Market Power and Asset Contractibility in Dynamic Insurance Contracts*

Alexander K. Karaivanov and Fernando M. Martin

February 16, 2016

Abstract

We study the roles of asset contractibility, market power and rate of return differentials in dynamic insurance when the contracting parties have limited commitment. We define, characterize and compute Markov-Perfect risk-sharing contracts with bargaining. These contracts significantly improve consumption smoothing and welfare relative to self-insurance through savings. Making savings decisions part of the contract (asset contractibility) implies sizable gains for both the insurers and the insured. The size and distribution of these gains depend critically on the insurers' market power. Finally, a rate of return advantage for insurers destroys surplus and is thus harmful to both contracting parties.

Keywords: optimal insurance, lack of commitment, Markov-perfect equilibrium, market power, asset contractibility.

JEL classification: D11, E21.

*Karaivanov acknowledges the financial support of the Social Sciences and Humanities Research Council of Canada. The views expressed in this paper do not necessarily reflect official positions of the Federal Reserve Bank of St. Louis, the Federal Reserve System, or the Board of Governors.

1 Introduction

Households face fluctuations in their incomes but desire stable consumption. Prime examples of shocks to income are variations in labor status and changes in health. Maintaining savings in liquid and low-risk assets, for instance in the form of government bonds or saving accounts, allows households to mitigate the impact of negative income shocks on their standard of living. Similarly, positive income shocks provide the opportunity to stock up savings to use in bad times. However, savings are an imperfect way to insure against idiosyncratic shocks; for instance, the return on a deposit does not increase because the depositor has been laid off or became sick. Hence, a natural way to complement self-insurance through savings is to contract with an insurer (private or government-run) willing to absorb an agent's individual risk. In a perfect world, the parties would sign a long-term contract that maximizes the surplus generated by the relationship and fully specifies the time paths of consumption and savings of the insured for all possible combinations of future income states.

In practice, however, economic actors often cannot or are legally barred from committing to a long-term contract. Take typical labor, housing, personal or property insurance contracts: costless renegotiation or switching providers is always possible, although sometimes only at fixed time-intervals. In addition, while insurers are frequently aware of an agent's net worth or assets, they may or may not have the ability to control private asset accumulation. The latter ability, however, can be key for the interplay between self-insurance and market- or government-provided third party insurance (e.g., Arnott and Stiglitz, 1991). As an example, government social security schemes (old-age insurance) usually have both voluntary and controlled/forced savings components. Various mixtures exist around the world.

We study the above issues and trade-offs in a multi-period risk-sharing setting, featuring a risk-neutral insurer and a risk-averse agent endowed with a stochastic income technology and the ability to save at a fixed rate of return. We assume that the parties cannot commit to a long-term contract—both the agent and the insurer can only commit to *one-period* risk-sharing contracts. In this setting, we show that there are still large gains from third-party insurance and from being able to make the agent's saving decisions part of the insurance contract.

Specifically, we model the interaction between the agent and insurer by assuming that they periodically bargain over the terms of the contract. Formally, we do so by adopting the solution concept of a Markov-perfect equilibrium, as in Maskin and Tirole (2001). This solution captures our notion of limited commitment, since contract terms are function of only payoff-relevant variables (in our setting, the agent's assets and the income realization), and the idea that "bygones are bygones", that is, the past does not matter beyond its effect on the current state.

We find that the agent's asset holdings are a key feature of Markov-perfect insurance contracts, as the assets determine the agent's endogenous outside option. Given that, we analyze the role of asset/savings contractibility by comparing the case of "contractible assets", when the insurer can fully control the agent's savings decisions, with the case of "non-contractible assets", in which the agent can privately decide on the amount of his savings, even though his asset holdings are observed by the insurer. In many situations, governments, insurance companies, banks and so on may have information about agents' assets but, for legal or other reasons, are unable to directly control agents' savings choices. In other situations, for example, social security, the opposite is true.

We show that asset contractibility affects the insurance contract terms and the degree of risk-sharing that can be achieved compared to self-insurance, except in the limit when insurance markets are perfectly competitive (free entry). Intuitively, whenever the insurer has market power (not necessarily monopoly power) and thus can generate positive profits from insuring the agent, private asset accumulation provides the agent with an instrument to "counter" the insurer by controlling his future outside option. Essentially, larger savings by the agent today imply a larger outside option tomorrow since the agent would be able to self-insure better. On the insurer's side, however, a larger outside option for the insured implies lower profits. We show that this misalignment of incentives between the contract sides, which has its origin in the commitment problem, causes a welfare loss to *both* sides when the agent's assets are non-contractible.

Numerically, we assess the degree to which the presence of third-party insurance improves the agents' welfare beyond what they can achieve on their own through savings. We show that the welfare gains for the poorest agents (zero assets) can be as high as 4.5% of their autarky consumption, per period. This number is significantly larger than the cost of business cycle fluctuations (about 0.1%), a common benchmark for welfare calculations in macroeconomic applications. In terms of the role of assets contractibility, the largest welfare loss if agent's savings are non-contractible is about 0.4% of autarky consumption per period.

We also find that the market power of insurance providers affects significantly the welfare gains that agents derive in Markov-perfect insurance contracts and, to a lesser extent, the welfare losses associated with agents' assets being non-contractible. The welfare gains from third-party insurance are strictly decreasing in the insurers' market power, whereas the welfare costs of asset non-contractibility peak at an intermediate value of market power, in between the monopolistic insurer case and perfectly competitive insurance markets.

Finally, our numerical results suggest that both insured and insurers are better off is there is no return on assets differential between them. A higher intertemporal return, or equivalently discount rate, for the insurer relative to the insured reduces the total surplus that can be generated in the risk-sharing relation. Furthermore, differences in the parties rates of return on assets amplify the distortions in the time-profiles of consumption and savings (relative to the equal return benchmark), which arise from the limited commitment friction.

The current paper builds upon and extends in several dimensions our previous analysis in Karaivanov and Martin (2015). In that paper, we introduced the idea of Markov-perfect insurance contracts and showed that limited commitment on the insurers' side is restrictive only when they have a rate of return advantage over agents with sufficiently large asset holdings. The limited commitment friction makes assets carried by the agent essential in a Markov-perfect equilibrium, as they cannot be replaced with promises of future transfers. In contrast, if the insurer and insured have equal rates of return on carrying assets over time, we showed that Markov-perfect insurance contracts result in an equivalent consumption time-path as a long term contract to which only the insurer can commit, since assets and promised utility are then inter-changeable. While we keep the basic idea of Markov-perfect insurance, here we differ in two important aspects. First, unlike in Karaivanov and Martin (2015), we allow agent's assets to be non-contractible. Second, instead of assuming an arbitrary asset-dependent but otherwise exogenous outside option for the agent, we endogenize the division of the gains from risk sharing by defining and analyzing a bargaining problem between the parties.

The paper also differs from the literature on optimal contracts with hidden savings (Allen, 1985; Cole and Kocherlakota, 2001; among others) which assumes that the principal has no ability to monitor the agent's assets. The main result in these papers is that no additional insurance over self-insurance may be possible, unlike here. On the technical side, our assumption of observable assets (even if non-contractible) helps us avoid dynamic adverse selection and the possible failure of the revelation principle with lack of commitment (Bester and Strausz, 2001), while still preserving the empirically relevant intertemporal implications of savings non-contractibility.

More generally, in the dynamic mechanism design literature, allowing agents to accumulate assets in a principal-agent relationship typically yields one of the following three results, depending on the specific assumptions made about the information or commitment structure: (i) agent's assets play no role (when the insurer can control the agent's consumption); (ii) assets eliminate the insurer's ability to smooth the agent's consumption beyond self-insurance (Allen, 1985; Cole and Kocherlakota, 2001); or (iii) the environment becomes highly intractable (Fernandes and Phelan, 2000; Doepke and Townsend, 2006). In contrast, we show that Markov-perfect insurance contracts result in simple dynamic programs with a single scalar state variable and avoid the curse of dimensionality, including with non-contractible savings.

2 The Environment

Consider an infinitely-lived risk-averse agent who maximizes discounted expected utility from consumption, c. The agent's flow utility is $u(c)$, with $u_{cc} < 0 < u_c(c)$ and u satisfying Inada conditions.[1] The agent discounts the future by factor $\beta \in (0, 1)$. Each period the agent receives an output/income endowment, which he can consume or save. Output is stochastic and takes the values $y^i > 0$ with probabilities $\pi^i \in (0, 1)$ for all $i = 1, \ldots, n$, with $n \geq 2$ and where $\sum_{i=1}^n \pi^i = 1$. Without loss of generality, let $y^1 < \ldots < y^n$.

The risk-averse agent would like to smooth consumption over output states and over time. We assume that the agent can carry assets a over time via a savings (storage) technology with fixed gross return $r \in (0, \beta^{-1})$. Let the set of feasible asset holdings be $\mathbb{A} = [0, \bar{a}]$, where $\bar{a} \in (0, \infty)$ is chosen to be sufficiently large so that it is not restrictive. In contrast, the lower bound on $\mathbb{A}$ is restrictive and represents a borrowing constraint. Assuming that assets cannot be negative means that the agent cannot borrow, that is, he can only save.

Suppose first that the agent had no access to insurance markets and therefore can only rely on self-insurance through savings—running up and down a buffer stock of assets as in Bewley (1977). In this situation, which we label autarky, the agent's optimal consumption and savings decisions depend on his accumulated assets and are contingent on the output realization. That is, given realized output y^i, the agent carries into the next period assets

[1] Throughout the paper we use subscripts to denote partial derivatives and primes for next-period values.

$a^i \geq 0$ and consumes $c^i \equiv ra + y^i - a^i$.

Formally, the agent's problem in autarky can be written recursively as:

$$\Omega(a) = \max_{\{a^i \geq 0\}_{i=1}^n} \sum_{i=1}^n \pi^i \left[u(ra + y^i - a^i) + \beta\Omega(a^i) \right]. \tag{1}$$

By standard arguments (e.g., Stokey, Lucas, and Prescott, 1989), our assumptions on u ensure that the autarky value function $\Omega(a)$ is continuously differentiable, strictly increasing, and strictly concave for all $a \in \mathbb{A}$. The autarky (self-insurance) problem is a standard "income fluctuation" problem, versions of which have been studied, for instance, in Schechtman and Escudero (1977) and Aiyagari (1994), among many others. The properties of the solution are well-known: imperfect consumption smoothing (c^i differs across states with different y^i); consumption, c^i and next period assets, a^i in each income state are increasing in current assets a; asset contraction (negative savings) in the lowest income state(s) and asset accumulation (positive savings) for some range of asset holdings in the highest income state(s).

Since the rate of return on assets is assumed smaller than the agent's discount rate, $r < \beta^{-1}$, the agent saves only to insure against consumption volatility.[2] In particular, there is a *precautionary* motive for saving due to the fact that the agent wants to mitigate the chance of ending up with zero assets, a situation in which he would be unable to self-insure against negative income shocks. Note that, since assets provide the same return in all output states, the agent is unable to insure perfectly against income fluctuations. Thus, there is a demand for additional insurance which we address in the next section.

3 Insurance

Suppose there exists a risk-neutral, profit-maximizing insurer. Throughout the paper we assume that the insurer can costlessly observe output realizations y^i and the agent's assets a. The insurer can borrow and lend, without restrictions, at gross rate $R > 1$. The insurer's future profits are also discounted at the rate R. The parameter R can have either a technological or preference interpretation. The special case $r = R$ can be thought of as the insurer being able to carry resources intertemporally using the same saving technology as the agent. If, instead, $R = \beta^{-1}$, we can think of the agent and insurer as having the same discount factor—a standard assumption in the literature. In general, we allow R to take any value in between these bounds, as stated in Assumption 1 below.

Assumption 1 *$0 < r \leq R \leq \beta^{-1}$, with $r < \beta^{-1}$, and $R > 1$.*

3.1 The agent's savings decision

Suppose the insurer, while observing the agent's assets a, cannot directly control the agent's savings decision, namely the choice of a'. We can think of the insurance arrangement between agent and insurer in any time period as the exchange of output y^i for gross

[2]That is, the agent would not save if output were constant over time.

transfer τ^i (this includes the insurance premium or payoff in the different states of the world). Transfers are allowed to depend on the agent's accumulated assets a, since assets affect how much insurance the agent demands.

Suppose the agent is offered insurance for the current period. What is his savings decision given transfers τ^i? Call period consumption $c^i \equiv ra + \tau^i - a'^i$, as implied by the insurance transfer τ^i, the gross return on the agent's current assets ra, and the agent's savings decision a'^i. Let $v(a'^i)$ denote the continuations value for the agent carrying assets a'^i into the next period. The function v is an equilibrium object which depends on all future agent-insurer interactions, which in turn depend on the level of assets carried into the future. The consumption/savings problem of the agent can then be written as follows

$$\max_{\{a'^i\}} \sum_i \pi^i [u(ra + \tau^i - a'^i) + \beta v(a'^i)]$$

With Lagrange multiplier $\xi^i \pi^i \geq 0$ associated with the non-borrowing constraint, $a'^i \geq 0$, the first-order conditions are

$$-u_c(ra + \tau^i - a'^i) + \beta v_a(a'^i) + \xi^i = 0,$$

for all $i = 1, \ldots, n$. In other words, given an insurance contract for the current period and anticipating future interactions (contracts) between the agent and the insurer, which yield continuation value v, the agent's savings decision is characterized by:

$$u_c(c^i) - \beta v_a(a'^i) \geq 0, \text{ with equality if } a'^i > 0. \tag{2}$$

When agent's savings are non-contractible, the insurer must take into account the agent's savings decision given by (2) when deciding on the insurance transfers τ^i. We call this the agent's incentive-compatibility constraint, as any insurance contract which allows the agent to make his own savings decisions must respect condition (2).

Below, we also consider the alternative case in which the agent's savings can be specified (enforced) as part of the insurance contract. In this case, inequality (2) does not restrict the design of the insurance contract offered to the agent.

3.2 Markov-Perfect Insurance

We assume that agent and insurer can bargain over the insurance terms each period. The insurance contract is negotiated every period since we assume a limited commitment friction—neither the agent, nor the insurer can commit to honor any agreement beyond the current period. This could be motivated by legal, regulatory, or market reasons—for example, in many real life situations (labor contracts, housing rental, home and car insurance, etc.) the parties are allowed to (costlessly) modify or re-negotiate the contract terms at fixed points of time (e.g., yearly).

If the parties do not reach an agreement, they revert to their respective outside option from then on. Of course, given the limited commitment friction, both parties know that any agreement spanning more than one period is subject to re-negotiation and cannot be committed to. The outside option for the agent is autarky, with value $\Omega(a)$ as derived above. The outside option of the insurer is zero profits.

To model the bargaining game between the agent and the insurer we adopt the Kalai (1977) solution, which picks a point on the utility possibility frontier depending on a single parameter, θ. This parameter can be interpreted as the agent's "bargaining power". Specifically, in Kalai's bargaining solution, a larger θ implies that the agent obtains surplus closer to his maximum feasible surplus while the insurer obtains surplus closer to his outside option. The converse is true for lower values of θ. The limiting case $\theta \to 1$ corresponds to the agent receiving his maximum possible surplus and the insurer receiving his outside option of zero profits. This can be interpreted as a market setting with perfect competition and free entry by insurers. In contrast, in the opposite limiting case, $\theta \to 0$, the agent receives his outside option, while the insurer receives maximum (monopoly) profits. Formally, the Kalai bargaining solution postulates a *proportional surplus-splitting rule*, which takes the form $(1-\theta)S^A = \theta S^I$, where S^A is the agent's surplus, defined as the difference between the agent's value in the contract and his outside option, and S^I is the insurer's surplus, defined analogously.

Let $\bar{y} \equiv \sum_{i=1}^n \pi^i y^i > 0$ denote expected output. The insurer's expected period profit is therefore $\bar{y} - \sum_{i=1}^n \pi^i \tau^i$. Equivalently, using $c^i = ra + \tau^i - a'^i$ we can re-write the insurer's profit in terms of agent's consumption and next-period assets as $\bar{y} + ra - \sum_{i=1}^n \pi^i(c^i + a'^i)$. The participation constraints of the contracting parties are therefore:

$$\sum_{i=1}^n \pi^i[u(c^i) + \beta v(a'^i)] \geq \Omega(a)$$

$$\bar{y} + ra - \sum_{i=1}^n \pi^i[c^i + a'^i - R^{-1}\Pi(a'^i)] \geq 0,$$

where v and Π denote the (endogenous) agent and insurer continuation payoffs, respectively, both as functions of the agent's asset holdings.

Assuming $\theta \in (0,1)$, we can write the insurance contract with Kalai bargaining as

$$\max_{\{c^i, a'^i \geq 0\}} \; \bar{y} + ra - \sum_{i=1}^n \pi^i[c^i + a'^i - R^{-1}\Pi(a'^i)]$$

subject to (2) for all $i = 1, \dots, n$ and

$$(1-\theta)[\sum_{i=1}^n \pi^i(u(c^i) + \beta v(a'^i)) - \Omega(a)] - \theta[\bar{y} + ra - \sum_{i=1}^n \pi^i(c^i + a'^i - R^{-1}\Pi(a'^i))] = 0. \quad (3)$$

Insurer's profits are maximized subject to the agent's incentive-compatibility constraint and the proportional surplus-splitting rule.

Since the insurer observes the output realization y^i and there are no private information issues or intra-temporal commitment problems, it is optimal that the agent receive full insurance, that is, $c^i = c$ for all i. Formally, this can be shown by taking the first-order conditions in the above problem with respect to c^i and noticing that they are fully symmetric with respect to i. Intuitively, the risk-averse agent is fully insured against his idiosyncratic income fluctuations and all income risk is absorbed by the risk-neutral insurer. Unlike in alternative settings, e.g., with moral hazard or adverse selection, here there are no gains from making agent's consumption state-contingent since output realizations are exogenous and not affected by any agent actions or type. Assuming a symmetric solution, we also obtain $a'^i = a'$ for all i. In this case, which we assume from now on, the insurance contract can be written as

$$\max_{c,a' \geq 0} \; \bar{y} + ra - c - a' + R^{-1}\Pi(a')$$

subject to

$$u_c(c) - \beta v_a(a') \geq 0, \text{ with equality if } a' > 0 \tag{4}$$

and

$$(1-\theta)\left[u(c) + \beta v(a') - \Omega(a)\right] - \theta\left[\bar{y} + ra - c - a' + R^{-1}\Pi(a')\right] = 0 \tag{5}$$

We formally define a *Markov-perfect equilibrium* and a *Markov-perfect insurance contract* in our setting as follows.

Definition 1 *Consider a risk-averse agent with autarky value* $\Omega(a)$*, as defined in* (1)*, and bargaining power* $\theta \in (0,1)$ *contracting with a risk-neutral insurer.*

(i) A ***Markov-perfect equilibrium (MPE)*** *is a set of functions* $\{\mathcal{C}, \mathcal{A}, v, \Pi\} : \mathbb{A} \to \mathbb{R} \times \mathbb{A} \times \mathbb{R} \times \mathbb{R}_+$ *defined such that, for all* $a \in \mathbb{A}$*:*

$$\{\mathcal{C}(a), \mathcal{A}(a)\} = \operatorname*{argmax}_{c,a' \geq 0} \; \bar{y} + ra - c - a' + R^{-1}\Pi(a')$$

subject to (4) *and* (5) *and where*

$$\begin{aligned} v(a) &= u(\mathcal{C}(a)) + \beta v(\mathcal{A}(a)) \\ \Pi(a) &= \bar{y} + ra - \mathcal{C}(a) - \mathcal{A}(a) + R^{-1}\Pi(\mathcal{A}(a)). \end{aligned}$$

(ii) For any $a \in \mathbb{A}$*, the* ***Markov-perfect contract*** *implied by an MPE is the transfer schedule:* $\mathcal{T}(a) \equiv ra - \mathcal{C}(a) - \mathcal{A}(a)$*.*

Solving for an MPE involves finding a fixed-point in the agent value v and the insurer's profit functions Π.

We briefly characterize the properties of the MPE with bargaining using the first-order conditions of the insurance problem. With Lagrange multipliers μ, λ and ζ associated with the constraints (4), (5), and $a' \geq 0$, respectively, the first-order conditions are:

$$-1 + \mu u_{cc}(c) + \lambda\{(1-\theta)u_c(c) + \theta\} = 0 \tag{6}$$

$$-1 + R^{-1}\Pi_a(a') - \mu\beta v_{aa}(a') + \lambda\{(1-\theta)\beta v_a(a') - \theta[-1 + R^{-1}\Pi_a(a')]\} + \zeta = 0. \tag{7}$$

The values of the Lagrange multipliers, specifically, whether they are zero or not, are critical to understanding the equilibrium properties.

Lemma 1 *In an MPE, the Lagrange multiplier on the surplus-splitting rule* (5) *is positive, that is,* $\lambda > 0$*.*

Proof. Re-arrange (6) as $\lambda\{(1-\theta)u_c(c) + \theta\} = 1 - \mu u_{cc}$. Given that $u_c > 0$ and $\theta > 0$, the sign of λ is the same as the sign of the right-hand side. Since (4) is an inequality constraint, $\mu \geq 0$. Thus, given $u_{cc} < 0$, the right-hand side of the previous expression is strictly positive, which implies $\lambda > 0$. ■

If, in addition, $\mu > 0$, then (4) implies an interior solution for future assets, and so, $\zeta = 0$.[3] Conditions (6) and (7) can then be solved to obtain the values of μ and λ. The optimal consumption and savings (c, a') implied by the Markov-perfect contract with an interior solution for assets are characterized by

$$u_c(c) = \beta v_a(a') \tag{8}$$

$$(1-\theta)[u(c) + \beta v(a') - \Omega(a)] = \theta[\bar{y} + ra - c - a' + R^{-1}\Pi(a')]. \tag{9}$$

We further describe the properties of the MPE insurance contracts numerically, in Section 4.1 below.

3.2.1 Discussion

If assets are contractible and there is a strictly positive rate of return differential between the parties (the case $R > r$), it would be optimal to have assets carried over time at the higher rate R. However, since in our setting the insurer cannot commit to future transfers, the only way it could take over all the agents's assets would be to appropriately compensate him today. This would imply inducing disproportionably high consumption today, which is not optimal for intertemporal smoothing reasons. This implies that the agent carries assets over time at the lower rate r. Note that the key problem is that the insurer is unable to commit to a long-term disbursement of the returns from assets via future transfers. In contrast, if the insurer could commit to an infinitely long contract, one can show that it is optimal to extract all agent's assets at the initial date (see Karaivanov and Martin, 2015 for details).

When assets are non-contractible, the agent can use savings to influence his future outside option, $\Omega(a')$. Hence, a conflict between the parties arises whenever the insurer has market power—the insurer would prefer if the agent held lower assets, which implies higher demand for market insurance by the agent due to his lower ability to self-insure and thus, higher profits for the insurer. In contrast, the agent would prefer larger future assets, a' which would raise his outside option, $\Omega(a')$ by providing a better ability to self-insure. The interplay of these incentives is illustrated in the numerical analysis below.

3.3 Special cases: monopoly and perfect competition

Above, we wrote the Markov-perfect insurance problem for any $\theta \in (0, 1)$. To gain more intuition about the properties of its solution, we describe what happens in two limiting cases, as θ goes to zero or one. The limiting case $\theta \to 0$ implies that the agent has no bargaining power and corresponds to the case of a monopolist insurer. Note that, as $\theta \to 0$, the surplus splitting rule (5) converges to $u(c) + \beta v(a') = \Omega(a)$. Since in an MPE, the agent's value is $v(a) = u(\mathcal{C}(a)) + \beta v(\mathcal{A}(a))$, it follows that $v(a) = \Omega(a)$, that is, the agent always receives present value equal to his outside option. In other words, when the agent faces a monopolist insurer, all the gains from the contract go to the insurer and the agent receives the same value as in autarky. Note that this applies regardless of whether

[3] Generically, an interior solution for asset choice implies $a' > 0$. However, it is possible to have an interior solution where $a' = 0$ and where the non-negativity constraint, although satisfied with equality, does not bind. In either case, $\zeta = 0$.

constraint (4) binds or not. However, as we show in the numerical analysis, the savings decision of the agent affects, in general, the profits that the insurer can extract.

The other limiting case, $\theta \to 1$, can be interpreted as the agent having maximum bargaining power (the insurer has zero bargaining power) and corresponds to the setting of perfect competition (free entry by insurers). Notice that as $\theta \to 1$, the surplus splitting rule (5) converges to $\bar{y} + ra - c - a' + R^{-1}\Pi(a') = 0$. Since in an MPE we have, $\Pi(a) = \bar{y} + ra - \mathcal{C}(a) - \mathcal{A}(a) + R^{-1}\Pi(\mathcal{A}(a))$, this implies $\Pi(a) = 0$, that is, the insurer receives zero expected present value profits. This holds for all asset levels $a \in \mathbb{A}$ and all periods. In turn, this implies that $\Pi(a) = \bar{y} + ra - \mathcal{C}(a) - \mathcal{A}(a) = 0$, or equivalently, $\Pi(a) = \bar{y} - \mathcal{T}(a) = 0$. In other words, if $\theta \to 1$, the insurer makes zero expected profits *per period*. Since this also implies that $\Pi_a(a) = 0$ for all $a \in \mathbb{A}$, as $\theta \to 1$ the first-order conditions (6) and (7) simplify to

$$
\begin{aligned}
-1 + \mu u_{cc}(c) + \lambda = 0 \\
-1 - \mu\beta v_{aa}(a') + \lambda = 0.
\end{aligned}
$$

As shown in Karaivanov and Martin (2015), Proposition 5, with free entry by insurers, the agent's value function $v(a)$ is strictly concave. Thus, $v_{aa} < 0$, which, together with $u_{cc} < 0$, implies that the above conditions are satisfied if and only if $\mu = 0$. Intuitively, when all the surplus from the risk-sharing contract goes to the agent, there is no misalignment between the insurer and the agent in how much assets to save and thus, the incentive-compatibility constraint (4) does not bind.

4 The Role of Asset Contractibility

4.1 Theoretical analysis

Does asset contractibility matter for the degree of insurance and the time-profiles of consumption and savings? In other words, how important is it for risk-sharing whether the insurer can or cannot bind the agent to a specific savings level? To answer these questions, we investigate whether the incentive-compatibility constraint (4) binds in an MPE, and under what conditions. If the constraint does not bind, then whether saving decisions can or cannot be contracted upon would not matter for risk-sharing. If the constraint does bind, however, then clearly the agent and the insurer have conflicting views of what savings should be. In the proposition below, we show that, in general, asset contractibility does matter for the contract terms.

Proposition 1 *In an MPE, if $\Pi_a(a') < 0$ for some $a \in \mathbb{A}$ such that $a' = \mathcal{A}(a) > 0$, then the incentive-compatibility constraint* (4) *binds, that is, the Lagrange multiplier μ is positive.*

Proof. Suppose $\mu = 0$. Then (6) and Lemma 1 imply $1 - \lambda\theta = \lambda(1-\theta)u_c(c) > 0$. Since $a' > 0$, we have $\zeta = 0$ and so we can rearrange (7) as

$$R^{-1}\Pi_a(a')(1-\lambda\theta) = \lambda(1-\theta)[u_c(c) - \beta v_a(a')]$$

The left-hand side is negative since, by assumption, $\Pi_a(a') < 0$ and since, as shown above, $1 - \lambda\theta > 0$. The right-hand side, however, is non-negative by (4), $\lambda > 0$ and $\theta \in (0, 1)$ – a contradiction. ■

The proposition above shows that, as long as insurer's profits are strictly decreasing in the agent's assets for some $a' > 0$ in $\mathbb{A}$ at which the agent is not borrowing-constrained, then Markov-perfect insurance contracts in which the insurer is able to specify and control agent savings (equivalently consumption) differ from Markov-perfect contracts in which the insurer is unable to do so. That is, asset contractibility matters for any asset level a satisfying the proposition conditions. Insurer's profits that monotonically decrease in the agent's assets (holding bargaining power θ constant) naturally arise, for example, if the agent's preferences exhibit decreasing absolute risk aversion (DARA). In that case richer agents have lower demand for market insurance (they can do more smoothing via their own assets) compared to poorer agents. The borrowing constraint $a' \geq 0$ is also less likely to bind for richer agents. See the numerical analysis section below for an illustration.

We can gain more intuition by looking at the special cases when θ approaches its bounds. As shown in Karaivanov and Martin (2013), in the monopolistic insurer case (the case $\theta \to 0$), if u is unbounded below and satisfies a mild technical condition, MPE contracts with and without asset contractibility differ and asset contractibility affects the insurer's profits. The reason is that the commitment friction creates a misalignment in the asset accumulation incentives of the contracting parties. Intuitively, the agent can use his ability to save privately to increase his outside option, since Ω is strictly increasing in a, ensuring higher future transfers, which counters the principal's desire, coming from profit-maximization, to drive the agent towards the lower utility bound $\Omega(0)$.

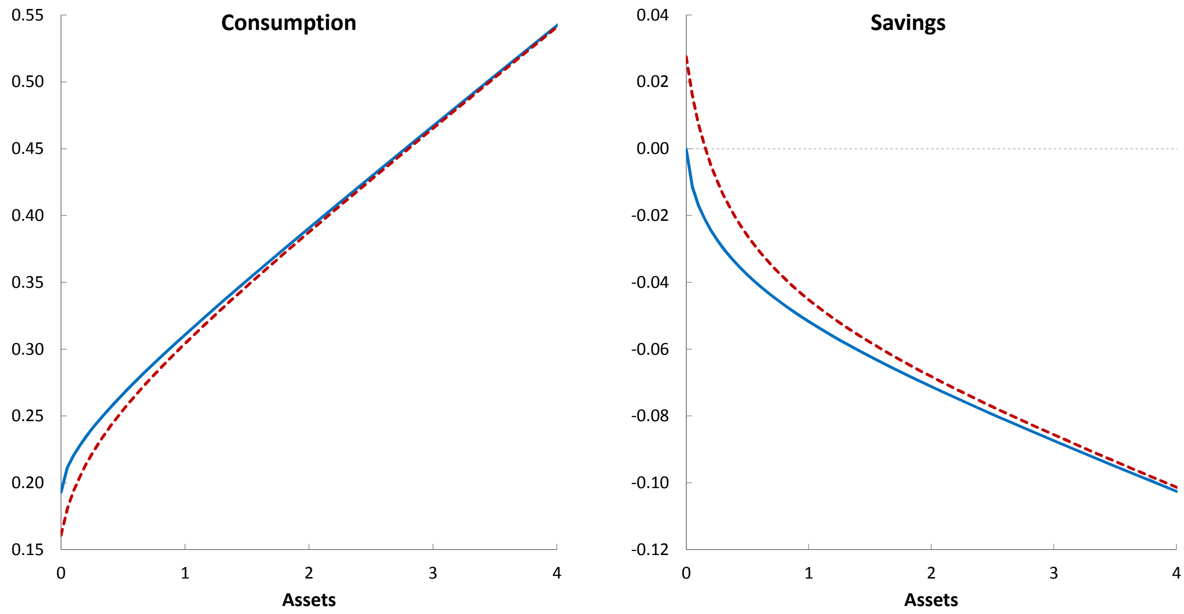
As $\theta \to 1$, the case of free entry by insurers, we showed above that the insurer makes zero expected profits *per period* for all assets levels a and that $\mu = 0$—the savings incentive-compatibility constraint (4) does not bind. In this case, since all of the surplus goes to the agent, the objectives of the two sides are perfectly aligned, and because the insurer makes zero expected profits per period, asset contractibility is irrelevant—the insurance contract is the same, regardless of whether the insurer can control the agent's savings. The result that, with free entry, the insurer makes zero profits per period is critical, as it does not allow the insurer to exploit his rate of return advantage when $r < R$ if assets are contractible.

4.2 Numerical analysis

We illustrate and quantify the effects of asset contractibility in Markov-perfect insurance contracts using a numerical simulation. We adopt the parameterization we used previously in Karaivanov and Martin (2015). Specifically, suppose $u(c) = \ln c$ and pick the following parameter values: $\beta = 0.93$, $r = 1.06$, $R = 1.07$, $y^1 = 0.1$, $y^2 = 0.3$ and $\pi^1 = \pi^2 = 0.5$. These parameters imply expected output $\bar{y} = 0.2$. For market power, we choose $\theta = 0.5$ as benchmark and analyze below the effects of varying it.

To compute the various cases we use the following method. We start off by computing the autarky problem. We use a discrete grid of 100 points for the asset space but allow all choice variables to take any admissible value. Cubic splines are used to interpolate between grid points. The upper bound for assets $\bar{a}$ is set to 5 which ensures that the asset accumulation functions always cross the 45-degree line (i.e., the upper bound is

Figure 1: Consumption and Savings



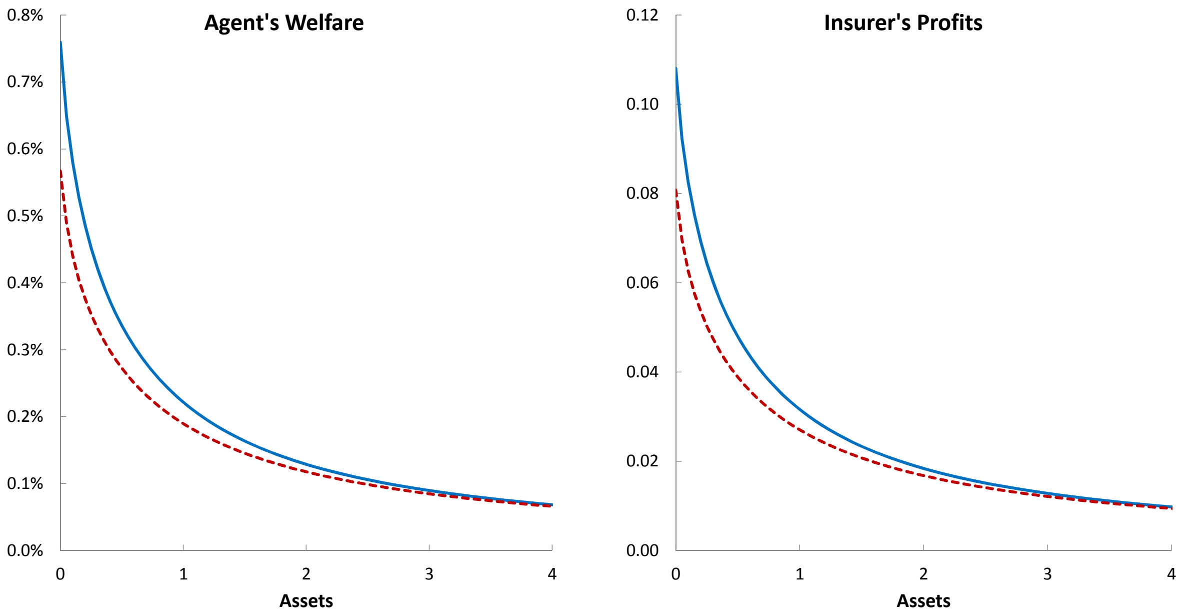
Note: The solid line corresponds to the case with contractible assets and the dashed line to the case when assets are not contractible.

never restrictive). Next we compute the Markov-perfect equilibrium assuming $\theta = 1$ (perfect competition), since in this case asset contractibility does not matter. We use the first-order conditions to the autarky and MPE problems to compute the numerical solutions for each case. Having solved the MPE with $\theta = 1$ we use it as the starting point to compute an MPE for other assumptions on market power and asset contractibility. These problems are solved using standard value function iteration methods.

Figure 1 displays the agent's consumption, c and *net savings*, $a' - a$, as a function of the agent's current asset level a. The solid line corresponds to the case with contractible assets, i.e., when constraint (4) is not imposed. The dashed line corresponds to the case when the agent's choice of a' is not contractible, that is, when constraint (4) is imposed. We see that the agent's consumption is strictly increasing in his assets while net savings are decreasing in assets. Allowing for the savings decision to be part of the insurance contract results in higher consumption and lower savings for the agent. Intuitively, when assets are contractible, the insurer wants to push agent's assets towards zero as this generates lower outside option and more profits. In addition, less assets are carried over time at the agent's rate of return r instead of the higher return R. The long-run implications of asset contractibility are also significantly different. When agent's assets are not contractible, if we start the agent with some initial assets a_0 and use the computed MPE to simulate the insurance contract for infinitely many periods, then the agent's assets converge in the limit to a positive value. This can be seen by the fact that the dashed-line in the right panel of Figure 1, which shows savings $a' - a$, is above zero for low enough asset values and below zero for sufficiently high asset levels. In contrast, when savings are contractible, the agent's assets converge to zero in finite time, as proven in Karaivanov and Martin (2015).

Figure 2 shows the implications of asset (non-)contractibility for the agent's wel-

Figure 2: Welfare and Profits



Note: The solid line corresponds to the case with contractible assets and the dashed line to the case when assets are not contractible.

fare and the insurer's profits. Agent welfare is measured as the per-period consumption equivalent compensation that the agent would require in autarky in order to be indifferent between remaining in autarky and accepting the insurance contract. Formally, for any $a \in \mathbb{A}$, we define the welfare gains as

$$\Delta(a) \equiv \exp\{(1-\beta)[v(a) - \Omega(a)]\} - 1.$$

Insurer's profits are measured as the expected net present value $\Pi(a)$, which is expressed in output units. As we can see, both the agent welfare and insurer's profits are strictly decreasing in agent's assets a. This is intuitive: at lower asset levels the agent is less able to self-insure and therefore benefits more from additional insurance. That is, the surplus generated in an insurance contract, which is proportionally split between the parties, is larger when the agent's wealth is lower.

Notice that the welfare gains for the agent in an MPE relative to self-insurance can be substantial: at the extreme, at zero assets (no ability to self-insure), they amount to almost 0.8% of consumption per period. The welfare gains are still significant at higher asset levels, converging towards 0.1% of autarky consumption per period, which is about the same number as the estimated cost of business cycles fluctuations for the average agent—see Lucas (1987). The welfare loss that arises if the agent's assets are non-contractible (the difference between the solid and dashed lines on the figure) can be large too: at zero assets, it is about 0.19%. This difference, however, becomes negligible at high asset levels.

Turning to the insurer's profits, we see that they are the largest when the insurer contracts with an agent with zero assets (given our log utility, this corresponds to the highest demand for insurance and lowest ability to self-insure). In this case, the net present value of profits equal 54% and 40% of the expected per-period output ($\bar{y} = 0.2$), for the cases with and without contractible assets, respectively. As we can see, being able to contract on the savings decision can also significantly boost insurer's profits, in addition to agent's welfare.

5 Extensions

5.1 Market power

We now analyze how the degree of the insurer's market power affects the results. That is, how do Markov-perfect insurance contracts change when we vary the bargaining power parameter θ? The proportional surplus-splitting rule, (5) directly implies that raising the agent's bargaining power θ strictly increases the agent's net surplus from market insurance, $v(a) - \Omega(a)$, relative to the insurer's present-value profits profits $\Pi(a)$.

Using the parameterization from the previous section, we quantify the effects of market power on the agent's welfare and the insurer's profits. Figure 3 shows the consumption equivalent compensation $\Delta(a)$ and the insurer's profits at zero assets, plotted as a function of the parameter θ. Recall that higher θ can be interpreted as *lower* market power for the insurer. As we see from the graph, unsurprisingly, the agent's welfare increases with his bargaining power, while the insurer's profits decrease. As we converge to a more competitive environment (higher θ), the agent's welfare increases considerably. In the extreme, at $\theta \to 1$ (perfectly competitive insurance market), the consumption equivalent compensation value of insurance in an MPE for an agent with zero wealth is about 4.5% of his autarky consumption, per period. At the other extreme, when $\theta \to 0$ (monopolistic insurer), the profits of an insurer facing an agent with zero wealth are the largest, with net present value about 65% of expected period output.

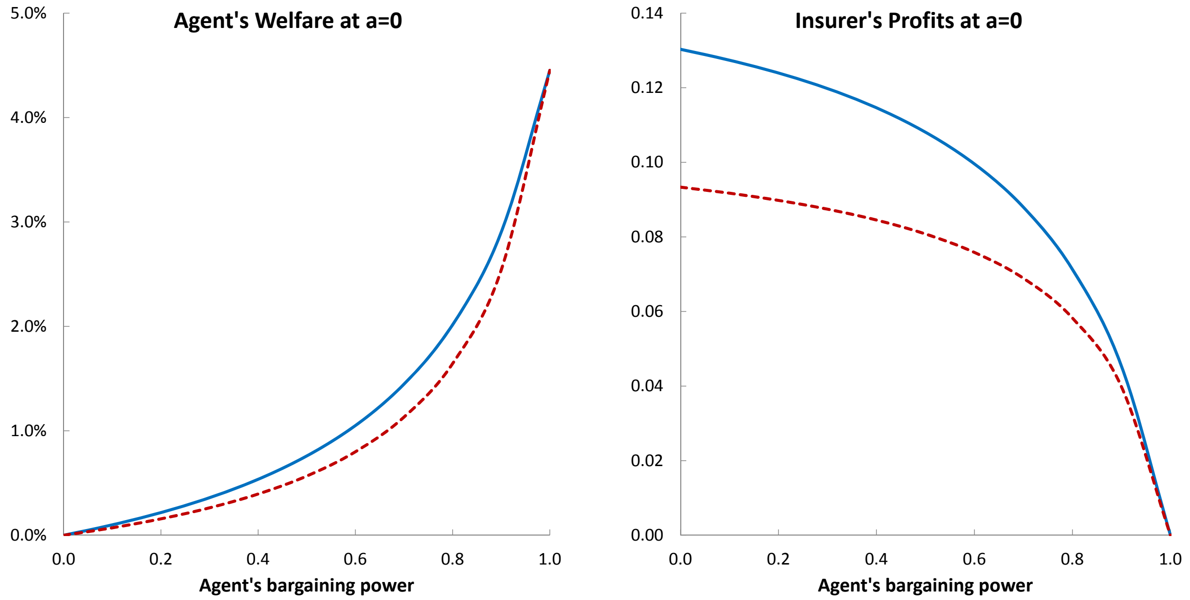
Figure 3 also shows that both the agent and the insurer lose (in terms of welfare or profits) when the agent's assets are not contractible over the whole range $\theta \in (0, 1)$. Interestingly, for the agent, the largest welfare loss from savings non-contractibility, equal to about 0.4% of autarky consumption, occurs at an interior value for the bargaining power parameter, at around $\theta = 0.8$. Remember that the agent cannot benefit from asset contractibility in the monopoly case ($\theta \to 0$) since in that case all gains from controlling the aget's assets go to the insurer. Also, as argued above, the agent does not benefit from asset contractibility in the case of perfect competition ($\theta \to 1$) since in that case the MPE with and without asset contractibility coincide (see Sections 3.3 and 4.1). For the insurer, the largest loss from asset non-contractibility occurs as $\theta \to 0$ (the monopoly case), with magnitude slightly higher than 14% of expected period output.

5.2 The Rate of Return R

We next analyze the effects of varying the insurer's intertemporal rate of return, R. Increasing R is equivalent to decreasing the factor by which future insurer's profits are discounted, that is, making the insurer more impatient. Note that there is no direct productivity effect of varying R as the agent's output technology, and hence total resources, are independent of R. In addition, the agent's autarky problem (1) remains the same.

Figure 4 plots the agent's welfare gains in an MPE relative to self-insurance, as mea-

Figure 3: Market Power



Note: The solid line corresponds to the case with contractible assets and the dashed line to the case when assets are not contractible.

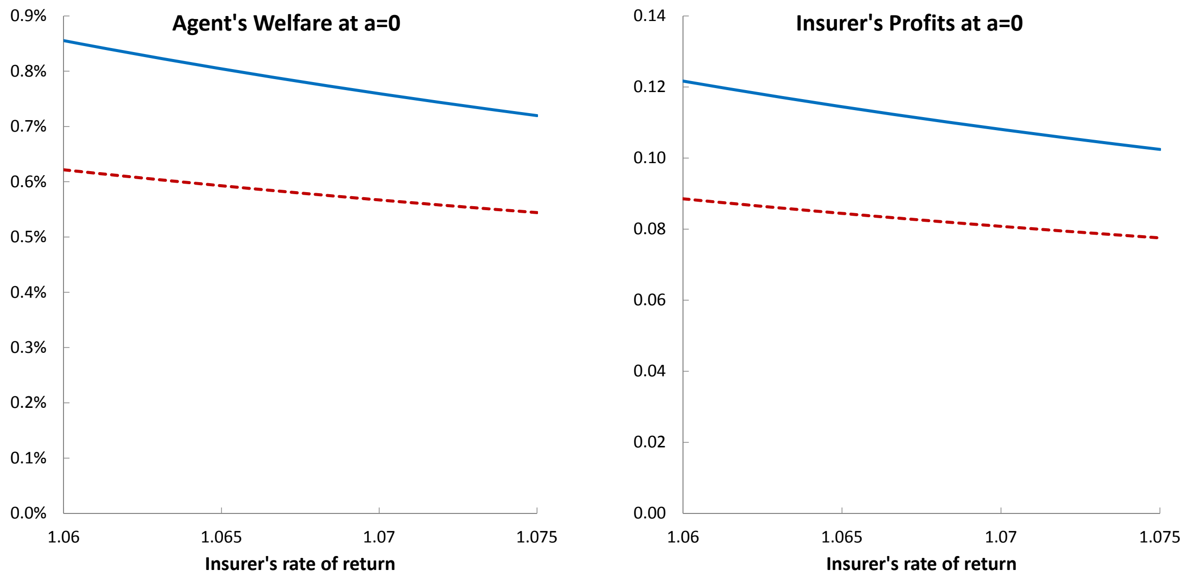
sured by $\Delta(a)$, and the insurer's present value profits $\Pi(a)$ as we vary R over its full range, from $R = r = 1.06$ to $R = 1/\beta \approx 1.075$. All other parameters, including the bargaining power θ, are held fixed at their respective benchmark values. In the interest of providing the clearest intuition for the results, we focus on the case of zero assets, $a = 0$. All other asset levels provide a similar qualitative picture (details are available upon request).

Two main results are evident from Figure 4. First, both the agent's welfare gains relative to autarky and the present value of the insurer's profits are strictly decreasing in R. The intuition for this result is found by looking at the direct effect of varying R on the agent's and insurer's surplus in the contract. If the decision variables c and a' were held fixed, the agent's surplus, $u(c) + \beta v(a') - \Omega(a)$ is constant in R, while the insurer's surplus, $\bar{y} + ra - c - a' + R^{-1}\Pi(a')$, is strictly decreasing in R. At $a = 0$, when assets are contractible we have $a' = 0$ and thus, when R increases, the only way to satisfy the proportional surplus-splitting constraint (5) is to decrease agent's consumption. When $a > 0$, savings decisions do vary with R and hence, there are further effects on welfare and profits.[4] Our numerical simulations show that, for the chosen parameters, the overall effect still goes in the same direction as in the case of zero assets. The difference in welfare gains as R varies can be substantial; for example, at zero assets, going from $R \approx 1/\beta$ to $R = r$ results in a welfare increase for the agent equivalent to 0.14% of his autarky consumption per period.

Second, Figure 4 shows that our results on the effects of asset (non-)contractibility

[4]In particular, the agent would prefer to contract with an insurer who has an intertemporal rate of return R closer to the agent's rate of return r as this mitigates the distortion in the time-profiles of consumption and savings arising from the commitment friction (see Karaivanov and Martin, 2015, Section 3.2 for additional details).

Figure 4: Rates of Return



Note: The solid line corresponds to the case with contractible assets and the dashed line to the case when assets are not contractible.

continue to hold for all admissible values of R. Making assets contractible increases both the agent's welfare and the insurer's profits (compare the dashed with the solid lines). Quantitatively, at zero assets, the welfare gains from making the agent's assets contractible are the highest at $R = r = 1.06$ and are equivalent to 0.23% of autarky consumption per period, compared to 0.19% at the benchmark value of $R = 1.07$ or 0.18% at $R \approx 1/\beta$.

6 Concluding remarks

We study the role of assets contractibility, market power and the rate of return differential between insured and insurers in a dynamic risk-sharing setting with a limited commitment friction. We find significant welfare effects along all three dimensions. Potential lessons from our analysis with relevance for actual insurance markets with commitment frictions similar to those we model indicate the desirability of increased competition, extending the ability to condition insurance terms on both the current assets and the savings of the insured, as well as mitigating the possibility of large return on assets differentials between insurance providers and households or firms.

References

[1] Aiyagari, S.R. (1994), "Uninsured Idiosyncratic Risk and Aggregate Saving", *Quarterly Journal of Economics* 109(3): 659–684.

[2] Arnott, R. and J. Stiglitz (1991), "Moral Hazard and Nonmarket Institutions: Dysfunctional Crowding Out of Peer Monitoring?", *American Economic Review*, 81(1): 179-90.

[3] Allen, F. (1985), "Repeated principal-agent relationships with lending and borrowing", *Economics Letters* 17(1-2): 27–31.

[4] Bester, H. and R. Strausz (2001), "Contracting with Imperfect Commitment and the Revelation Principle: The Single Agent Case", *Econometrica* 69(4): 1077–98.

[5] Bewley (1977), "The Permanent Income Hypothesis: A Theoretical Formulation", *Journal of Economic Theory* 16, p.252-92.

[6] Cole, H. and N. Kocherlakota (2001), "Efficient Allocations with Hidden Income and Hidden Storage", *Review of Economic Studies* 68(3): 523–42.

[7] Doepke, M. and R. Townsend (2006), "Dynamic mechanism design with hidden income and hidden actions", *Journal of Economic Theory* 126(1): 235–85.

[8] Fernandes, A. and C. Phelan (2000), "A Recursive Formulation for Repeated Agency with History Dependence", *Journal of Economic Theory* 91: 223–47.

[9] Kalai, E. (1977), "Proportional solutions to bargaining situations: Interpersonal utility comparisons", *Econometrica* 45: 1623–30.

[10] Karaivanov, A. and F. Martin (2013), "Dynamic Optimal Insurance and Lack of Commitment", Working Papers 2011-029, Federal Reserve Bank of St. Louis.

[11] Karaivanov, A. and F. Martin (2015), "Dynamic Optimal Insurance and Lack of Commitment", *Review of Economic Dynamics* 18(2): 287–305.

[12] Maskin, E. and J. Tirole (2001), "Markov Perfect Equilibrium", *Journal of Economic Theory* 100(2): 191–219.

[13] Schechtman, J. and V. Escudero (1977), "Some Results on an Infinite Fluctuation Problem", *Journal of Economic Theory* 16: 151–66.

[14] Stokey, N., R. Lucas and E. Prescott (1989), *Recursive Methods in Economic Dynamics*, Harvard University Press, Cambridge, MA.