0} u^i(G, H, c) = \infty$, *similarly for H and* c_i; *and (iii) both G and H are normal goods for each agent.*[8]

The above assumption encompasses a wide variety of utility functions used in the literature, and all our qualitative results hold for any functional

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[6]Who might be United Way, the Red Cross, the elders in a commune, the village headman, the head of a religious community, etc.

[7]Although we derive our results under a utilitarian social welfare function, it will become clear that the main findings also go through under any function featuring a small enough degree of inequality aversion.

[8]This is a standard assumption in the private provision literature (e.g., see Bergstrom et al. 1986, p. 32). Alternatively, Proposition 1 obtains (the proof is available from the authors) if we assume that u_{13}, $u_{23} \geq 0$ and $u_{33} < 0$ on $\Re^3_{++}$, which, however, imposes cardinality conditions on u.

forms satisfying it. We also analyze the special cases of additive separability and homothetic preferences. This enables us to clarify further the intuition behind our results. Finally, under Cobb–Douglas utility, $u = \alpha \ln G + \beta \ln H + \ln c$ (used by Cornes and Itaya 2004), we obtain a closed-form characterization of the contribution equilibria (see Section 5).

The first-order conditions in the separate provision case are

$$
\begin{aligned}
&[g_1]\, u_1^1(G, H, M_1 - g_1 - h_1) \le u_3^1(G, H, M_1 - g_1 - h_1),\\
&[h_1]\, u_2^1(G, H, M_1 - g_1 - h_1) \le u_3^1(G, H, M_1 - g_1 - h_1),\\
&[g_2]\, u_1^2(G, H, M_2 - g_2 - h_2) \le u_3^2(G, H, M_2 - g_2 - h_2),\\
&[h_2]\, u_2^2(G, H, M_2 - g_2 - h_2) \le u_3^2(G, H, M_2 - g_2 - h_2),
\end{aligned}
\tag{1}
$$

where the inequalities are strict when the agent is not contributing, that is, g_i or h_i is zero. The subscripts denote partial derivatives and the superscripts denote agents. Similarly, the first-order conditions under joint provision are

$$
\begin{aligned}
&[z_1]\ \lambda u_1^1(\lambda Z, (1-\lambda)Z, M_1 - z_1) + (1-\lambda) u_2^1(\lambda Z, (1-\lambda)Z, M_1 - z_1)\\
&\quad \le u_3^1(\lambda Z, (1-\lambda)Z, M_1 - z_1),\\
&[z_2]\ \lambda u_1^2(\lambda Z, (1-\lambda)Z, M_2 - z_2) + (1-\lambda) u_2^2(\lambda Z, (1-\lambda)Z, M_2 - z_2)\\
&\quad \le u_3^2(\lambda Z, (1-\lambda)Z, M_2 - z_2),
\end{aligned}
$$

again with strict inequalities when an agent is not contributing.

Notice that when u^i is homothetic the individual contributions, z_1 and z_2, are independent of λ since the demand for the private good, $M_i - z_i$, is a constant fraction of M_i no matter how the rest of the wealth is split on the other two goods (the Engel curve is a straight line through the origin). As we show following, this property enables us to obtain a very easy analytical characterization of the contribution equilibrium. In general, the contributions z_1 and z_2 will depend on the sharing rule, λ, which is an additional factor creating a wedge between the provision levels under the joint and separate schemes. The main result of the paper that social welfare under joint provision can be higher than under separate provision remains robust under these alternative specifications.

3. Homogeneous Preferences for Public Goods

We first characterize the equilibria that arise when the agents' preferences toward the two public goods are homogenous, i.e., $u^1(G, H, c) = u^2(G, H, c) = u(G, H, c)$. We then compare the welfare consequences in the separate provision case and the joint provision case. Without loss of generality assume $M_1 \ge M_2$.

PROPOSITION 1: *Under preference homogeneity,*

(a) *there exist only two types of equilibria under both the separate and joint provision schemes:*
   (i) *both agents contribute to both goods if their wealths are not too unequal;*
   (ii) *only the richer agent contributes if agents' wealths are sufficiently unequal;*

(b) *when both agents contribute, their utilities at the equilibrium allocation are equalized.*

Proof: (a) Start with the SP case. We will prove that either all four FOCs hold at equality (i.e., both agents contribute positive amounts to each public good) or only agent 1's FOCs hold at equality (i.e., the poorer agent 2 does not contribute). The latter case is an equilibrium whenever M_2 is small enough relative to M_1 such that $u_3(G, H, M_2) > u_3(G, H, M_1 - g_1^S - h_1^S)$ where g_1^S and h_1^S solve agent 1's first-order conditions (as equalities) at $G = g_1$ and $H = h_1$, that is, when the poorer agent's marginal utility of an extra unit private consumption is higher than that of contributing this unit to either public good given the richer agent's contributions.[9]

Suppose, on the contrary, that the richer agent does not contribute in an equilibrium, i.e., $z_1 \equiv g_1 + h_2 = 0$. Then, it must be that $z_2 > 0$ with $g_2, h_2 > 0$. Proceeding as in Bergstrom, Blume, and Varian (1986) hereafter BBV, it is convenient to rewrite the consumer's problem as

$$\max_{c_i, G, H} u^i(G, H, c_i)$$
$$\text{s.t. } c_i + G + H = M_i + G_{-i} + H_{-i},$$
$$G \geq G_{-i} \text{ and } H \geq H_{-i},$$

which, ignoring the inequalities, is the standard problem of a consumer with income $M_i + G_{-i} + H_{-i}$ who consumes the three goods, G, H, and c. The concavity, Inada conditions, and normality in Assumption 1 ensure the existence of strictly increasing demand functions $f_G(\cdot)$ and $f_H(\cdot)$ representing the amount of G and H the consumer would choose given her "full income" on the right-hand side, $M_i + G_{-i} + H_{-i}$ if the inequality constraints do not bind. Denote by $\phi_G(\cdot)$ and $\phi_H(\cdot)$ the inverse functions of f_G and f_H respectively. As in fact 4 on p.37 of BBV, the above implies that, in equilibrium, for $i = 1, 2$ we have $f_G(M_i + G_{-i} + H_{-i}) \leq G$ with equality if $g_i > 0$ and similarly, $f_H(M_i + G_{-i} + H_{-i}) \leq H$ with equality if $h_i > 0$. This is equivalent to

[9]We derive the exact condition on M_1 and M_2 for the Cobb–Douglas case in Section 5.

$$M_i - z_i \le \phi_G(G) - G - H \quad \text{with equality if } g_i > 0,$$
$$M_i - z_i \le \phi_H(H) - G - H \quad \text{with equality if } h_i > 0. \qquad (2)$$

Now if agent 1 does not contribute ($z_1 = 0$) but agent 2 does ($z_2 > 0$), the above inequalities are violated since $M_1 \ge M_2$. Hence, it must be that $z_1 > 0$, i.e., the richer agent always contributes in equilibrium. Next, note that it is impossible that agent 1 contributes to only one of the goods but not the other, e.g., $g_1 > 0$ but $h_1 = 0$. Since $H > 0$, it must be $h_2 > 0$. Now if also $g_2 > 0$, (2) implies $c_1 = c_2$ (since $c_i = M_i - z_i$) so agent 1 must contribute to H too. If instead $g_2 = 0$, the first inequality in (2) implies $c_1 > c_2$ but the second implies $c_2 > c_1$, which is a contradiction.

Finally, we show that when $z_2 > 0$ it is impossible that agent 2 contributes to one public good but not the other. This would imply again that $c_1 = c_2$ in equilibrium so $u_j^1 = u_j^2$ for $j = 1, 2, 3$, i.e., if $z_2 > 0$ agent 2 must contribute to both goods. The only remaining possibility is $z_2 = 0$ in which case both inequalities above are strict and $c_2 < c_1$ in equilibrium. This happens if M_2 is very small relative to M_1.

We can use the same method in the JP case to rewrite the consumer's problem as one of choosing c_i and Z. The assumed normality of both public goods implies that there exists a strictly increasing demand function for Z, $f_Z(\cdot)$ with argument the agent's full income and its inverse, $\phi_Z(Z)$. Thus, as before, we have in equilibrium (see fact 4 in BBV)

$$M_i - z_i \le \phi_Z(Z) - Z \ \text{ with equality if } z_i > 0.$$

We see immediately that we cannot have $z_2 > 0$ but $z_1 = 0$ so either both agents contribute or only the richer one (if M_2 is small). Also, note that if both contribute then $c_1 = c_2$.

(b) The result follows directly from the fact that the agents consume the same amount of both public goods and the private good in an equilibrium where both contribute (see part (a)). ■

Proposition 1 shows that, when both agents have identical preferences, there does not exist an equilibrium in which they contribute to different public goods—if an agent is a contributor overall, he must contribute to *both* public goods. This is crucial for the next results as it means that joint provision can (at least from a feasibility point of view for the moment) implement the separate provision allocation. Clearly, this would not have been possible for an SP equilibrium in which, for instance, agent 1 contributes only to G while agent 2 contributes only to H. As we see from the proof, the intuition behind this result comes from the assumed (weak) complementarity between the public and private goods and the strict concavity in the private good. The proposition statement is easiest to understand in the separable preferences case: $u(G, H, c) = f(G, H) + v(c)$. The reason that only the richer agent contributes when wealth inequality is high comes from the fact that given the public good character of G and H the poorer agent cannot equalize her marginal utilities even when spending all her income on the private good.

The next result shows that it is always possible to replicate the SP equilibrium outcome under a joint provision scheme.

PROPOSITION 2: *For each possible equilibrium outcome of the separate provision regime* (G^S, H^S, c_1^S, c_2^S), *under the sharing rule* $\lambda^S = \frac{G^S}{G^S+H^S}$ *the same outcome is equilibrium under the joint provision scheme.*

Proof: We know from Proposition 1 that in any equilibrium of the (SP) regime, either (i) $z_1 > 0$, $z_2 > 0$ (both agents contribute to each public good) or (ii) $z_1 > 0$, $z_2 = 0$ (only the richer agent contributes to both public goods). Consider case (i). We know that for $i = 1, 2$, $u_1^i = u_2^i = u_3^i$ (all four FOCs hold at equality). It follows that, evaluated at the SP equilibrium allocation,

$$\lambda u_1^i + (1-\lambda)u_2^i = u_3^i$$

for any λ. The SP FOCs, (1) imply that $(\lambda^S, z_1^S, z_2^S)$ where $z_1^S = g_1^S + g_2^S$ and $z_2^S = h_1^S + h_2^S$ satisfy the JP FOCs. Clearly then, under the sharing rule λ^S, the equilibrium outcome of the (SP) regime is also an equilibrium under joint provision. The other case is analogous. ■

The above result clearly implies that we can always choose λ so that the JP scheme achieves the same total provision level and the same aggregate welfare as the SP allocation. The next step is to see whether bundling the contributions can actually improve over the separate contribution equilibrium. We will look at this question from several angles—total contributions, Z as well as individual, $u(G, H, c_i)$, and joint welfare (defined as the sum of individual utilities). Let $W^J(\lambda)$ denotes the equilibrium social surplus under JP. We have

$$W^J(\lambda) = \sum_{i=1,2} u\big(\lambda Z^*(\lambda), (1-\lambda)Z^*(\lambda), M_i - z_i^*(\lambda)\big),$$

where $Z^*(\lambda)$, $z_1^*(\lambda)$, and $z_2^*(\lambda)$ are the equilibrium contributions as functions of λ . Taking the derivative with respect to λ and using that the richer agent always contributes, and Proposition 1(b), we obtain

$$\frac{dW^J(\lambda)}{d\lambda} = \big(u_1^1 - u_2^1 + u_1^2 - u_2^2\big)Z^*(\lambda) + \big(\lambda u_1^2 + (1-\lambda)u_2^2\big)\frac{dZ^*(\lambda)}{d\lambda}, \quad (3)$$

where once again we use superscripts only to denote differences in the function arguments. In the case when agent 2 does not contribute, we have $z_2^* = 0$, so $z_1^* = Z^*$.

PROPOSITION 3:

(a) *In the case of general nonhomothetic preferences satisfying (A1), there exists a sharing rule* λ^* *that yields higher social surplus under JP compared to that under SP and a sharing rule* λ^{**} *that yields higher total provision. Moreover, there can exist sharing rules such that both social surplus and total contributions are higher under JP compared to under SP.*

(b) *In the case of homothetic preferences separable in the private good, the sharing rule* λ^* *maximizing social surplus under JP also maximizes total provision,* Z *and* λ^* *equals the implied "SP share,"* $\lambda^S = \frac{G^S}{G^S+H^S}$, *where* G^S *and* H^S *are the equilibrium provision levels under separate provision.*

Proof: (a) In this general case z_i depend on the share λ. Evaluate the derivative (3) at $\lambda = \lambda^S$. We have

$$\frac{dW^J(\lambda)}{d\lambda}\bigg|_{\lambda=\lambda^S} = 2\big(u_1^2 - u_2^2\big)Z^*(\lambda^S) + \big(\lambda^S u_1^2 + (1-\lambda^S)u_2^2\big)\frac{dZ^*}{d\lambda}\bigg|_{\lambda=\lambda^S}. \quad (4)$$

In the case where both agents contribute the first term is zero so we get

$$\frac{dW^J(\lambda)}{d\lambda}\bigg|_{\lambda=\lambda^S} = u_3^2\frac{dZ^*}{d\lambda}\bigg|_{\lambda=\lambda^S}.$$

Now only if $\frac{dZ^J}{d\lambda}|_{\lambda=\lambda^S} = 0$ (which happens for u homothetic, for example) and because of the local concavity, we have that welfare is maximized at λ^S. In general, however, if $\frac{dZ^J}{d\lambda}|_{\lambda=\lambda^S} \neq 0$ then some other sharing rule λ will maximize total provision, Z under JP, e.g., if the derivative is positive then some $\lambda^* > \lambda^S$ will maximize Z^J and also (since $u_3 > 0$) the same λ^* will maximize W^J (given that $\frac{dZ^*}{d\lambda}|_{\lambda=\lambda^J} = 0$). The opposite is true if the derivative of Z is negative. Clearly, there exists a whole interval of values for the share in a local neighborhood of λ^S that yields higher contributions and aggregate welfare than the SP regime.

When only the richer agent contributes, the previous argument applies also to the case of utility separable in private consumption since then $u_1 = u_2 > 0$. In the nonseparable case the sign of the derivative (4) cannot be determined but, again, in general it will be different from zero at λ^S so choosing a different share (higher than λ^S if the derivative is positive and lower if it is negative) would increase welfare. In this nonseparable case, the shares maximizing welfare and total provisions do not have to be the same.

To complete the argument, we need to check whether $\frac{dZ^J}{d\lambda}|_{\lambda=\lambda^S}$ is not identically zero for $\lambda = \lambda^S$ for general u. Take the separable case, for example. Consider the equilibrium where both players contribute under JP (the other case is analogous). Differentiate the FOCs with respect to λ and solve for $\frac{dZ^*}{d\lambda} = \frac{dz_1^*}{d\lambda} + \frac{dz_2^*}{d\lambda}$ to obtain

$$\frac{dZ^*}{d\lambda} = \frac{2\left[f_2 - f_1 + (1-\lambda) Z^* f_{22} - \lambda Z^* f_{11}\right]}{2\left[\lambda^2 f_{11} + (1-\lambda)^2 f_{22}\right] + v''\left(M_1 - z_1^*\right) + v''\left(M_2 - z_2^*\right)}.$$

Clearly this expression is not identically zero for $\lambda = \lambda^S$ (at the SP equilibrium) unless for some reason $(1-\lambda) Z^S f_{22} = \lambda\ Z^S f_{11}$. The sufficient condition is even more involved in the nonseparable case.

(b) In the homothetic preferences case, we have $\frac{dZ_1^*(\lambda)}{d\lambda} = \frac{dZ_2^*(\lambda)}{d\lambda} = \frac{dZ^*(\lambda)}{d\lambda} = 0$ as the optimal contribution is independent of λ (the fraction of income spent on the private good is a constant depending only on preferences). Evaluate the derivative (3) at $\lambda = \lambda^S$, i.e., at the SP equilibrium allocation. Because of the separability of preferences and the SP FOCs $u_j^1 = u_j^2 = v_j(G^S, H^S)$, $j = 1, 2$, so the first term in the derivative is always zero. Hence it is clear that the whole derivative is zero. Under our assumptions W^J is concave in λ ($\frac{\partial^2 W^J}{\partial \lambda^2} < 0$ since u_{11}, u_{22}, and $-u_{12} < 0$), thus λ^S maximizes social surplus under JP. Since in this case it is also true that $\frac{dZ^*}{d\lambda} = 0$, λ^S also maximizes total contribution. Notice also that in the case where both agents contribute we do not need the separability (we just need $\frac{dZ^*(\lambda)}{d\lambda} = 0$) to get the result because of Proposition 1(b). ■

The result that JP in general could improve over SP in both total provision or aggregate welfare sense is the main result of the paper. In the case where both agents contribute (low wealth inequality) or, alternatively, in all equilibria if preferences take the separable form, the optimal sharing rule λ^* that maximizes social welfare under JP leads *simultaneously* to higher aggregate contributions and joint welfare. Note that when both agents contribute in equilibrium, Proposition 1(b) implies that bundling can actually raise welfare in Pareto sense as well.

In addition, we have shown that the optimal sharing rule under JP also maximizes the total contribution that can be obtained. Notice that even though both agents have the same preferences over the desired mix of public goods, it is possible that an agency that can bundle contributions provide the goods in a ratio different to what both agents prefer. The reason is that by increasing provision through bundling the agency can achieve higher joint welfare. By choosing the sharing rule appropriately, bundling can be welfare improving since tying agents' contributions partially alleviates the free-riding problem. Only under homothetic preferences where there is independence of contributions on the share, given the identical preferences, manipulating the sharing rule cannot generate welfare gains.

4. Heterogeneous Preferences for Public Goods

Assume now that the agents have different preferences. We will show that the degree of wealth inequality between the agents is crucial for whether bundling can achieve a social welfare improvement. For our assumed class of preferences we show that, under sufficiently high wealth inequality, joint welfare is higher under joint provision compared to under separate provision. In contrast, when the two agents have equal wealths, we demonstrate that they can be worse off under joint provision as it distorts the agents' contributions to a suboptimal allocation. In general it is hard to characterize all equilibria or the exact conditions under which they occur with general preferences. That is why our general results are complemented by Section 5 where we provide a complete analytical and graphical analysis for the case of Cobb–Douglas preferences where some additional interesting findings can be obtained.

4.1. High Wealth Inequality

Assume that M_2 is sufficiently smaller than M_1 so that agent 2's optimal choice is not to contribute in equilibrium under both the separate and joint provision regimes. The Inada assumptions on u ($u(\cdot,\cdot,0)=\infty$) and continuity guarantee the existence of a range of values[10] for M_2 with a left bound of 0 where this property must hold. The Inada conditions also imply that the richer agent (agent 1) must contribute to both public goods in any such equilibria. For example, in the separate provision case this would hold for M_1 and M_2 such that

$$u_3^2\big(g_1^S, h_1^S, M_2\big) > u_1^2\big(g_1^S, h_1^S, M_2\big) \text{ and } u_3^2\big(g_1^S, h_1^S, M_2\big) > u_2^2\big(g_1^S, h_1^S, M_2\big),$$

where g_1^S and h_1^S are the solutions to

$$\begin{aligned} u_1^1(g_1, h_1, M_1 - g_1 - h_1) &= u_2^1(g_1, h_1, M_1 - g_1 - h_1) \\ &= u_3^1(g_1, h_1, M_1 - g_1 - h_1). \end{aligned}$$

Similar conditions can be derived for the JP case. The exact conditions on M_1 and M_2 needed are derived for Cobb–Douglas preferences in Section 5. It is crucial to notice that Proposition 2 still applies in this case, that is, the SP allocation can be replicated as equilibrium under joint provision by setting $\lambda = \lambda^S = \frac{g_1^S}{g_1^S + h_1^S}$. Given this, we prove the following result for high wealth inequality.

PROPOSITION 4: *Suppose u^1 and u^2 satisfy A1. Under preference heterogeneity and sufficiently high wealth inequality:*

(a) there exist sharing rules (not necessarily the same) under joint provision that lead to higher social surplus and (weakly) higher total contributions compared to under separate provision;

[10] The respective intervals may be different in general for the SP and JP schemes in which case we take the smaller one.

(b) *the joint surplus maximizing share, λ^*, under joint provision is in general different from the implied "share", λ^S, thus the contribution allocations differ between the two regimes. It is possible that $Z^*(\lambda^*) < Z^S$;*

(c) *social welfare under JP at λ^* is higher than social welfare under SP with the richer agent being worse off while the poor being better off.*

Proof: (a) and (b) The logic of the proof is very similar to the homogeneous nonseparable preferences case when only the richer agent contributes. As before, we obtain that

$$\frac{dW^J(\lambda)}{d\lambda} = \left(u_1^1 - u_2^1 + u_1^2 - u_2^2\right) Z^*(\lambda) + \left(\lambda u_1^2 + (1-\lambda) u_2^2\right) \frac{dZ^*(\lambda)}{d\lambda}.$$

Evaluating the above at $\lambda = \lambda^S$ (defined as before) we have $u_1^1 = u_2^1$. Inspecting the above expression it is clear that it will not be equal to zero in general; thus, the optimal share λ^* that makes it equal to zero will be different from λ^S. Once again (see the proof of Proposition 3) we can show that $\frac{dZ^*(\lambda^S)}{d\lambda}$ is not identically zero.[11] If $\frac{dZ^*(\lambda^S)}{d\lambda} > (<)\, 0$ an increase in λ will generate higher (lower) total provisions. The sign of $\frac{dW^J(\lambda^S)}{d\lambda}$ is unclear even if we know the sign of $\frac{dZ^*(\lambda^S)}{d\lambda}$, so we cannot say that the same λ can lead to *both* higher provision and higher welfare as in Proposition 3. However, it is clear that generically $\frac{dW^J(\lambda^S)}{d\lambda} \neq 0$, so JP will definitely lead to higher welfare than SP. Note also that it is possible that at λ^* social welfare be higher under JP compared to under SP but total contributions to be lower. In the homothetic separable preferences case, we have that total contributions under JP are maximized at λ^S but still social welfare can be improved by choosing a different sharing rule.

(c) To see that both agents cannot be made better off under JP at λ^*, remember that only the richer agent contributes in this case. His utility maximization problem under SP can then be written as

$$\max_{\lambda, Z} u^1 = u(\lambda Z, (1-\lambda) Z, M_1 - Z), \tag{5}$$

where we have replaced the original choice variables g_1 and h_1 by $\lambda = \frac{g_1}{g_1 + h_1}$ and $Z = g_1 + h_1$. Clearly, for a given λ, the problem of agent 1 under joint provision is a constrained version of the above problem, so he would be always weakly worse off under JP as long as agent 2 does not contribute. Given that $W^J(\lambda^*)$ is higher than W^S, it must be the case that the poorer agent is made better off by bundling. ■

Once again we obtain that joint provision can lead to increased social welfare. However, in contrast to the homogeneous preferences case, we cannot

[11] It will be in the homothetic case.

guarantee that this will be coupled with increased contributions as well—Z^* can well be lower. This result demonstrates that, under preference heterogeneity and when there are multiple pure public goods, a *policy of public good bundling can be welfare improving despite a (weakly) lower total contribution level* (see more on that in Section 5).

The last part of the proposition shows that the generated welfare improvement is of redistributive character, i.e., joint provision does not generate a Pareto improvement over the SP allocation. The proposition also shows that the optimal sharing rule under joint provision will be in general different from that under separate provision. For example, if $u^i = F^i(G) + F^j(H) + v(c_i)$, $i, j = 1, 2$, $i \neq j$, and $\lambda^S < 1/2$ (agent 1 likes good G less and agent 2 likes good H more, e.g., $F^{1\prime}(x) < F^{2\prime}(x)$, for all x) then the optimal sharing rule under JP moves provision away from the richer agent's preferred mix implying different equilibrium provision allocations between SP and JP.[12]

The welfare result occurs because the more egalitarian sharing rule under JP plays a redistribution function tilting the provision levels of the two goods more in line with agent 2's preferences given that most of the resources come from the richer agent. This decreases the welfare of agent 1. The welfare effect for agent 2 is ambiguous in general because the direct positive effect of a preferable public good mix may not be enough to offset the negative effect of the reduced public good contributions. It turns out, however, that for sufficiently high wealth inequality the positive effect dominates the negative and agent 2 is better off. We show in Section 5 that the general flavor of these results carries through even in equilibria where agent 2 contributes.

4.2. Low Wealth Inequality

In this section we compare the properties of the contribution equilibria under joint and separate provisions when the two agents have equal wealths, $M_1 = M_2 = M$, and contrast them with the high inequality results from above. By continuity all qualitative results carry over for a sufficiently low degree of wealth inequality. We show that under an egalitarian wealth distribution bundling can *decrease* total provision and that both agents could in fact be *worse off* under joint provision.

Concentrate on the equilibrium such that agent 1 contributes only to good H and 2 only to good G. Notice that such an equilibrium would not occur for high wealth inequality due to reasons stated above. On the other hand, it would be the typical case when agents are equally wealthy but have different preferences in the way that each "prefers" (i.e., has a higher marginal valuation for) a different public good for any provision level (see below for an example of sufficient conditions on u that ensure this). This type of equilibrium has the natural interpretation of agents having "taste" for different

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[12]Full details on this example are available from the authors.

public goods—e.g., some like parks, others like safety. We obtain the following result.

PROPOSITION 5: *For $M_1 = M_2$, in an equilibrium where each agent contributes only to a different public good, total provision under JP is strictly lower than that under SP.*

Proof: Define λ^S as before and look at the JP FOCs at $\lambda = \lambda^S$. It is clear that at $\lambda = \lambda^S$ the SP allocation *does not* satisfy these FOCs. Actually, we have that the LHS of each JP FOC is *smaller* (for any λ) than its RHS at the SP equilibrium G^S, H^S (because of the SP FOCs two of which hold with inequality given the type of equilibrium we look at). This implies that at the JP equilibrium (for any λ) we must have $z_1^J < z_1^S = H^S$ and $z_2^J < z_2^S = G^S$. But that means that Z^J will be necessarily strictly smaller than Z^S. Thus in this case JP *decreases* total contribution unambiguously. ■

Unfortunately, without making extra assumptions we cannot say more. Next we impose some additional sufficient conditions on u (apart from A1) and show that

In the low inequality case, it is possible that both agents are worse off under bundling.

Suppose that $u^i(G, H, c)$ is separable in the three goods and u^1 is a "mirror image" of u^2, i.e., $u^1(G, H) = F_1(G) + F_2(H) + v(c_1)$ and $u^2(G, H) = F_2(G) + F_1(H) + v(c_2)$. Suppose also that $F_1'(x) < F_2'(x)$ for all x, i.e., for a given provision level the two agents always "prefer" different public goods—agent 1 prefers H while agent 2 prefers G where by "prefer" we mean obtaining higher marginal utility per unit contributed. We will only look for symmetric equilibria.

Start with separate provision. When $M_1 = M_2$, inspection of (1) shows that only two of the FOCs can hold at equality, that is, each agent contributes only to the public good he values more:

$$F_1'(G) < F_2'(H) = v'(M - H) \quad \text{and} \quad F_1'(H) < F_2'(G) = v'(M - G).$$

The above implies that in equilibrium $G^* = H^* = Z^S/2$, where Z^S solves $F_2'(\frac{Z}{2}) = v'(M - \frac{Z}{2})$.

Under joint provision, symmetry implies that both FOCs must hold at equality and $z_1 = z_2 = \frac{Z^*}{2}$. Notice that both agents earn the same utility in equilibrium. Under separate provision we have

$$W^S = 2\left[F_1\left(\frac{Z^S}{2}\right) + F_2\left(\frac{Z^S}{2}\right) + v\left(M - \frac{Z^S}{2}\right)\right]$$

while social surplus under joint provision is

$$W^J = 2\left[F_1\left(\frac{Z^*}{2}\right) + F_2\left(\frac{Z^*}{2}\right) + v\left(M - \frac{Z^*}{2}\right)\right].$$

The latter must be lower than the former since $F_2'(\frac{Z^*}{2}) > v'(M - \frac{Z^*}{2})$, i.e., contributing more to the preferred good is optimal for each agent under SP.

Bundling can be welfare reducing at low levels of inequality since it forces the agents to contribute to both goods while optimally they would contribute only to one of them. This acts as a tax, lowering the marginal utility of contribution for each agent, which leads to less amount contributed and lower individual and social welfare.

4.3. Discussion

The intuition why bundling can increase social surplus, especially at high wealth inequality is that, when the poorer agent does not contribute, the more equal sharing of total contributions benefits her by more than the loss of welfare of the richer agent and joint surplus actually increases with joint provision. Basically, what happens under high inequality and SP is that the richer agent "has his way" in terms of the mix of public goods provided (the poorer agent cannot offset this since he has very little wealth to start with) and this leaves agent 2 worse off compared to under joint provision which features more preferable (from the viewpoint of the poorer agent) provision levels ratio. By "catering for the poorer agent" joint provision is thus able to improve social welfare under high wealth inequality by undoing to some extent the provision bias toward the richer agent's preferences coming from the inequality in resources.

The results that bundled contribution can never lead to a Pareto improvement over separate contributions and that actually at low levels of inequality joint provision can be welfare reducing to both agents relative to separate provision indicate that a joint provision policy can be defended on social optimality grounds only for high levels of wealth inequality and in a transferable utility framework.

An important implication of our results is the difference between maximizing total provision and maximizing social welfare in the second best environments considered here. Remember that it is possible to have *higher* total contribution but *lower* social welfare under separate compared to under joint provision. This has policy significance—wealth redistributions aimed to increase total contributions may actually lead to lower social welfare and vice versa. While it is clear that both the SP and JP regimes feature suboptimal contribution and social surplus levels, one needs to be mindful of the possible inconsistency of policies aimed to raising contribution levels versus those aimed at raising joint welfare as neither of these necessarily implies the other.

Our results were derived in a two-player framework for simplicity, but it is clear that they would hold in more general situations with higher number of potential contributors. For example, the high inequality scenario in which we have shown that bundling can raise social welfare can be easily reproduced with more than two agents as long as their wealths are unequal enough so that only the richest agent contributes. Similarly, our findings remain valid by considering economies consisting of a big number of replicas of our original two agents and looking at equilibria where the same agents are treated symmetrically. Of course, there exist more general cases with $n > 2$ players that our results may not automatically extend to. Nevertheless, the main intuition that bundling could improve social welfare by changing the provision mix toward the first best should still remain valid. Still, a more complete formal analysis of the multiple agent case could be a fruitful extension and reveal additional interesting results.

Remember that in the nonhomothetic case total contributions collected under bundling depend on the announced share, λ. We saw above that setting $\lambda = \lambda^S$ could lead to higher total contributed amount than under the optimal JP share λ^*. In our analysis we have assumed that the public good provider is able to fully commit to the preannounced sharing rule.[13] If this were not the case, it is possible to have a *time-inconsistency problem*—the provider may announce λ^S to obtain more contributions but then actually share according to λ^*, which will lead to higher social welfare. A potential extension of our results in this direction, perhaps using arguments as in Kydland and Prescott (1977) can be of interest.

5. Example—the Cobb–Douglas Case

In this section we assume that preferences take the Cobb–Douglas form (which satisfies A1)

$$u^i(G, H, c_i) = \alpha_i \ln G + \beta_i \ln H + \ln c_i.$$

A complete analytical characterization of the separate and joint provision equilibria is possible in this case. We use it to further illustrate the intuition and sharpen our general results from the previous sections. In the homogeneous preferences case we set $\alpha_1 = \alpha_2 = \alpha$ and $\beta_1 = \beta_2 = \beta$, while heterogeneous preferences are represented by $\alpha_1 = \beta_1 = \alpha$ and $\beta_1 = \alpha_2 = \beta$. Without loss of generality we assume $\beta \geq \alpha$ thus good H is "liked more" by both agents under homogeneous preferences, while with heterogeneous preferences agent 1 likes H more but agent 2 likes G more. Notice also that the Cobb–Douglas

[13]In the real world, credible commitment can be achieved for example due to concern for long run reputation. Charities that do not honor their promised sharing rule are likely to lose public patronage. There may also be legal implications; for example, a recent Wall Street Journal article (WSJ, Feb 6, 2006, Page A1, "Poisoned Ivy: Lawsuit Alleges Princeton Misused a Family's Big Gift") describes a lawsuit filed by a private donor for using the gift in a manner inconsistent with the donor's intentions.

form is homothetic and separable; thus, the equilibria z_1 and z_2 under joint provision are independent of λ, which considerably simplifies the analysis.

In the homogeneous preferences case, Proposition 2 applies, so the SP equilibrium is equivalent to the JP equilibrium under the optimal λ which in this case is easily shown to be $\lambda^* = \frac{\alpha}{\alpha+\beta}$. The optimal sharing rule λ maximizing social surplus is independent of the equilibrium provision and agents' endowments. From the FOCs one can derive that both agents contribute under both SP and JP as long as $M_2 > \frac{M_1}{1+\alpha+\beta}$.

Concentrate now on the heterogeneous preferences case where we can obtain a sharper characterization of the resulting equilibria than in the general analysis above. We suppose $\beta > \alpha$ and maintain our assumption that $M_1 \geq M_2$. Under SP it is easy to see from (1) that at least one of the agent will not contribute to at least one of the goods in equilibrium. Under JP it is easy to show that the optimal sharing rule that will be chosen by the public goods provider in this case is $\lambda^* = 1/2$, that is, to split total contribution in half[14] between the two public goods. The intuition comes from the fact that agents are treated equally by the provider who maximizes the sum of utilities and the fact that agent 1's preferences are a "mirror image" of agent 2's preferences with respect to G and H. Looking at the first-order conditions, there are two possible types of equilibria: one in which only the richer agent contributes and one in which both contribute.

Solving the linear systems of FOCs for the SP and JP cases is an easy but tedious task so we omit the details.[15] Tables A1 and A2 fully characterize the solutions under separate and joint provision in all possible types of equilibria as well as the parameter restrictions under which the latter occur.

For given preference heterogeneity, the nature of the SP equilibrium which obtains depends on the relative inequality in agents' resource levels M_1 and M_2. For highly diverse wealths only the richer agent contributes to both goods while when wealths are relatively similar each agent contributes only to the good they like more.

Since the Cobb–Douglas function is a particular example of homothetic preferences satisfying A1, we know that all results established above still apply. However, in addition to this, we are now able to provide a complete characterization of the contribution equilibria for *any degree* of wealth inequality to supplement the limiting results (high and low inequality) from the previous section. The analytical results obtained computationally are illustrated in Figure 1. The figure describes how total contributions and welfare vary as function of M_1 holding total wealth $M = 1$ fixed between perfect equality $(M_1 = 1/2)$ and perfect inequality $(M_1 = 1)$.

The following proposition summarizes our results for the case of Cobb–Douglas preferences.

[14]Simple algebra shows that the same equal sharing rule is optimal under the first best.
[15]However, they are readily available from the authors upon request.

PROPOSITION 6 (Cobb–Douglas preferences): *Under preference heterogeneity,*

(a) *Total contributions under SP, Z^S, are (weakly) higher than those under JP, Z^J, for any M_1, M_2.*

(b) *There exists a threshold level of wealth inequality such that social surplus under SP is strictly higher than social surplus under JP for inequality below the threshold and the opposite is true for inequality above the threshold.*

(c) *For any level of inequality, the welfare of the richer agent is lower under JP compared to that under SP. For high enough wealth inequality, the welfare of the poorer agent (agent 2) is higher under JP compared to that under SP.*

Proof: See Appendix.

The intuition why joint provision cannot achieve higher total contribution level compared to separate provision is that the equal sharing rule distorts the agents' incentives to contribute. The richer agent is worse off under joint provision for any degree of wealth inequality. Notice that the welfare result in (b) does not require that agent 2 be a noncontributor (as assumed in Section 4) — Figure 1 shows that it is possible that both agents contribute under both regimes and still $W^J > W^S$. Notice that SP could result in *higher total contributions but lower welfare* (for intermediate degree of inequality). This result demonstrates that, under preference heterogeneity and when there are multiple pure public goods, a policy of public good bundling can be welfare improving despite a (weakly) lower total contribution level.

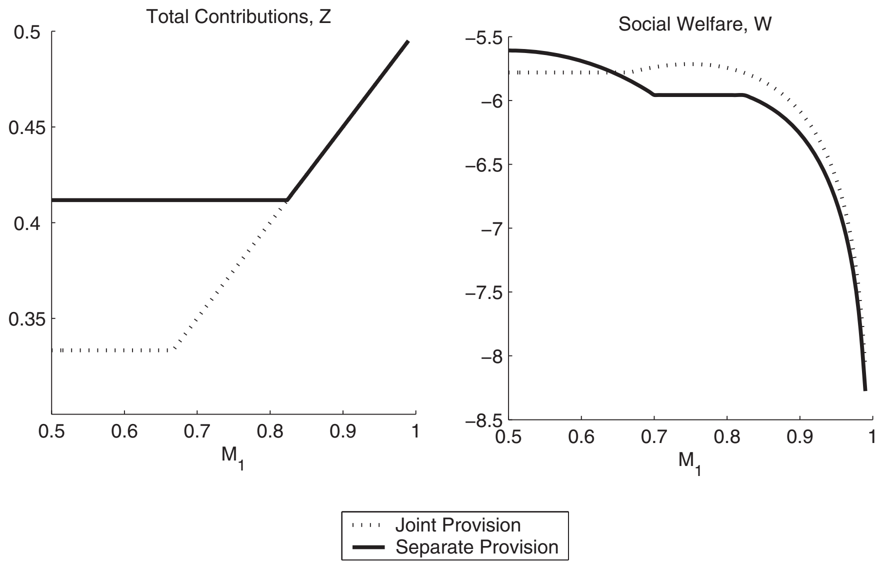

Figure 1: Cobb–Douglas case, wealth inequality effects ($M = 1$, $\alpha = 0.3$, $\beta = 0.7$)

6. Conclusions

There are many real-life situations where agents contribute voluntarily to more than one public goods. In this paper we examine such a scenario and study the effects of different contribution mechanisms on public good provision and welfare. Once there are multiple public goods, voluntary contributions suffer from inefficiencies of scale as well as mix. Hence the role of preference and wealth heterogeneity is crucial (see also Clark and Kanbur 2004). Our analysis explores whether a simple solution, namely bundling of the contributions to the public goods could help the situation. The results suggest that while scale inefficiencies may worsen under bundling, the provision mix can be improved, leading to greater aggregate welfare. We find that this holds in the case when agents have identical preferences or when wealth heterogeneity is high.

In deriving our results, we do not assume any informational advantage for the providers (who determine the share in which the public good will be divided in the joint provision case). It might well be the case that they might have more relevant information that the donors do not possess. In that scenario it is even more likely that the bundling scheme will be preferred since the onus of information acquisition then falls on the donors in the separate provision scheme.

Despite the fact that we work out a purely theoretical and hence abstract example, we believe that our results have potential policy implications for situations where agents contribute simultaneously to multiple public goods. We have shown that wealth redistributions aimed to increase total contributions may actually lead to lower social welfare and vice versa. With wealth heterogeneity, bundling public good contributions was demonstrated to be socially optimal only for high levels of inequality, thus different contributions schemes may have to be chosen in different situations depending on the degree of preference and resource heterogeneity among agents. Our analysis thus has potential implications for the design of institutions that function on voluntarily donated funds. Charities, nongovernmental organizations, even research institutions rely on public donations, which are then channeled into various uses which have public goods characteristics. This paper provides an (admittedly simplified) framework that can be used to understand when it might be optimal to use bundling and when to have use-specific contributions.

The fact that bundling cannot generate a Pareto improvement over separate provision has potentially interesting political economy implications. Remember that the rich agent is actually worse off under joint provision. Depending on whether the median voter in a democratic political system or a dictator otherwise could be classified as “rich” or “poor” in the context of our stylized model, such social welfare improving policies may or may not be implementable in practice.

The value of analyzing potential problems pertaining to private provision of public goods can be especially high in development economy contexts where government failure and institutional weaknesses often limit public provision (Besley and Ghatak 2004). The importance of studying the effects of inequality in the wealth or land distribution on the public good provision process has been widely acknowledged in recent applied work (e.g., Bardhan et al. 2006, Bardhan and Dayton-Johnson 2002). We contribute to this literature by proposing an additional mechanism (not relying on direct wealth redistribution), which might lead to social welfare enhancement in unequal societies where public goods are privately provided.

In terms of future research, it could be useful to study the welfare effects of bundling under different public goods technologies, as in many real world situations, for example, best-shot, weaker, or weakest link (Cornes 1993). One might also investigate the role of dynamics in a multiple public goods setting (Gradstein 1992 and Varian 1994) or the implications of incomplete information, unlike our common knowledge framework.

Appendix

Proof of Proposition 6:

(a) Follows immediately since $\frac{\beta}{1+\beta} \geq \frac{\alpha+\beta}{2+\alpha+\beta}$ (see Tables A.1 and A.2).

(b) We show first that at perfect equality, i.e., $M_1 = M_2 = M/2$, we have $W^S \geq W^J$ with equality only if $\alpha = \beta$. To see this notice that $W^{S.\text{III}} \geq W^{J.\text{II}}$ is equivalent to $(\alpha+\beta)\ln\frac{\beta}{1+\beta} + \ln\frac{1}{1+\beta} \geq (\alpha+\beta)\ln\frac{\alpha+\beta}{2+\alpha+\beta} + \ln\frac{2}{2+\alpha+\beta}$. Call $\alpha+\beta = A$. Notice that the right-hand side is a function of A only while the left-hand side is decreasing in α holding A fixed. Thus, for given A, the left-hand side is minimized at $\alpha = \beta = A/2$ and then it actually equals the right-hand side. Thus for any A and any $\alpha < \beta$ the left-hand side is strictly larger than the right hand side, i.e., $W^S > W^J$ at perfect wealth equality.

Now consider the case of high wealth inequality, i.e., $M_2 \leq \frac{\alpha M_1}{\beta(1+\alpha+\beta)}$ or $M_1 \geq \frac{\beta(1+\alpha+\beta)M}{(1+\beta)(\alpha+\beta)}$ (cases S.I and J.I). Notice that $\text{sign}(W^J - W^S)$ is then equal to $\text{sign}(\frac{(\alpha+\beta)^2}{4} - \alpha\beta)$ and the latter is always positive as long as $\alpha < \beta$. Observe that this also implies that $W^J > W^S$ for $M_2 \leq \frac{\alpha M_1}{\beta}$, i.e., for $M_1 \geq \frac{\beta M}{\alpha+\beta}$ (fact 1), since $W^{S.\text{II}}$ equals $W^{S.\text{I}}$ evaluated at $M_2 = \frac{\alpha M_1}{\beta(1+\alpha+\beta)}$ holding M fixed. Now look at W^S and W^J as a function of M_1 holding M fixed. Notice that W^S is strictly decreasing in M_1 for case S.I., i.e., $M_1 \in [\frac{M}{2}, \frac{\beta M}{\alpha+\beta}]$ starting at a level higher than W^J (fact 2) and then constant for $M_1 \in [\frac{\beta M}{\alpha+\beta}, \frac{\beta(1+\alpha+\beta)M}{(1+\beta)(\alpha+\beta)}]$ (case S.II.). In contrast, W^J is constant for all $M_1 \in [\frac{M}{2}, \frac{(1+\alpha+\beta)M}{2+\alpha+\beta}]$ (case J.II) while under case J.I W^J is initially *increasing*[16] in M_1 for $M_1 \in [\frac{(1+\alpha+\beta)M}{2+\alpha+\beta}, \frac{1+2(\alpha+\beta)}{2(1+\alpha+\beta)}M]$[17] and then decreasing in M_1 (fact 3). Given facts 1–3 one can easily convince oneself that W^J and W^S plotted as function of M_1 holding M

Table A.1. Public good provision and social surplus under separate provision

Case	Parameter restriction	Provision, Z^S	Social surplus, W^S
S.I	$0 \leq M_2 \leq \frac{\alpha M}{(1+\beta)(\alpha+\beta)}$	$\frac{(\alpha+\beta)M_1}{1+\alpha+\beta}$	$(\alpha+\beta)\ln\frac{\alpha\beta M_1^2}{(1+\alpha+\beta)^2} + \ln\frac{M_1 M_2}{1+\alpha+\beta}$
S.II	$\frac{\alpha M}{(1+\beta)(\alpha+\beta)} < M_2 < \frac{\alpha M}{\alpha+\beta}$	$\frac{\beta M}{1+\beta}$	$(\alpha+\beta)\ln\frac{\alpha\beta^3 M^2}{(1+\beta)^2(\alpha+\beta)^2} + \ln\frac{\alpha\beta M^2}{(1+\beta)^2(\alpha+\beta)^2}$
S.III	$\frac{\alpha M}{\alpha+\beta} \leq M_2 \leq \frac{M}{2}$	$\frac{\beta M}{1+\beta}$	$(\alpha+\beta)\ln\frac{\beta^2 M_1 M_2}{(1+\beta)^2} + \ln\frac{M_1 M_2}{(1+\beta)^2}$

Table A.2. Individual and total contributions under joint provision

Case	Parameter restriction	Provision, Z^J	Social surplus, W^J
J.I	$0 \leq M_2 \leq \frac{M}{2+\alpha+\beta}$	$\frac{(\alpha+\beta)M_1}{1+\alpha+\beta}$	$2(\alpha+\beta)\ln\frac{(\alpha+\beta)M_1}{2(1+\alpha+\beta)} + \ln\frac{M_1 M_2}{1+\alpha+\beta}$
J.II	$\frac{M}{2+\alpha+\beta} < M_2 \leq \frac{M}{2}$	$\frac{(\alpha+\beta)M}{2+\alpha+\beta}$	$2(\alpha+\beta)\ln\frac{(\alpha+\beta)M}{2(2+\alpha+\beta)} + 2\ln\frac{M}{2+\alpha+\beta}$

constant must cross at a unique point $M_1 = m \in (\frac{M}{2}, \frac{\beta M}{\alpha+\beta})$, which is the threshold from the proposition statement.

(c) Under S.I., J.I must hold so $u_1^{\mathrm{J.I}} \leq u_1^{\mathrm{S.I}}$ is equivalent to

$$\frac{\alpha \ln \alpha + \beta \ln \beta}{2} \geq \frac{\alpha+\beta}{2} \ln\left(\frac{\alpha+\beta}{2}\right), \tag{6}$$

which is true since the function $x \ln x$ is convex for $x > 0$. Under S.II. we need to show that $u_1^{\mathrm{S.II}}$ is bigger or equal to $u^J{}_2$ achieved in both cases J.I and J.II since both are possible. Since under S.II we have that $M \geq \frac{(1+\beta)(\alpha+\beta)M_1}{\beta(1+\alpha+\beta)}$, we have $u_1^{\mathrm{S.II}} \geq \alpha \ln \frac{\alpha M_1}{1+\alpha+\beta} + \beta \ln \frac{\beta M_1}{1+\alpha+\beta} + \ln \frac{M_1}{1+\alpha+\beta} = u_1^{\mathrm{S.I}}$ and we already know this is not smaller than u_1^J under J.I. Now suppose S.II and J.II hold instead. Given that $\frac{\beta}{(1+\beta)(\alpha+\beta)} \geq \frac{1}{2+\alpha+\beta}$ we have $u_1^{\mathrm{S.II}} \geq \alpha \ln \frac{\alpha M}{2+\alpha+\beta} + \beta \ln \frac{\beta M}{2+\alpha+\beta} + \ln \frac{M}{2+\alpha+\beta} \geq u_1^{\mathrm{J.II}}$ because of (6). Now, suppose S.III. and J.I hold. Given that $\frac{1}{1+\beta} \geq \frac{1}{1+\alpha+\beta}$, we have $u_1^{\text{S.III}} \geq \alpha \ln \frac{\alpha M_1}{1+\alpha+\beta} + \beta \ln \frac{\beta M_1}{1+\alpha+\beta} + \ln \frac{M_1}{1+\alpha+\beta} \geq u_1^{\text{J.I}}$ because of (6). The same logic works under S.III and J.II because $\frac{1}{1+\beta} > \frac{1}{2+\alpha+\beta}$. The fact that the poor agent must be better off under high inequality follows from part (b). ■

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