Development Dynamics with Credit Rationing and Occupational Choice

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July, 2004

Abstract

The paper presents a stylized general equilibrium model of a developing economy in which the wealth distribution, the interest rate, and the wage rate are endogenous and interact dynamically. A credit market imperfection stemming from limited commitment results in allocative inefficiency due to credit rationing and occupational choice constraints. Credit rationing is shown to persist as the economy develops. We argue that wealth inequality in the above framework could be detrimental for economic development, providing a room for welfare improving redistribution policies. The model is shown to be capable of matching both general empirical regularities pertaining to developing economies as well as actual macroeconomic data from Thailand.

Keywords: occupational choice, credit rationing, economic development

JEL Classification: O12, O16, D58

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1 Introduction

Recently there has been a considerable interest in the economic development literature to study the dynamics of the wealth distribution. It is well known that, in a standard Solow-style complete markets setting, the distribution of wealth among agents is irrelevant - long-run output, the interest rate and the wage rate are all determined uniquely by total wealth and savings behavior, independently of the initial or subsequent wealth distributions. This changes completely, however, when financial market imperfections result in credit rationing, i.e. when some agents are willing to borrow but unable to obtain loans. In such environments, not only total wealth but the whole wealth distribution affects the resulting outcome both in the short and in the long run. This, in some cases, can leave room for welfare improving redistribution policies.

A particularly important issue is how the degree of wealth inequality is related to the process of economic growth and development. The relationship may be bidirectional: on the one hand a given degree of current inequality may affect the development process in the future but, on the other hand, as the economy grows, inequality may increase or decrease. Under incomplete markets the degree of inequality inherent in the wealth distribution typically also affects the equilibrium wage and interest rates, which can lead to complicated development dynamics characterized with multiple equilibria, hysteresis or cycles.\footnote{See Galor and Zeira (1993), Banerjee and Newman (1993), Piketty (1997), Lloyd-Ellis and Bernhardt (2000).}

The main purpose of this paper is to propose a computable general equilibrium setting in which the wealth distribution, the wage rate and the interest rate interact dynamically in an occupational choice framework. As a result we create a stylized working model of a developing economy using which we can fully describe the static and dynamic characteristics of basic economic aggregates such as income, investment, wages, interest rates, capital and labor shares, entrepreneurship, inequality, etc. The model implications are then compared to several stylized facts from the literature characterizing the process of economic development. In addition, our methodology allows us to introduce various types of tax, employment or investment regulatory policies and analyze their static and dynamic impact on the economy. We also test the empirical content of the model using actual data from Thailand.

Various theoretical models\footnote{Piketty (1997), Aghion and Bolton (1997), Lloyd-Ellis and Bernhardt (2000), Banerjee and Newman (1993), Ghatak and Jiang (2002) and Gine and Townsend (2004).} have stressed the importance of incomplete information and limited commitment in the credit market and their occupational choice implications in generating a development process characterized with inequality and, in some cases, 'poverty traps', i.e. equilibria in which the economy fails to grow out of an initial underdeveloped technological state. Piketty (1997) and Aghion and Bolton (1997) concentrate on the dual dynamics of the wealth distribution and the interest rate in models with no labor market. Limited liability and unobservability of effort induce credit rationing based on inherited wealth. The interest rate is determined endogenously and depends on the distribution of wealth. As wealth accumulates, the demand for credit declines while supply rises leading to a decrease in the interest rate and the disappearance of credit rationing as the economy matures. In contrast, Banerjee and Newman (1993) and Ghatak and Jiang (2002) study the interaction between the wealth distribution and the wage rate in an occupational choice framework with an exogenous interest rate. Agents choose among three occupations: worker, entrepreneur or subsister. Once again, there is a credit market imperfection induced by the assumption that agents can renege on their debt and get away unpunished with some positive probability. This, together with a non-convexity in the technology available to the entrepreneurs, implies that the long-run distribution of wealth may depend on the initial conditions.

All four papers discussed above concentrate on the long run wealth distribution investigating the
conditions under which it is unique or multiplicities arise. As such they devote insufficient attention to issues of empirical interest such as the dynamics of macroeconomic variables. In addition, in all of them either the credit or the labor market are exogenous. Lloyd-Ellis and Bernhardt (2000) make the first step towards creating a dynamic occupational choice model capable of characterizing the evolution of key macroeconomic aggregates during the process of development. Still, they lack an endogenous credit market, focusing instead on agents' entrepreneurial efficiency as an additional source of heterogeneity. Their results depend crucially on the heterogeneity of entrepreneurial ability and the strong assumption that ability is uncorrelated with wealth. When efficient entrepreneurs are relatively abundant, a ‘traditional’ development process emerges (e.g. Lewis, 1954, Fei and Ranis, 1966), where the evolution of macroeconomic variables follows certain well-established empirical regularities and inequality initially increases and then decreases in Kuznets (1955) style. If, instead, efficient entrepreneurs are relatively scarce, the model generates cyclical behavior. The Lloyd-Ellis and Bernhardt (2000) model has been recently augmented by Gine and Townsend (2004) who attach to it an exogenously expanding credit sector and then calibrate and estimate it with developing country data.

This paper is inspired by and complements all of the above providing several new insights. First, we demonstrate that fully endogenizing both the labor and credit markets is very important since the interaction between the interest and wage rates is crucial for the development dynamics. Second, our results show that credit rationing is not necessarily a temporary phenomenon occurring only at certain stages of the development process, but instead can persist as the economy grows. Thus, in contrast to Piketty (1997) and Aghion and Bolton (1997), credit constraints may be present also in developed economies as confirmed by several empirical studies

3. Third, relative to Banerjee and Newman (1993) we develop further the degree of realism in the endogenous labor market, relaxing certain indivisibilities and introducing valued leisure, which makes the labor supply elastic and allows the equilibrium wage rate to move continuously as opposed to taking one of two values. Finally, compared to the relatively complex setting of Lloyd-Ellis and Bernhardt (2000), the model proposed here is extremely easy to compute while at the same time preserving the capability to generate plausible development dynamics. This feature of our approach makes it extremely well-suited for computationally intensive purposes such as calibration, maximum likelihood estimation, and structural empirical work in general.

The paper characterizes an equilibrium development process driven by the endogenous interaction of the distribution of wealth with credit constraints and occupational choice. Individuals, in contrast to Lloyd-Ellis and Bernhardt (2000), differ only in their inherited wealth. Nevertheless, the model succeeds in matching the basic empirical regularities related to development, demonstrating that the extra degree of freedom inherent in ability heterogeneity is not indispensable for obtaining realistic development dynamics. This result is extremely valuable in terms of empirical work since entrepreneurial ability is notoriously hard to measure and may be correlated to an unknown extent with entrepreneurial wealth.

Agents in the model choose whether to work as entrepreneurs or wage workers. The credit market is incomplete because of limited enforceability of credit contracts. All agents who have wealths below some threshold would choose to renege on their loans and thus are refused lending. Initially, because of the credit rationing, few agents are wealthy enough to be able to borrow and become entrepreneurs, so the economy is poor, wages and incomes are low, and credit is scarce resulting in a high interest rate. Over time wealth gradually accumulates, allowing more people to become entrepreneurs, the interest rate goes down, wages are bid up, profits rise and income inequality increases. However, as the economy continues to grow, wages rise enough so that profits begin to fall and the wealth inequality starts to decline, though aggregate income and wealth continue to increase. The credit rationing threshold initially goes down while the effect of the decrease in the interest rate is strong enough but

3For instance, Evans and Jovanovic (1989) and Holtz-Eakin, Joulfaian and Rosen (1994).
later climbs back up as wages grow and thus entrepreneurs need more resources to start up a business. The interaction between the credit and labor markets works in two opposite directions through the credit rationing mechanism. On the one hand higher interest rates mean higher borrowed amounts which implies a bigger number of constrained agents who become workers, while on the other hand high wages imply high wage bills, i.e. increased absolute levels of rationing. This implication of the model is new to the literature and uncovers an interesting linkage between the credit and labor markets.

The benchmark development process described in the paper should be viewed as highly stylized. Our analysis concentrates predominantly on qualitative predictions and shows only the main forces at play in the short and in the long run. As in Lloyd-Ellis and Bernhardt (2000), we show that our results match some important stylized facts associated with development\(^4\): (i) wages are initially low but then rise monotonically, (ii) development is accompanied by an increase in entrepreneurial activity, (iii) the labor share of income rises with per capita income, (iv) the capital-output ratio initially rises, then levels off and may decline as wages rise, (v) income growth rates are high in the early stages of development then decline as the economy matures, (vi) income inequality rises in Lorenz sense during the early phases of development then gradually falls.\(^5\)

Because the existence of credit rationing does not allow all willing agents to become entrepreneurs, the equilibrium which obtains does not, in general, feature an efficient scale of entrepreneurial activity and there could be room for a welfare-improving redistributionary policy. One example for such a policy is considered in section 5, namely the introduction of a proportional inheritance tax with its revenue equally distributed among all agents. As a result, the economy grows at a higher rate and reaches higher levels of income and wealth compared to those observed before the introduction of the tax. In addition, inequality is substantially reduced. Yet, the goal of the re-distribution should not be ex-post equality, since at least early on, some agents should receive relatively large bequests to avoid credit rationing.

The paper proceeds as follows. Section 2 contains a detailed description of the model. Section 3 discusses some theoretical results and defines and characterizes the competitive equilibrium. The following section concentrates on the numerical simulation of the model, describing in detail the computational algorithm, the simulation results and comparative statics. Section 5 deals with extensions investigating the results of introducing a redistributive tax policy and calibrating the model using macroeconomic data from Thailand. Section 6 concludes and provides potential directions for future research. All proofs can be found in the appendix.

\section{The Model}

\subsection{General Setup and Timing}

Consider an economy with a discrete, infinite time horizon, \(t = 0, 1, 2\ldots\) and a constant population of a continuum of dynasties. Each period only a single member of each dynasty is active. She lives for only one period and then has one child. Each person comes with an endowment, \(w\) which is a bequest inherited from her parent. The initial \((t = 0)\) endowment (wealth) distribution is given by \(G_0(w)\).

In every period the only source of heterogeneity among the agents is their wealth. Thus the state of the economy in a given period, \(t\) is the current distribution of wealth, represented by the function

\footnotesize
\begin{itemize}
\item \(^4\)See also Maddison (1989).
\item \(^5\)This is the statement of the famous Kuznets hypothesis. For empirical evidence supporting it, see Paukert, 1973; Ahluwalia, 1976; Lydall, 1979; Summers, Kravis and Heston, 1984; Lindert and Williamson, 1985 and Williamson, 1991.
\end{itemize}
$G_t(w)$. Aggregate wealth at time $t$ is then:

$$W_t = \int wdG_t(w)$$

Each agent has complete information about her own wealth and the aggregate distribution of wealth $G_t(w)$. Agents are endowed with one unit of labor and can choose among two possible occupations: an entrepreneurial activity involving hiring labor, or working for a wage. They earn an income $y_t$ which, in the end of each period, is divided into consumption $c_t$ and bequest $b_t$.

Each time period consists of three stages: occupational choice, production and consumption. In the beginning of the period an agent can become an entrepreneur or a worker. This choice is potentially constrained by the existence of credit rationing, requiring one to possess a certain minimum amount of wealth in order to be eligible to borrow and finance an enterprise. This constraint is known by the agents before making their occupational and consumption decisions. Production begins after the agents have chosen their occupations: those who become entrepreneurs use their inheritances plus borrowed funds (if able to obtain such) to establish firms and hire workers at the market wage rate. Those who become workers put their inherited wealth with a competitive financial intermediary (a bank) which provides credit to the borrowing entrepreneurs. At this time the workers also choose how many labor hours to supply. When this stage is over, each agent decides how much to consume and bequeath, has a single child and dies. A new period begins, with the bequests forming the new initial wealth distribution.

2.2 Preferences

Agents are risk neutral and maximize income, $y$ minus disutility of labor:

$$U = y - \frac{l^2}{2\mu}$$

where $0 \leq l \leq 1$ is hours worked and $\mu > 0$. For simplicity, it is assumed that, in order to get any output with positive probability, entrepreneurs have to put in their total labor endowment, $l = 1$. Workers, however, supply labor elastically. Taking into account that if the current wage is $v$, worker’s income is $vl$, it is straightforward to verify that the optimal choice of $l$ is

$$l^* = \min\{1, \mu v\}$$

i.e. the labor supply is upward sloping until it reaches its maximum possible level of 1.

As in the classic Solow (1956) model (and also in all the papers on occupational choice listed in the introduction), we assume that each period a fixed fraction $s$ of end-of-period income is saved, i.e. $b_t = sy_t$. A common interpretation of this assumption is that, each agent maximizes a Cobb-Douglas utility defined directly on consumption and bequest (Andreoni, 1989), $U = Ac^{1-s}b^s - \frac{l^2}{2\mu}$, so that indirect utility for income is just $U = y - \frac{l^2}{2\mu}$, and optimal consumption and bequest are $c = (1-s)y$, and $b = sy$.

2.3 Technology

The entrepreneurial production technology, $y = f(k,l)$ involves two inputs - capital, $k$ and labor, $l$. There is a fixed minimum capital outlay required to start an enterprise, $k = I$. We also assume for
simplicity that each entrepreneur must hire at least \( m \) workers. Output is uncertain and takes the following values:

\[
y = \begin{cases} 
q & \text{with probability } p \\
0 & \text{with probability } 1 - p \\
0 & \text{if } k \geq I \text{ and } l \geq m \\
0 & \text{otherwise}
\end{cases}
\]

### 2.4 Credit Market and Rationing

There exists a competitive financial intermediary (bank) which can take deposits and lend to the agents. The bank can observe the wealth level and input choice of a potential borrower. However, as in Banerjee and Newman (1993), it is possible that a borrower renege on a loan contract and flee with the borrowed amount. This limited commitment problem generates credit rationing in equilibrium. If a borrower reneges, she is caught with probability \( \pi \) and suffers a utility penalty of \( F \) (interpreted for instance as imprisonment).

The bank provides loans by offering a financial contract, specifying repayments \((R_f, R_s)\), contingent on project failure or success. Assuming perfect competition and free entry, it earns zero profits in equilibrium. Let us also assume that workers must be paid the current wage \( v \), whatever the outcome of the entrepreneur’s project is, so the money needed for wage payments must be available in advance. Thus, a borrower with wealth \( w < I + mv \) who is willing to become an entrepreneur would request from the bank a loan of \( I + mv - w \). Since, if the project fails, the outcome is 0, the optimal lending contract must have:

\[
R_f = 0, \quad R_s = \left( \frac{r}{p} \right) (I + mv - w),
\]

where \( r \) is the gross interest rate in the economy.

In addition, the loan contract must be incentive compatible, i.e. the bank would not lend to agents who will renege in equilibrium. A borrower will not renege if and only if her expected utility of doing so is higher than that of running away with the money, i.e. if:

\[
pq - r(I + mv - w) \geq pq - p\pi F \quad \text{(ICC)}
\]

which is equivalent to:

\[
w \geq I + mv - \frac{\pi p F}{r} \equiv \hat{w}(r, v)
\]

The above inequality defines a threshold wealth level such that all agents with \( w < \hat{w}(r, v) \) will be refused credit by the bank. If such an agent strictly prefers to be an entrepreneur, the above implies that she is credit rationed. The intuition about why poorer agents are credit rationed is simple: it is exactly those agents who have to pay back the largest amounts and hence have the highest incentives to renege. Alternatively, we can interpret the debt contract as the agent supplying her wealth, \( w \) as a collateral for a loan of \( I + mv \) issued by the bank. The credit rationing condition then says that only agents with high enough collateral will be given access to credit.

Notice that the credit rationing threshold, \( \hat{w} \) depends on both the interest rate, \( r \) and the wage rate, \( v \). This result has important implications for the development dynamics occurring in the model economy. In particular, it is the reason why, as the economy grows, credit rationing does not necessarily have to disappear. The threshold is decreasing in \( p, \pi \) and \( F \) since they increase either the probability or the size of the utility punishment when caught. Notice also that \( \hat{w}(r, v) < I + mv \), i.e. a rationed agent cannot self-finance the production process.

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\(^6\)In contrast, the credit rationing threshold depends only on the interest rate in Banerjee and Newman (1993), Piketty (1997) and Aghion and Bolton (1997) and on exogenous parameters only in Ghatak and Jiang (2004) and Galor and Zeira (1993).
The existence of credit rationing implies that some agents will be financially constrained due to the inefficiency in the credit market. Such agents are willing to borrow but are refused credit because they do not have sufficient wealth to guarantee that they will perform on their loan. Thus, they are "forced" to become workers, which results in an inefficiency in production. This feature of the model relates it to the liquidity constraints literature, e.g. Evans and Jovanovic (1988), who empirically test and confirm the existence similar credit constraints in small business data. Additional supporting evidence is provided by Holtz-Eakin, Joulfaian and Rosen (1994), who test for liquidity constraints using data on inheritances to avoid endogeneity and selection bias problems. Feder et al. (1991) find evidence for binding credit constraints in a developing country context using data from China. These results show that credit constraints exist even in developed economies. Our findings match this observation and in this way differ from Aghion and Bolton (1997) or Piketty (1997) who postulate that credit rationing disappears as the economy develops.

If an entrepreneur has enough wealth to manage without borrowing, i.e. if \( w \geq I + mv \), she invests \( I + mv \) into the production project and puts the rest in the bank earning a safe return of \( r \) per unit. Her expected utility is thus \( U^E(w, r, v) \equiv pq + r(w - I - mv) - 1/2\mu \).

If an agent borrows, i.e. if \( \hat{w}(r, v) \leq w < I + mv \), she earns expected utility of \( pq - pr(p(I + mv - w) - 1/2\mu = U^E(w, r, v) \) as well.

A worker earns a wage \( v \), works \( l^* \) hours and deposits all her wealth in the bank, thus obtaining utility of \( U^W(vl^* + rw - (l^*)^2/2\mu) \).

Given the current \( r \) and \( v \), each agent chooses an occupation which provides her with maximum utility. The interest and wage rate equate the supply and demand for credit and labor, generated by the agents’ optimal decisions. More specifically, \( r \) and \( v \) must solve the following two market clearing conditions, where \( w^* = I + mv \):

\[
\int_{\hat{w}(r,v)}^{w^*}(w^*(v) - w)dG_t(w) = \int_{w^*(v)}^{\infty}(w - w^*(v))dG_t(w) + \int_0^{\hat{w}(r,v)} wdG_t(w);
\]

or, equivalently:

\[
(credit) \quad [G_t(\infty) - G_t(\hat{w}(r,v))]w^*(v) = W_t \quad (1)
\]

and

\[
(labor) \quad [G_t(\infty) - G_t(\hat{w}(r,v))]m = l^*G_t(\hat{w}(r,v)); \quad (2)
\]

Because of the presence of credit rationing, the market clearing \( r \) and \( v \) depend on the whole distribution of current wealth.

### 2.5 Realized Incomes and Bequests

In the end of each period an agent with initial wealth \( w \) has a realized income \( Y \) given as follows:

- for a borrower:
  \[ Y = Y^B = \begin{cases} 
  q - (r/p)(w^* - w) & \text{with probability } p \\
  0 & \text{with probability } 1 - p
  \end{cases} \]

- for an unconstrained non-borrower:
  \[ Y = Y^{NB} = \begin{cases} 
  q - r(w^* - w) & \text{with probability } p \\
  r(w - w^*) & \text{with probability } 1 - p
  \end{cases} \]

i.e. she gets her deposited money back since she is a net lender and thus represents no risk for the bank. This is more realistic than Piketty’s (1997) assumption that agents lose all money they have deposited when they fail.

- for a worker:

\[ Y = Y^W(vl^* + rw - (l^*)^2/2\mu) \]
\[ Y = Y^W = vl^* + rw \]

Each agent then consumes a fixed fraction \((1 - s)\) of her income and passes the rest \((s)\) as a bequest to her child, for whom it represents initial wealth.

## 3 Theoretical Results

This section presents the main theoretical results implied by the model. We start by specifying and solving the agents’ optimization problem. Next, we define a competitive equilibrium for the model economy and prove the existence and uniqueness of market clearing wage and interest rates. The section concludes with the characterization of a long-run stationary wealth distribution with credit rationing. Certain necessary conditions for the existence of such distribution are also discussed.

### 3.1 Occupational Choice

Agents differ only in their starting wealth, \(w\) and thus all differences in occupational choices and input usage or supply are driven by the heterogeneity of the wealth distribution and the credit rationing mechanism. The agents take their bequest, \(r\) and \(v\) as given, calculate the utility outcome of choosing each occupation and opt for the one which delivers maximum utility. When making their choice they also take into account the level of \(\hat{w}\), the minimum wealth level above which one is eligible for a credit from the bank, i.e. they may be constrained in their occupational choice.

If an agent opts to become a worker, she chooses the optimal number of hours to supply. As we saw in section 2, the individual labor supply is given by \(l^* = \min\{1, \mu v\}\). With respect to the factor markets, we then have that each worker supplies \(w\) units of credit and \(l^*\) units of labor. There are two types of entrepreneurs. Agents with \(\hat{w} \leq w < w^*\) are net borrowers from the bank, whereas those with \(w > w^*\) are net creditors. In terms of credit market, we can think of the first type of agents (‘borrowing entrepreneurs’) as demanding \(I + mv - w\) units of credit. Similarly, the second type (‘non-borrowing entrepreneurs’) can be thought of supplying \(w - I - mv\) units of credit. All entrepreneurs demand \(m\) units of worker’s labor\(^7\).

After the occupational choices are made, inputs are hired and production occurs. At the end of the period agents collect their realized final wealth, \(Y\) and decide how much to consume, \(C\) and bequeath, \(B\). Remember that given our assumption, we have:

\[ C = sY \quad \text{and} \quad B = (1 - s)Y \]

Note that the agent’s problem is separable, i.e. the occupational choice stage which determines \(Y\) is independent of the consumption and bequest decision. Given \(r\) and \(v\), if the final wealth of two agents is the same, they bequeath and consume the same amounts, even when they had chosen different occupations. Thus, the optimization problem can be separated in two stages: first, choose the occupation which yields the maximum possible utility and second, solve for the optimal consumption and bequest given the end-of-period wealth obtained in the first stage.

Each agent can pick one of the two possible occupations. However, some agents are restricted in their occupational choice as not everyone is able to obtain a loan and become an entrepreneur. As a result, there could be three types of agents in the model economy. Let \(l^*_i\) and \(l^*_D\) be the supply and demand for labor, \(c^*_S\) and \(c^*_D\) supply and demand for credit, and \(\hat{u}(w, r, v)\) be the expected utility for type \(i = 1, 2, 3\) defined as follows:

\(^7\)They can be also thought of supplying and demanding themselves the 1 unit of entrepreneurial labor they implement but this is irrelevant for the labor market equilibrium.
1. Non-borrowing entrepreneurs: \( l^1_S = 0, l^1_D = m, c^1_S = w - I - mv, c^1_D = 0, u^1(r, v, w) = U^E(w, r, v) \).

2. Borrowing entrepreneurs: \( l^2_S = 0, l^2_D = m, c^2_S = 0, c^2_D = I + mv - w, u^2(r, v, w) = U^E(w, r, v) \).

3. Workers: \( l^3_S = l^* (v), l^3_D = 0, c^3_S = w, c^3_D = 0, u^3(r, v, w) = U^W(w, r, v) \).

It turns out that the choice between being a worker or an entrepreneur does not depend on the agent’s wealth \( w \), but is driven, instead by the credit constraint. This implies the following:

**Proposition 1**

(a) For a given \( r \), there exists a unique maximum possible value for the equilibrium wage rate, \( \bar{v}(r) \) at which agents are just indifferent between being workers or entrepreneurs and which is given by the solution to:

\[
v = \frac{pg - rI + (l^* (v))^2 / 2\mu}{l^* (v) + rm}
\]

(b) \( \bar{v}(r) \) is decreasing in \( r \).

Note that the above result implies that we must have \( v \leq \bar{v}(r) \) in equilibrium, since otherwise all entrepreneurs will switch to being workers. Also, note that for \( v < \bar{v}(r) \) every agent prefers to be an entrepreneur but some of them are rationed out of the credit market and thus have to become workers.

## 3.2 Competitive Equilibrium

We now define a competitive equilibrium for the model economy and prove the existence and uniqueness of market clearing interest and wage rates. We need to introduce some further notation. Let \( T_i^t(w) \) denote the fraction of the population that become type \( i \) in equilibrium, \( i = 1, 2, 3 \) (see the list above). Given \( G_t \) and the market equilibrium at time \( t \), we know the market clearing wage and interest rates and each agent’s final wealth, \( Y \) and bequest, \( B \). Using the latter we can construct \( G_{t+1} \). This is done using the transition function for the distribution of inheritances described below. Let \( \Gamma_t \) denote the support of \( G_t, \Gamma = \bigcup_{t=1}^{\infty} \Gamma_t \) and \( Z \equiv \Gamma \times [1, \infty) \times R \). Let also \( \xi \) be a random variable, taking the values 0 or 1 and such that:

\[
\text{prob}(\xi = 1 \mid r, v, w) = \begin{cases} p & \text{if } i \in \{1, 2\} \\ 1 & \text{if } i \in \{3\}, \end{cases}
\]

where \( i \) is the occupation type of the agent.

**Definition: Competitive Equilibrium**

Given an initial wealth distribution \( G_0(w) \), a competitive equilibrium for the model economy consists of:

- sequences of wages \( \{v_t\}^\infty_{t=1} \) and interest rates \( \{r_t\}^\infty_{t=1} \)
- sequences of distributions of initial wealths \( \{G_t\}^\infty_{t=1} \) and their supports \( \{\Gamma_t\}^\infty_{t=1} \)
- a sequence of occupational types, \( \{T_i^t\}^\infty_{t=1}, i=3 \)
- a time invariant occupational choice function \( i(r, v, w) : Z \to \{1, 2, 3\} \)
- a time invariant final wealth function \( Y(r, v, w, \bar{\xi}) : Z \times \{0, 1\} \to R_+ \), where \( \bar{\xi} \) is a \( 1 \times 3 \) vector, the elements of which are independent draws of the random variable \( \xi \) for each agent type, according to (3)
- time invariant bequest and consumption functions $B : R_+ \to R_+$ and $C : R_+ \to R_+$
- time invariant labor and credit demand and supply functions $l_D^j, l_S^j, c_D^j, c_S^j : Z \to R_+$, as described in section 3.2.

such that:

- given $v_t, r_t$ the functions $i, Y, B$ and $C$ solve the maximization problem (CP) for each agent with wealth $w$, for all $t \geq 1$
- given the occupational decisions of the agents and the distribution of inheritances $G_t$:
  - the labor market clears at $v_t, r_t$:
    $$ \sum_{j=1}^{3} \int l_D^j dG_t(w) = \sum_{j=1}^{3} \int l_S^j dG_t(w), \quad \text{(LM)} $$
    where $\Delta_{jt} \subset \Gamma$ are defined as: $\Delta_{jt} = \{ w : i(r_t, v_t, w) = j \}$ and satisfy:
    $$ \sum_{j=1}^{3} \int dG_t(w) = 1 $$
  - the credit market clears at $v_t, r_t$:
    $$ \sum_{j=1}^{3} \int c_D^j dG_t(w) = \sum_{j=1}^{3} \int c_S^j dG_t(w), \quad \text{(CM)} $$
  - the sequence of supports $\{ \Gamma_t \}_{t=1}^{\infty}$ evolves according to:
    $$ \Gamma_{t+1} = \Gamma_{t+1}(\Gamma_t | r_t, v_t) = \{ w' \in R_+ : w' = B(Y(w, r_t, v_t, \bar{\xi}_t)), \forall w \in \Gamma_t \} $$
  - the next period wealth distribution is determined according to:
    $$ G_{t+1}(M) = G_t(N), $$
    where $N = \{ w \in \Gamma_t : B(Y(w, v_t, r_t)) \in M \}$ for all $M \in \sigma(\Gamma_{t+1})$, with $\sigma(\Gamma_{t+1})$ being the $\sigma$-algebra generated by $\Gamma_{t+1}$ as defined above.

Note that the equilibrium wage and interest rate depend on the whole distribution of wealth in the economy at the beginning of each period. Thus, the current distribution $G_t(w)$ affects the wage and interest rates $r_t$ and $v_t$, which in turn determine next period’s wealth and so on. This can be expressed as:

$$ G_{t+1} = G_{t+1}(G_t), \quad r_t = r_t(G_t) \text{ and } v_t = v_t(G_t) $$

The above equations define a non-linear transition function in the state variable, $G_t$, despite the fact that, from the point of view of each individual dynasty, its wealth follows a linear stationary Markov process. Because of this non-linearity one cannot apply standard techniques to establish existence, uniqueness and global stability of an ergodic measure on the wealth space and interpret it as the long run distribution of wealth. Instead, the non-linearity of the wealth transition function generates the possibility of multiple or no stationary steady states and/or path dependence as in Banerjee and Newman (1993), Piketty (1997) or Lloyd-Ellis and Bernhardt (2000).

We are interested in economies that exhibit credit rationing in equilibrium. To ensure that, we need $\hat{w}(r, v) > 0$ at the market clearing $r$ and $v$. A sufficient condition for this is given in the next:
Lemma 1

A sufficient condition for credit rationing to exist in equilibrium is: (A1) \( I > \pi p F \).

The above condition is very intuitive - it basically states that the fixed capital outlay needed to start a business is higher than the expected utility penalty of reneging on a contract, implying that agents with low enough wealth would always prefer to renege. From now on we assume that (A1) holds.

The next results provide further characterization of the competitive equilibrium defined above.

Lemma 2

The credit rationing threshold, \( \hat{w} \) is increasing in \( r \) and \( v \), i.e. \( \frac{\partial \hat{w}}{\partial r} > 0 \) and \( \frac{\partial \hat{w}}{\partial v} > 0 \).

The intuition is that, as the interest and wage rates increase, the cost of providing the necessary resources to start business goes up which increases agents’ incentive to renege. This implies that a higher wealth level is needed to guarantee the increased credit required. Unlike in Aghion and Bolton (1997), the result in Lemma 2 is consistent with the previous literature on credit rationing (e.g. Stiglitz and Weiss, 1981 and Bernanke and Gertler, 1989) which predicts that rationing is more likely when the cost of capital is high.

Given the current wealth distribution, \( G_t(w) \) and the chosen functional forms, we can describe the labor and credit market clearing conditions determining the interest and wage rates in the economy. Denote by \( \lambda_t(w) \) the measure of agents in \( G_t(w) \) with wealth less than \( w \) and let \( N \) be the total measure of agents. The following three equations can be used to solve for \( r_t \) and \( v_t \) (see (1) and (2)):

\[
(N - \lambda_t(\hat{w}(r_t, v_t)))(I + mv_t) = W_t \quad \text{(credit market)} \tag{5}
\]

\[
m\lambda_t(\hat{w})(v_t) = N - \lambda_t(\hat{w}(r_t, v_t)) \quad \text{(labor market)} \tag{6}
\]

We continue by establishing existence and uniqueness of market clearing interest and wage rates in any given period.

Proposition 2

For a given non-degenerate wealth distribution \( G_t(w) \) there exist unique market clearing interest and wage rates \( r_t \) and \( v_t \).

The above proposition guarantees that, under our assumptions, we will be able to solve for the equilibrium \( r \) and \( v \) in each period, which then can be used to compute next period’s wealth distribution using the transition functions defined above. Thus the proposition enables us to simulate and characterize the behavior of the model economy through time.

3.3 Dynamics and the Long-Run

Having described the static equilibrium within each period, let us now turn to characterizing the model dynamics concentrating on the case when they lead to a stationary long-run wealth distribution. Remember that, because of the non-linear state transition function, in general nothing guarantees that such distribution will exist so in this section we make certain simplifying assumptions to ensure that. A necessary condition for the existence of an invariant limiting distribution of wealth is that \( r \) and \( v \) are stationary, i.e. the market clearing wage and interest rate converge to some constant values \( r^* \) and \( v^* \). We first provide a heuristic characterization of an invariant distribution in our setting which is followed by a formal analysis of the conditions for its existence.
We start with deriving the transition functions for individual wealth under credit rationing. Let for simplicity $\mu$ be such that $l^* = \mu v^*$, i.e. we have an interior solution for the worker’s supply of labor. Consider a worker with initial wealth $w$. Her child’s initial wealth is then given by:

$$w' = s(\mu v^* + r^w)$$  \hspace{1cm} (7)

The above equation defines the transition function for each worker lineage’s individual wealth: a straight line with slope $sr^*$. We assume that $sr^* < 1$, so wealth does not grow without bound. From we see that if a person should remain a worker forever her wealth converges to:

$$\tilde{w}(r^*, v^*) = \frac{s\mu v^*}{1 - r^s},$$  \hspace{1cm} (8)

which is the intersection of the transition line in (7) with the 45-degree line.

Now consider an entrepreneur with $w < \hat{w}$, i.e., a borrower. For such person the wealth transition equation is:

$$w' = \begin{cases} 
0 & \text{with probability } 1 - p \\
 s(q - (r^*/p)(w^* - w)) & \text{with probability } p 
\end{cases} \hspace{1cm} (9)
$$

i.e. it can be represented by two lines - a horizontal one at 0 and one with slope $sr^*/p$. Assume that:

$$sr^*/p < 1 \hspace{1cm} (A2)$$

in order to preclude growth without bound. This also implies that $sr^* < 1$. The agent’s wealth next period is determined by one of the two branches of the above transition function, depending on success or failure of the entrepreneurial project. Observe that the slope of the upper (success) branch is larger than that of the worker’s transition function.

Finally, consider a person with $w \geq \hat{w}$, i.e., a self-financed entrepreneur. The wealth transition function for such agent is given by:

$$w' = \begin{cases} 
 sr^*(w - w^*) & \text{with probability } 1 - p \\
 s(q - r(w^* - w)) & \text{with probability } p 
\end{cases} \hspace{1cm} (10)$$

The transition equations described above are plotted on fig. 1. In equilibrium, all agents with wealth below $\hat{w}(r^*, v^*)$ are workers, all agents with $w \in [\hat{w}, w^*)$ are borrowing entrepreneurs, and the agents with $w \geq w^*$ are self-financed entrepreneurs. We know from our previous analysis that every agent who is not credit constrained would choose to be an entrepreneur. Thus, if we wish to represent graphically the overall individual wealth transition function for levels of $w$, it will coincide with the transition function of a worker for $w < \hat{w}(r^*, v^*)$, then with the one of a borrower for $\hat{w} \leq w < w^*$, and finally with the function given by (10) for $w > w^*$. By the Law of Large Numbers, a fraction $1 - p$ of all entrepreneurs fail each period and thus their children’s wealth dynamics are governed by the lower branches of the transition equations (9) – (10), whereas the rest succeed and their children receive bequests in accordance with the upper branches.

Having characterized the transition functions for individual wealth, let us now turn to describing some of the properties of the stationary wealth distribution, $G^*$, for which $(r^*, v^*)$ is a long run steady state of the dynamic system in (5), i.e. such that: $r^* = r(G^*)$, $v^* = v(G^*)$ and $G^* = G^*(G^*)$.

**Proposition 3**

In a stationary long-run wealth distribution $G^*$ as defined above it must be true that:

$$\hat{w}(r^*, v^*) \geq \hat{w}(R^*, v^*)$$  \hspace{1cm} (11)

\[\text{8Remember we have defined } w^* = I + mv.\]
The idea behind the proposition is that, in order to sustain a stationary equilibrium and hence constant fractions of each type of agents, there must exist upward interclass mobility (i.e. workers becoming entrepreneurs) to balance the downward ‘leakage’ from the entrepreneurial class due to project failures.

Notice that, within the stationary distribution, individual wealths move around as dynasties shift places within and across the different classes. To make things clearer, consider the example in fig. 2. Let us look at an agent whose parent’s project has failed having been a borrowing entrepreneur, thus leaving his child a bequest of 0. The child of the latter starts with a wealth of \( w_1 \), i.e. she is still being credit rationed and hence a worker. The next heir of the same dynasty starts with \( w_2 > \bar{w} \), so she chooses optimally to be an entrepreneur and borrow. Next, she either fails and we are back where we started, or accumulates more wealth, which moves the next dynasty member to \( w_3 \), etc. In this way, as time passes, each lineage goes through the process of accumulating wealth, then someone down the line goes bankrupt and the whole process starts anew. No one, even the richest lineage, is insured against becoming poor at some point of time, i.e. each dynasty has its good and bad times over and over again.

Now, think of not only an individual lineage but a whole cohort of them, moving through the distribution as described above. Once the cohort enters the entrepreneurial class, in each following period only a fraction \( p \) of its members remain there. Thus, the stationary distribution’s density consists of big spikes near 0 (failed agents), \( w_1 \) (workers), \( w_2 \), and progressively smaller spikes at \( w_3 \), \( w_4 \), etc.

To summarize: the stationary distribution is a result of the interaction of two forces working in the opposite directions. On the one hand, project failures cause interclass mobility which represents a movement across the distribution and across occupation types while, on the other hand, the fact that (A2) is assumed to hold, causes the convergence of wealth levels for people staying within a type.

Figure 3, which is the result of a numerical simulation of the model, confirms the intuition provided here by demonstrating an example of how a uniform initial wealth distribution gradually converges to the spiky shape suggested above.

Let us now turn to a formal characterization of the conditions for existence of an invariant distribution of wealth in our setup.

**Lemma 3**

*There exists a wealth level, \( \bar{w} \) and a time period, \( T \) such that \( \Gamma_t \subseteq [0, \bar{w}] \) for \( t \geq T \).*

The lemma states that, after some finite time \( T \), everyone’s wealth is bounded by the finite value \( \bar{w} \). The intuition for this result should be clear given assumption (A2) which rules out unbounded wealth growth.

In general, we can write down the transition function for individual wealth as:

\[
 w_{t+1} = f(w_t, \xi_t)
\]

where \( f(w_t, \xi_t) = B(Y(w_t, \bar{\xi}_t)) \) at \( r^* \) and \( v^* \). For example if \( w_t \in \Delta_1 \) (self-financed), we have:

\[
 f(w, 0) = sr^*(w - w^*) \\
 f(w, 1) = s(q - r^*w^* + r^*w)
\]

Note that \( f \) is linear in \( w \) for all types of agents. In addition, for all \( i \in \{1, 2, 3\} \), \( f \) is (weakly) increasing in \( w \). Let \( V = [0, \bar{w}] \) and let \( \sigma(V) \) be the set of all Borel subsets of \( V \). Define \( P : V \times \sigma(V) \rightarrow \)

---

9In fact, these are not spikes but spike-like spurts because of the different timing of failures.
\[ P(w, M) = \text{prob}(f(w, \xi) \in M), \ M \in \sigma(V) \]

Finally, let also \( T^*G(.) \) be the Markov transformation of \( G \) defined as:

\[ T^*G(M) = \int P(w, M)dG(w) \]

**Definition (Invariant Distribution)**

A distribution \( G \) on \( V \) is invariant for \( P \) if, for all Borel subsets \( M \in V \), we have:

\[ T^*G(M) = G(M) \]

Now we are ready to state the main result of this section which is the counterpart of Proposition 3 in Piketty (1997).

**Proposition 4**

For each possible stationary interest and wage rates, i.e. \( r_{t+1} = r_t = r^* \) and \( v_{t+1} = v_t = v^* \), \( \forall t \geq t_0 \), there exists a unique invariant distribution \( G^*(w) \).

The intuition why an invariant distribution of wealth exists whenever there exists time \( t_0 \) such that \( r_{t+1} = r_t = r^* \) and \( v_{t+1} = v_t = v^* \), \( \forall t \geq t_0 \) is that once a lineage’s wealth becomes less than \( \bar{w} \), it can never again go above this value. In addition, any measurable subset of \( V \) (i.e. any subset of \( \bigcup_{i=1}^{3} \Delta_i \)) is visited an infinite number of times by the lineage’s wealth, \( w_t \).

It remains an open question, however, whether and under what conditions, there exist stationary values for the wage and interest rate. Since the admissible \( r^* \)’s are bounded from below by 1 and the admissible \( v^* \)’s are bounded from above by \( \bar{v} \) (1), a sufficient condition to ensure the existence of limiting stationary values is the monotonicity of the sequences \( \{r_t\}_{t=1}^{\infty} \) and \( \{v_t\}_{t=1}^{\infty} \). If \( \{r_t\}_{t=1}^{\infty} \) is monotonously decreasing in time and \( \{v_t\}_{t=1}^{\infty} \) is monotonically increasing, then a standard mathematical result about convergence of bounded monotonic sequences gives us the existence of the desired limiting stationary values \( r^* \) and \( v^* \) and thus, by the above proposition, the existence of a unique invariant wealth distribution as in Aghion and Bolton (1997). There is a big difference, however, between our result and that of Aghion and Bolton, due to the fact that the model economy here exhibits credit rationing in the long-run, in contrast to theirs and, in addition, the credit rationing threshold depends on both \( r \) and \( v \) in our model.

A formal proof of the monotonicity of the sequences for \( r \) and \( v \) is intractable due to the reasons discussed above, and this property will not hold for all parameter values. However, numerical simulations show that for a high enough saving rate, so that the aggregate wealth of the economy grows in time, \( r_t \) indeed decreases and \( v_t \) increases monotonically in time.

To conclude, in general nothing guarantees that the model economy will converge to a stationary equilibrium. We saw above that if such an equilibrium exists it has to satisfy certain restrictive conditions. This should not be disappointing, however, since the main objective of the paper is to characterize the process of economic development, i.e. the transition path generated by the model dynamics.
4 Numerical Simulation

In this section we describe the algorithm and the results of a numerical simulation of our model. As we saw in the previous section, it is hard to derive analytical results about the model dynamics due to the complexity and non-linearity of the state transition function. That is why, to get an understanding of the dynamics, we simulate the economy initially under a benchmark specification, and then under several departures from it in order to provide comparative statics results. The simulations show that the model is able to match in a satisfactory way the stylized facts and empirical regularities characterizing developing economies listed in the introduction.

4.1 The Algorithm

Let us start by describing the numerical algorithm used to solve and simulate the model. Remember from above that the agent’s occupational choice problem is completely separable from the bequest-consumption one, with the latter producing simple linear rules. The ”hard” part thus is solving for the equilibrium interest and wage rate. To achieve that, we use the already derived optimal decision rules for an agent with wealth \( w \) facing an interest rate \( r \) and a wage rate \( v \). In addition, for computational reasons, we compute a discrete version of the model substituting the continuum of agents with a large finite population.

Computational Algorithm

Step 1

Before we actually start simulating the dynamics of the economy we need to specify the following:

1.1. the total number of agents, \( N \).

1.2. the initial wealth distribution, \( G_0(w) \). In the benchmark specification we take a discrete uniform distribution on \([0, 2]\), i.e. 200 equally spaced points on this interval.

1.3. parameter values for \( I, m, s, q, p, \pi, F, \mu \) and \( \varepsilon \) (error tolerance) - see table 1 below. The parameter values are chosen in a way so that \( l^*(v) = \mu v \) for all simulated time periods although it is clear that the same algorithm can be used when \( l^* = 1 \) as well.

Step 2

Solving for the equilibrium number of entrepreneurs, \( n^e \):

2.1. Start at some \( n^e \), e.g. \( N/2 \).

2.2. Use the market clearing conditions for labor and credit to obtain an equation for \( n^e \). That is, express \( v \) from the credit market clearing condition (5) and substitute it in the labor market equation, (6) to get:

\[
(N - n^e)\mu \left( \frac{W}{n^e} - I \right) = n^e
\]  

2.3. Use a bisection method with tolerance \( \varepsilon \) to solve the above equation for \( n^e \) and then round the solution to the nearest integer, \( \tilde{n}^e \).

Step 3

3.1. Plug \( \tilde{n}^e \) obtained above back into the credit market equilibrium condition to obtain the equilibrium wage rate: \( v = \frac{W - \tilde{n}^e I}{m\tilde{n}^e} \).

3.2. Sort the agents in ascending order of their wealth and use \( \tilde{n}^e \) to find the wealth threshold level \( \tilde{w} \) such that exactly \( \tilde{n}^e \) agents have wealths higher than \( \tilde{w} \). In particular, since we cannot use a continuum of agents in the simulations, I use \( \tilde{w} = (w_{\tilde{n}^e} + w_{\tilde{n}^e + 1})/2 \), where \( w_i \) denotes \( i \)-th highest wealth in \( G(w) \).

3.3. Use the threshold level, \( \tilde{w} \) from above to solve for the equilibrium interest rate \( r = \frac{p\pi F}{W/\tilde{n}^e - \tilde{w}} \).
Step 4
4.1. Compute the utility levels for each type of agents.
4.2. Compute the demands and supplies for credit and labor for each individual, using the results in sections 2 and 3.
4.3. Compute the aggregate demand and supply for labor and credit.

Step 5
Compute the realized income for each agent using the equations in section 2. For each entrepreneur success or failure are determined by simultaneously drawing random numbers from a uniform distribution on [0, 1]. If the number drawn is less than $p$ the entrepreneur succeeds, otherwise she fails. Having computed the realized final incomes, compute the bequests, the distribution of which represents next period’s initial wealth distribution.

Step 6
Repeat steps 2 through 5 for the desired number of periods.

Step 7
Due to the stochasticity of output, repeat the above simulation a large number of times (1000) in a Monte Carlo fashion and report averages across draws.

The idea behind the algorithm is to use the inherent separability in the determination of the interest and wage rate from the market clearing conditions and the credit rationing constraint. Note that we first solve the system of market clearing equations in $n^e$ and $v$ and then use the functional relationship between $n^e$ and $r$ implied by the rationing threshold, $\hat{w}$ to obtain the equilibrium interest rate, $r$. Then, having found $r$ and $v$ we can proceed to compute agents’ utilities, supplies and demands, as well as their bequests which represent next period’s initial wealth distribution.\(^{10}\)

<table>
<thead>
<tr>
<th>Table 1 - Benchmark Specification</th>
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<tbody>
<tr>
<td>Parameter</td>
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<td>$\mu$</td>
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<tr>
<td>$I$</td>
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<td>$p$</td>
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<td>$\pi$</td>
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4.2 Results
This section presents the results obtained by numerical simulations of the model under different parameter configurations. We will concentrate mainly on the benchmark specification (parameter values are given in Table 1), characterizing the dynamics of several major macroeconomic variables. In addition, we explore some comparative statics by varying the model parameter values around the benchmark.

The macroeconomic aggregates which the model is capable of generating and which characterize the equilibrium of the economy in each period are: the gross interest rate, $r$, the wage rate, $v$, the total end-of-period wealth, $W$, the aggregate amounts of capital and labor used, $K$ and $L$, the GDP (computed as the difference between end-of-period and beginning-of-period wealth), $Y$, the GDP’s growth rate, $\gamma_Y$, the income share of labor, $vL/Y$, the fraction of entrepreneurs in the economy, $\phi^E$, the Gini coefficient (gini) used as a measure of income inequality, the credit rationing threshold, $\hat{w}$, the growth rate of end-of-period wealth, $\gamma_W$, and the capital-output ratio, $K/Y$.

\(^{10}\) A Matlab program code implementing the above algorithm is available from the author upon request.
4.2.1 Benchmark Specification

The results from the numerical simulation at the benchmark parameters are presented in table 2 and fig. 4. We briefly characterize the behavior of each of the economic variables listed above, verifying whether the model succeeds in matching the empirical regularities presented in the beginning of the paper.

Looking at table 2 we see that both total end-of-period wealth, $W$ and income, $Y$ grow on average over the 25 simulated periods\(^{11}\), with the former increasing at a slightly higher rate. The growth of GDP and total wealth is also matched by growing capital utilization. The accumulation of wealth by the economy increases the availability of credit over time and leads to a decline in the equilibrium net interest rate from 48.9\% to approximately 7\%. At the initial stages of development most agents are relatively poor and, as such, they are rationed out of the credit market since, because of the high interest rate, $\hat{w}$ is relatively high. The credit rationing forces these agents to become workers which results in a relatively abundant supply of labor and a low wage rate. As the economy grows, agents gradually accumulate wealth and more of them become eligible for credit: first because they have more collateralizable assets and second, because $\hat{w}$ falls as $r$ decreases. This leads to a decrease in the equilibrium share of workers from 90\% to 58\% and the wage is bid up to .37 in period 25 which represents a raise of more than 7 times relative to the initial level. The wage rate behavior exhibited in the model thus matches fact (i) from the list in section 1.

Since in our model everyone who is not worker is an entrepreneur, fact (ii) is also confirmed, due to the decline in the number of workers as the economy grows. Wages increase at a higher rate than that of the decrease in the number of workers and this leads initially to a steady increase in the wage labor share of income by period 20, followed by levelling off as the economy matures which matches fact (iii). The capital-output ratio at first slightly rises on average, then declines a bit in the last periods: fact (iv). Fact (v) is also matched by the model economy. This can be seen most clearly in the behavior of the growth rate of income, $\gamma_Y$, which is above 10\% in the first seven periods but after that averages only about 3\%. Total wealth also exhibits higher growth rates in the initial stages of development.

Last but not least comes the confirmation of fact (vi): the model economy exhibits a Kuznets curve, with inequality increasing sharply in the early periods (from .33 to .82) and then going down to a much lower level (.51). The reason for this pattern is the following: at first while wages are relatively low, the few entrepreneurs in the economy earn a lot in profits and interest income on the high wealth they possess thus income inequality worsens. As the economy accumulates wealth, profits and interest incomes decline, wages grow and the income gap between workers and entrepreneurs is reduced.

Finally, let us also comment on the behavior of the credit rationing constraint, $\hat{w}$ which, as we saw, is key to the model and quite different compared to the previous literature. As we know from section 3, $\hat{w}$ is increasing in $r$ and $v$. At first, due to the decline in the interest rate caused by the increase in credit supply, $\hat{w}$ goes down (from .18 to .02), allowing more agents to enter the entrepreneurial occupation. This bids the wage up and that eventually offsets the effect of the declining interest rate. As a result, with $v$ increasing in the later stages, $\hat{w}$ goes up again although it does not reach its initial value. This prediction of our model, driven by the dependence of the credit rationing constraint on the wage is lacking from the previous occupational choice literature. It causes the rationing threshold to persist and even grow in absolute value instead of disappearing as the economy develops. Remember, however, that aggregate income also grows, so measured as a percentage of average initial income, the rationing threshold actually declines. This is illustrated in the declining number of workers even after $\hat{w}$ starts to increase.

\(^{11}\)To facilitate the presentation of the results we do not report the numbers for all 25 simulated periods. The complete simulation results are available from the author upon request.
4.2.2 Comparative Statics

This section explores some comparative statics of the model. We consider in turn changes in each of the parameter values and the initial wealth distribution and study the resulting implications for the simulated economy.

Tables 3 considers a decrease in the saving rate, $s$, to 0.31. As in Aghion and Bolton (1997), the saving rate is of crucial importance for the model dynamics since it determines the fraction of final wealth carried over to the next period by each dynasty. A lower saving rate compared to the benchmark results in lower levels and growth rates for total wealth, $W$, and income, $Y$. Moreover, if $s$ is sufficiently low, it can cause a sustained decline in economic performance resembling a recession. Income inequality rises fast initially but then, unlike in the benchmark, remains at a very high level due to the large gap between the incomes of the few very rich business owners and those of the majority of poor workers.

Consider now a change in the initial wealth distribution to a more unequal one chosen in such way that the total wealth in the economy is kept at the same level as in the benchmark case - see table 4. The increase in initial inequality seems not to affect too much the long-run levels of income, wealth, investment and labor supply relative to the benchmark. Notice, however, that the economy growth rate is affected negatively by the increase in inequality, especially in the latter periods confirming the intuition from the literature that the initial wealth distribution can have sustained long run effects unlike in the classic Solow model. We also see that the higher initial inequality actually leads to higher average levels of wealth and income in the first periods, implying that a certain degree of inequality can be good at least in the short run. The empirical regularities of development are once again confirmed. The only notable difference is the sustained decline in the Gini coefficient - it does not rise in the early stages as it already starts at a high level.

The rest of the comparative statics are summarized in table 5 where we show the main effects of varying the remaining structural parameters of the model. A 10% increase in the necessary start-up cost $I$ from 1 to 1.1 results in a decline in the average income and wealth growth rates, together with lower attained levels of $Y$, $W$, and $K$, compared to the benchmark. In addition, the interest and wage rates, the labor share and the number of entrepreneurs are also lower. The main reason for these effects is that less agents can afford to become entrepreneurs and the lower demand for labor by their firms repercussions across the whole economy by lowering the wage earnings of the workers. Since the latter represent the majority of the population, this leads to a slower growth and lower income levels. Inequality increases since only the richest agents become entrepreneurs and earn high profits (due to the low wage levels), whereas the majority of the population consists of poor workers. As in Banerjee and Newman (1993) this suggests that it is more important to have more "middle class" workers than few entrepreneurs making huge profits, i.e. in this sense inequality can be detrimental for the development process. The empirical regularities given by facts (i)-(vi) from section 1 continue to hold despite the different levels of the economic aggregates.

A decrease in the probability of success, $p$, to 0.85 results in a higher probability of failure and hence lower levels of the income and wealth aggregates (about 60% of their benchmark counterparts). The growth rates of GDP and $W$ are also significantly lower with aggregate investment, $K$ following suit. The interest and wage rates are also lower driven by the lower demands for credit and labor. Income inequality remains high, together with the number of workers employed.

A 10% increase in the labor supply parameter, $\mu$, to 2.2 does not lead to any dramatic changes in the economy compared to the benchmark. There is a slight increase in the income and wealth growth

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12 The full results are readily available from the author.
13 The idea that inequality can be harmful for the economic development has received significant attention in the recent literature (see Benabou, 1996 for an excellent review).
rates and levels, caused by the fact that for a given wage workers now supply more labor hours \((L)\) is about 6-7\% higher on average). The fact that workers earn more leads to a higher rationing threshold and a lower long-run degree of inequality.

An increase in \(\pi\) (the probability of punishing a reneging borrower) from .5 to .55 also has almost no effect on most the economic aggregates except the interest rate. It is interesting to note that the negative direct effect of the increase in \(\pi\) on \(\hat{w}\) is offset by the general equilibrium effect working through the higher interest rate. The reason is that the equilibrium number of entrepreneurs is determined only by the wage rate, which implies that \(r\) must adjust to offset changes in \(\pi\) or \(F\) in order to achieve the implied level of \(\hat{w}\). Since \(\pi\) and \(F\) enter multiplicatively and symmetrically in the model equations, it is clear that everything said above would also apply for an increase in \(F\). The qualitative implications of the model with regards to the stylized development facts remain robust to changes in these two parameters.

Finally, a decrease in the high output level, \(q\) leads, as expected, to lower levels and growth rates and results similar to those observed at \(I = 1.1\) - an economy with a high number of workers, low wages and persisting inequality. The relative poverty of this economy compared to the benchmark is also reflected in the behavior of the credit threshold, \(\hat{w}\) - after plunging in the beginning it stays at a relatively low level throughout all periods as now workers have on average much less wealth than the entrepreneurs. Still, despite the low levels, the stylized facts from the introduction remain evident in the results.

To conclude: the comparative statics confirm the robustness of the model results to moderate changes in the benchmark parameters and thus support its potential usefulness as a tool to study and analyze various policies related to developing economy contexts. We consider one such policy in the next section. Furthermore, the relative robustness of the model implications can be a valuable quality when taking its predictions to the data in a structural way. The main advantage of structural empirical methods is that they allow the researcher to evaluate quantitatively various counterfactuals preserving a common basis of comparison.

5 Extensions

5.1 Introduction of a Redistributionary Tax Policy

In order to investigate further the relationship between wealth inequality and economic development, we consider the introduction of a redistributionary policy consisting of a flat rate tax on bequests, combined with a lump sum subsidy. The subsidy is equal for all agents and is fully financed by the tax revenue. Thus, the proposed policy is effectively equivalent to a progressive tax on income redistributing wealth from the rich to the poor. We show that the proposed policy leads to an increase in the growth rates and levels of income and wealth in the economy in comparison with the benchmark case. Thus, because of the inefficiency in the credit market, a well-guided redistributionary policy is capable of bringing an improvement in welfare.

To clarify matters, consider a person with end-of-period wealth \(Y_i\). Before the introduction of the tax policy her child’s initial wealth would have been \(w' = sY_i\). Under the tax scheme, however, bequests are taxed so the above relation becomes:

\[
\hat{w}' = (1 - \tau)sY_i + S, \tag{13}
\]

where \(S\) is the lump sum subsidy and \(\tau\) is the tax rate (equal to 20\% in our simulation). The government
budget is assumed to balance each period which requires:

$$NS = \int \tau sY_i di$$  \hspace{1cm} (14)

In (14), $S$ corresponds to the tax revenue collected from a person with the average final income, thus the children of agents with incomes less than the average receive a net subsidy, whereas those of agents with higher than average incomes are taxed.

Let us now turn to the implications of the above policy in our benchmark specification (table 6 and fig. 4). Given our previous findings that inequality can be harmful for economic development we should expect that redistribution, by lowering inequality, could improve economic performance. This is exactly what happens. The average growth rates of GDP and final wealth rise respectively from 6.8% to 7% and from 7% to 7.2%. The beneficial effect is also evident in the levels data - GDP’s final level is about 5% higher. The same is true for total wealth and hence consumption which under risk neutrality translates to a positive effect on welfare. Wages also reach higher levels, showing that labor income is a main driving force of economic growth in the model. All other economic variables provide further evidence for the success of the tax policy - entrepreneurial activity, investment, and the labor share are higher than their benchmark values. The empirical regularities form section 1 are once again confirmed. Notice, however, that the Kuznets curve is much less pronounced with inequality reaching significantly lower maximum levels. The reason is, of course, the progressive character of the tax policy, which by design aims at reducing inequality.

5.2 Calibration of the Model with Thai Data

As pointed out in the introduction, the emphasis of the paper has been to demonstrate the capability of our model in matching qualitatively a variety of stylized facts and empirical regularities related to the process of development. We have seen that, despite its computational simplicity, our theoretical setting has performed well in this task and our results are relatively robust to changes in the structural parameters. Below we show that, in addition to its realistic qualitative predictions, our model also has a sufficient quantitative content. As in Gine and Townsend (2004) we calibrate the model using data from Thailand and demonstrate that there exist parameter values for which the model fits well the time series of income growth, inequality, and the fraction of entrepreneurs exhibited by the Thai economy in the period 1976-1996.

Two data sources are used in the calibration exercise. Macroeconomic time series data on income growth, inequality and entrepreneurship comes from Thai national statistics. In order to simulate the model we also need data on the initial wealth distribution. For that purpose we follow Gine and Townsend (2004) and use wealth data for 1976 constructed by Jeong (1999) and based on household data from the Thai Socio-Economic Survey (SES).\textsuperscript{14}

Given the initial wealth distribution, we look for values of the structural parameters of the model that generate time series for income growth, inequality and entrepreneurship which match best the corresponding data. More specifically, we fix $\mu = 2$, $\pi = 0.5$ and $m = 1$ as in the benchmark and search over the remaining five parameters:\textsuperscript{15} $I$, $p$, $q$, $F$, $s$ to minimize a mean-squared error criterion. Formally, to perform the calibration we use as a metric the normalized equally weighted sum of period-by-period squared deviations of the three time series generated by the model from the data:

\textsuperscript{14}We encourage the reader to refer to Gine and Townsend (2004) for a detailed description of the data.\textsuperscript{15}The best match occurs for the best combination of parameters and shocks to the agents’ projects thus, a suitable vector of shocks is also searched for implicitly by the calibration procedure. The justification is that the empirical observations are treated as a particular outcome of a stochastic data generating process identified by the structural parameters.
\[ \frac{1}{3} \sum_{j=1}^{3} \sum_{t=1976}^{1996} \left( \frac{z_{jt}^{\text{sim}}}{\mu_{jt}^{\text{data}}} - z_{jt}^{\text{data}} \right)^2 \]

where \( z_{jt} \) denotes the variable \( j \) (one of the three time series being matched) at time \( t \) and \( \mu_{jt}^{\text{data}} \) is the mean of the variable \( z_{jt}^{\text{data}} \).

The results from the calibration are exhibited in fig. 5. We see that model matches extremely well the behavior of the growth rate of income in the data tracing closely its ups and downs over the 21 years in the sample. The degree of inequality measured by the Gini coefficient is also matched relatively well, although the model overpredicts the level of inequality relative to the data\(^{16}\) especially in the later periods. Notice, however, that we use the model with no redistributive taxes in the calibration exercise while some degree of redistribution is likely to be present in the actual data. Putting the panels of fig. 5 together we observe that the Thai economy has experienced significant growth coupled with increasing inequality. This has important policy implications related to poverty, safety nets, etc. which however remain outside the scope of this study. What we stress here instead, is that our model is capable of generating an environment featuring growth with increasing inequality without relying on exogenous factors as, for instance, in Gine and Townsend (2004) whose main results and ability to fit the data seem to be driven to a large extent by the exogenously expanded financial intermediation sector. Finally, the model does not perform so well on the fraction of entrepreneurs, initially understating and then overstating their number relative to the data. Still, the obtained numbers are not completely off the map - notice that the average across time periods is almost the same between the simulation and the data.

The calibrated values of the model parameters are reported in table 7 below:

<table>
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<th>Parameter Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter Values</td>
</tr>
<tr>
<td>( I )</td>
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<tr>
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</table>

The calibrated savings rate of 27% matches well the empirical reality from Thailand where savings as a percentage of national income have been around 22% between 1976-1986 and subsequently increased to 32% in the high growth period of the early 90s. There are no natural counterparts to the rest of the parameters but they do not look unreasonable in view of the model structure.

We should stress once again that the value of the calibration exercise is mostly illustrative showing the model’s ability to make reasonable quantitative predictions. Viewed as such, this exercise has no formal empirical content and little predictive power since it is not clear whether there exist other parameter configurations that match the data equally well. Structural techniques, for example based on maximum likelihood estimation as employed in Gine and Townsend (2004), Paulson and Townsend (2003) and Karaivanov (2004) should be viewed as a natural way to formalize and extend our results.

6 Conclusion

This paper contributes to the growing literature on the interaction of economic development and income inequality in an incomplete markets setting. A credit market imperfection due to limited commitment

\(^{16}\)To match the different reported levels of inequality for 1976 between the SES initial wealth distribution and the macro data, we have adjusted the actual numbers from the macro data accordingly, i.e. the solid line in the bottom panel of fig. 5 is actually a vertically shifted version of its national statistics counterpart. What is important for our purposes is that this does not affect the slope and shape of the inequality curve.
results in credit rationing, which affects agents’ occupational choices, forcing the less wealthy to become workers, since they do not have enough assets to qualify for a loan. As the economy develops, agents accumulate wealth, which diminishes the relative importance of the credit constraints, the number of entrepreneurs increases, and income grows. However, credit rationing is shown to persist even at the later stages of development due to increasing wage bills.

The model economy matches qualitatively several empirical regularities observed in the developing countries, demonstrating that the introduction of heterogeneous entrepreneurial efficiency as in Lloyd-Ellis and Bernhardt (2000) or Gine and Townsend (2004) is not indispensable once both the credit and labor markets are fully endogenized and labor supply is made elastic. The calibration of the model with Thai data demonstrated its ability to match the reality of a developing economy quantitatively as well. In this way our results could be useful for the systematic formation and evaluation of development policies where new theoretical models, as Townsend (1997) claims, are badly needed.

The paper shows that inequality is an important determinant of economic growth when market imperfections are present. We have argued that wealth inequality could be detrimental for economic development and demonstrated how a suitably designed inequality-reducing policy in the form of a progressive inheritance tax can improve economic performance. Thus our findings call for policy intervention, which is in stark contrast to, for example, Greenwood and Jovanovic (1990), where the competitive equilibrium outcome is Pareto optimal. Our results are consistent with Benabou’s (1996) thesis that inequality is perhaps the answer to the apparent puzzle raised by Lucas (1993) about the similarity between South Korea and the Philippines in the early 1960s with respect to all major economic aggregates and their subsequent strikingly different economic performances. Benabou claims that the explanation lies in the fact that the initial income distribution when development started was considerably more unequal in the Philippines which had a Gini coefficient 17% higher than that of South Korea.

Note, however, that the message that inequality can be detrimental for economic development should not be taken to extremes. As Townsend (1997) observes, and as we have seen in the simulations, some inequality can be necessary, at least in the initial stages of development, especially if the aggregate wealth in the economy is insufficient for everyone to be able to operate at efficient production scale. In such case, a certain degree of inequality ensures that growth is sustained and the economy escapes the ‘poverty trap’ in which it would have fallen if everyone were equally poor.

It should be acknowledged that the structure of the credit market in the model economy is perhaps overly simplistic. For example, Lehnert (1998) has shown that allowing agents to endogenously form rotating saving and credit associations (rosacas) in which borrowers pool their wealth and a fraction of them chosen by lottery receives credit, represents a Pareto improvement in the presence of credit rationing. Similarly, Ghatak and Guinnane (1999) have shown that introducing joint liability lending can also alleviate the credit constraints. Such financial institutions and mechanisms are indeed observed in many developing countries and incorporating them into a dynamic occupational choice model like ours can be a fruitful venue for further research. Another possible extension of our results could be introducing risk aversion, which will have non-trivial consequences given the incompleteness of financial markets17. Subject to data availability, yet another possibility might be to go beyond the simple calibration exercise and formally estimate the model using structural methods as in Gine and Townsend (2004) or Paulson and Townsend (2003).

7 Appendix

Proof of Proposition 1

(a) The wage and interest rate combination at which an agent is just indifferent between becoming an entrepreneur or a worker is given by \( u^W(r, v) = u^E(r, v, w) \) which is equivalent to:

\[
vl^*(v) + rw - \frac{l^*(v)^2}{2\mu} = pq + r(w - I - mv) - \frac{1}{2\mu}
\]

or

\[
v = \frac{pq - rI + \frac{l^*(v)^2}{2\mu} - \frac{1}{2\mu}}{l^*(v) + rm}
\] (15)

To show that the above equation has a unique solution, consider the two possible cases. First, let \( l^*(v) = \mu v \). Then (15) is equivalent to:

\[
\frac{1}{2\mu}v^2 + rmv - pq + rI + \frac{1}{2\mu} = 0
\] (16)

Since \( \frac{1}{2\mu} > 0 \) and \( rm > 0 \) the equation cannot have two positive roots, so the value of \( \bar{v} \) is indeed unique. The second case is \( l^*(v) = 1 \), where \( \bar{v}(r) = \frac{pq - rI}{1 + rm} \). The equilibrium wage rate cannot exceed \( \bar{v} \) as this would mean that all agents will be willing to be workers, implying an equilibrium wage of 0.

(b) Proof is trivial and hence omitted.

Proof of Lemma 1

We have that \( \bar{w}(r, v) = I + mv - \frac{\pi pF}{r} > I - \pi pF \) as \( v > 0 \) and \( r \geq 1 \). But then the condition in (A1) is equivalent to \( \bar{w}(r, v) > 0 \) for whatever values \( r \) and \( v \) might take, i.e. there will always be some credit rationed agents in equilibrium.

Proof of Proposition 2

Assume first that \( l^*(v) = \mu v \). Let us denote the measure of entrepreneurs in the economy by \( n_e = N - \lambda(\bar{w}(r, v)) \) and express the wage rate from the credit market clearing equation (5) in terms of \( n_e \):

\[
v = \frac{W}{n_e} - I
\]

Plug the above expression into the labor market clearing equation (6) to obtain:

\[
\mu(N - n_e)(\frac{W}{n_e} - I) = n_e
\] (17)

The above equation has a unique solution for \( n_e \) as its left hand side (lhs) is strictly decreasing and its right hand side (rhs) is strictly increasing in \( n_e \). In addition, as \( n_e \to 0 \) the lhs goes to infinity, whereas the rhs is 0, and as \( n_e \to N \) the rhs goes to 0 whereas the rhs is \( N \). We can solve (17) for the unique equilibrium mass of entrepreneurs \( n_e \), which implies a unique value for \( v \). To solve for the interest rate, remember that \( \lambda(\bar{w}) = N - n_e \). If the wealth distribution \( G \) is non-degenerate this equation has a unique solution for \( \bar{w} \), which then can be used together with the value of \( v \) computed above to solve for the unique equilibrium interest rate using that \( r = \frac{\pi pF}{I + mv - \bar{w}(r, v)} \). In the case when \( l^* = 1 \) we can solve (6) for \( n_e \) and then obtain \( v \) from (5) and \( r \) in the same way as before.
Proof of Proposition 3

Assume the opposite, i.e. \( \bar{w} > \hat{w} \). Then, if an agent is a worker, her wealth converges to \( \bar{w} \). Therefore, in the long run all workers must have wealth \( \bar{w} \) and there is no upward class mobility (workers becoming entrepreneurs). However, in every period a measure \( 1 - p \) of the borrowing entrepreneurs’ projects fail and their children start next period with an initial wealth of 0. As time goes by, these lineages’ wealth will converge to \( \bar{w} \) and settle there. Clearly, as this process continues, the measure of entrepreneurs declines monotonically and the measure of workers increases, which changes the equilibrium interest and wage rates and reduces \( \hat{w} \) until eventually it gets lower than \( \bar{w} \) - a contradiction. Thus, such situation cannot be a stationary equilibrium.

Proof of Lemma 3

Consider a self-financing individual who experiences an infinite sequence of success (\( \xi_1 = 1 \)). Her individual transition function for wealth is then given by:

\[
    w' = s(f p - R w + R w)
\]

This equation has a fixed point \( w^* f(R, v) = \frac{s(f p - R w)}{1 - s R} \), holding the assumption that \( 1 - s R > 0 \). Thus, for any agent with individual wealth level \( w_0 \), the wealth of her lineage will converge (both from above or below) to \( w^* f \) if success persists, or it will fall strictly below \( w^* f \) if a failure occurs. Since under our assumptions self-financed entrepreneurs are the agents with highest wealth, the wealth of all other lineages will be also lower than \( w^* f \). Then we can take \( \bar{w} = w^* f + \varepsilon \) for some \( \varepsilon > 0 \) and by the definition of convergence there will exist a \( T \) s.t. \( \forall t \geq T, w_t < \bar{w} \).

Proof of Proposition 4

Lemma A1 (Monotonicity of P)

The transition function \( P(w, M) \) is increasing in \( w \) in first order stochastic dominance sense, i.e. \( \forall (w, w') \in V^2, w \leq w' \) we have \( P(w', [0, x]) \leq P(w, [0, x]) \) for all \( x \in V \).

Proof of Lemma A1

The lemma says in words that the higher one’s initial wealth, the higher is the probability of reaching a final wealth above some \( x \). Consider two cases:

1. \( w, w' \in \Delta_j \) for some \( j \), i.e. both agents optimally choose to be of the same type. Then, since \( f \) is increasing in \( w \) the result in the lemma is obviously true, remembering the definition of \( P \).

2. \( w \in \Delta_j, w' \in \Delta_{j'} j \neq j' \), i.e. agents choose different occupations. Remember from section 3, that the higher one’s wealth, the broader the set of types one can choose from, i.e. in our case type \( j \) is feasible for the agent with wealth \( w' \) but was not chosen because it was found suboptimal. Thus, the agent with \( w' \) can match any choice of the other agent, i.e. any final expected wealth level that \( w \) can achieve, but in some cases can do better, so the result in the lemma holds again. \( \square \)

Lemma A2 (Monotone Mixing Condition)

The monotonic transition function \( P \) satisfies the following property: \( \forall w^* \in \bigcup_{j=1}^{3} \Delta_j \) there exists an integer \( m \) such that \( P^m(0, [w^*, \bar{w}]) > 0 \) and \( P^m(\bar{w}, [0, w^*]) > 0 \), where \( P^m(w, M) \) denotes the probability of reaching \( M \) from \( w \) in \( m \) generations.

The mechanism behind Lemma A2 was explained in the example in section 3.4. In words, it states that even the poorest individual will have lineage wealth above \( w^* \) after \( m \) consequent successes and even the richest lineage’s wealth can fall below \( w^* \) after \( m \) consequent failures.
Proof of Lemma A2

Take some \( w^* \in V \). Then since \( f(w, 1) - w \) is always strictly positive on \([0, w^*] \) and remains strictly bounded by the positive number \( a = f(\bar{w}, 1) \) (see fig. 2), there exists \( n_1 \) s.t. \( f^{(n)}(0, 1) > w^* \) for \( n \geq n_1 \), where \( f^{(n)} \) denotes the \( n \)-th iterate of \( f(\cdot, 1) \). Similarly, \( w - f(w, 0) \) is strictly positive on \([w^*, \bar{w}] \) and remains uniformly bounded from below by some positive number \( b \) (see again fig. 3). Thus, \( \exists n_0 \) s.t. \( f^{(n)}(\bar{w}, 0) < w^* \) for \( n \geq n_0 \). Take \( m = \max\{n_0, n_1\} \). We have:

\[
P^m(0, [w^*, \bar{w}]) \geq q^m > 0
\]

since the minimum probability of success is \( q \) for any \( w \). Similarly:

\[
P^m(\bar{w}, [0, w^*]) \geq (1 - q)^m > 0
\]

since the maximum probability of failure is \( 1 - q \) for all \( w \). The second inequality holds because when borrowers fail their final wealth is 0, i.e. the interval \([0, w^*] \) can be reached for all \( w^* \in V \).

We are now ready to prove the proposition. The existence of an invariant distribution \( G \) for the Markov process defined by \( P(w, M) \) follows immediately from the monotonicity of \( P \) established in Lemma A1 and from Corollary 4 in Hopenhayn and Prescott’s (1992). The uniqueness of the invariant distribution follows from the monotonicity of \( P \), its monotone mixing property (Lemma A2) and Hopenhayn and Prescott’s Theorem 2 and Corollary 2.

References


Figure 1 - Individual Wealth Transition Lines

Figure 2 - Dynasty Wealth Dynamics
Figure 3 - Wealth Distribution Dynamics

Note: Wealth is on the horizontal axis and mass of agents is on the vertical axis
Figure 4 - Dynamics of Economic Indicators

- Interest rate
- Wage rate
- Rationing Threshold
- Nr. of Entrepreneurs
- Inequality
- GDP
- Labor share
- Capital used
- Total Wealth

**Legend:**
- benchmark (no tax)
- with tax
Figure 5 - Calibration of the Model to Thai Data

GDP growth rate

Fraction of entrepreneurs

Inequality (Gini coefficient)
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<th>Period</th>
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<th>v</th>
<th>W</th>
<th>K</th>
<th>Y</th>
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**Table 3 - Comparative Statics, s=0.31**

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**Table 4 - Comparative Statics, Different Initial Wealth Distribution**

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<td>0.36</td>
<td>327.06</td>
<td>83.69</td>
<td>213.17</td>
<td>0.49</td>
<td>0.14</td>
<td>0.42</td>
<td>0.52</td>
<td>0.14</td>
<td>0.45</td>
<td>0.39</td>
</tr>
<tr>
<td>25</td>
<td>1.07</td>
<td>0.37</td>
<td>337.49</td>
<td>85.54</td>
<td>219.83</td>
<td>5.77</td>
<td>0.15</td>
<td>0.43</td>
<td>0.50</td>
<td>0.15</td>
<td>6.04</td>
<td>0.39</td>
</tr>
</tbody>
</table>

Note: The numbers in bold at the bottom of the income and wealth growth columns represent the average growth rate over the 25 periods.
### Table 5 - Comparative Statics

<table>
<thead>
<tr>
<th>Changes</th>
<th>Effects (compared to benchmark)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. A 10% increase in start-up costs, I</td>
<td>50% decline in long run wealth level &lt;br&gt;lower levels of W, K, L &lt;br&gt;27% decline in the average income growth rate &lt;br&gt;lower labor share, fraction of entrepreneurs &lt;br&gt;increase in inequality &lt;br&gt;lower absolute rationing threshold in the long run</td>
</tr>
<tr>
<td>2. A decrease in the probability of success, p to 0.85</td>
<td>lower levels of Y, W, K, L &lt;br&gt;lower growth rates of wealth and income &lt;br&gt;lower interest and wage rates &lt;br&gt;increase in inequality &lt;br&gt;lower absolute rationing threshold in the long run</td>
</tr>
<tr>
<td>3. A 10% increase in the labor supply parameter, µ</td>
<td>no significant changes in economic aggregates &lt;br&gt;slight increase in L &lt;br&gt;slight increase in the income and wealth growth &lt;br&gt;higher absolute rationing threshold &lt;br&gt;decrease in inequality</td>
</tr>
<tr>
<td>4. A 10% increase in the probability of punishment, π</td>
<td>almost no effect on most economic variables &lt;br&gt;an increase in the interest rate</td>
</tr>
<tr>
<td>5. A 10% decrease in the high output level, q</td>
<td>lower levels and growth rates &lt;br&gt;decrease in the wage rate &lt;br&gt;decrease in the fraction of entrepreneurs &lt;br&gt;increase in inequality &lt;br&gt;lower absolute rationing threshold</td>
</tr>
</tbody>
</table>

### Table 6 - Effects of A Redistributive Policy

<table>
<thead>
<tr>
<th>Period</th>
<th>r</th>
<th>v</th>
<th>W</th>
<th>K</th>
<th>L</th>
<th>Y</th>
<th>γY %</th>
<th>vL/Y</th>
<th>φE</th>
<th>gini</th>
<th>ĥ</th>
<th>γW %</th>
<th>K/Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.49</td>
<td>0.06</td>
<td>69.54</td>
<td>19.00</td>
<td>19.00</td>
<td>49.44</td>
<td>20.97</td>
<td>0.02</td>
<td>0.10</td>
<td>0.33</td>
<td>0.18</td>
<td>20.87</td>
<td>0.39</td>
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<tr>
<td>5</td>
<td>1.25</td>
<td>0.11</td>
<td>135.58</td>
<td>36.77</td>
<td>36.77</td>
<td>94.57</td>
<td>14.45</td>
<td>0.04</td>
<td>0.18</td>
<td>0.59</td>
<td>0.07</td>
<td>14.70</td>
<td>0.39</td>
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<tr>
<td>10</td>
<td>1.21</td>
<td>0.22</td>
<td>229.05</td>
<td>60.92</td>
<td>60.92</td>
<td>154.75</td>
<td>7.22</td>
<td>0.09</td>
<td>0.30</td>
<td>0.50</td>
<td>0.14</td>
<td>7.41</td>
<td>0.40</td>
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<tr>
<td>15</td>
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<td>0.31</td>
<td>297.39</td>
<td>76.71</td>
<td>76.71</td>
<td>196.64</td>
<td>2.34</td>
<td>0.12</td>
<td>0.38</td>
<td>0.41</td>
<td>0.22</td>
<td>2.65</td>
<td>0.39</td>
</tr>
<tr>
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<td>0.35</td>
<td>323.82</td>
<td>82.77</td>
<td>82.77</td>
<td>211.82</td>
<td>1.96</td>
<td>0.14</td>
<td>0.41</td>
<td>0.37</td>
<td>0.27</td>
<td>1.65</td>
<td>0.39</td>
</tr>
<tr>
<td>25</td>
<td>1.20</td>
<td>0.39</td>
<td>351.20</td>
<td>87.38</td>
<td>87.38</td>
<td>229.76</td>
<td>7.00</td>
<td>0.15</td>
<td>0.44</td>
<td>0.35</td>
<td>0.30</td>
<td>7.24</td>
<td>0.38</td>
</tr>
</tbody>
</table>

Note: The numbers in bold at the bottom of the income and wealth growth columns represent the average growth rate over the 25 periods.