Econometrica Supplementary Material

SUPPLEMENT TO "DYNAMIC FINANCIAL CONSTRAINTS: DISTINGUISHING MECHANISM DESIGN FROM EXOGENOUSLY INCOMPLETE REGIMES": ONLINE APPENDIX

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S-1. MORAL HAZARD WITH UNOBSERVED CAPITAL AND INVESTMENT

SUPPOSE AN AGENT'S EFFORT IS UNOBSERVABLE as in the moral hazard (MH) regime and, in addition, the principal also cannot observe the agent's current capital stock, k, and the choice for next period, k'. The unobserved state k adds a dynamic adverse selection problem to the moral hazard problems arising from the two unobserved actions, z and k'.

To write this setting as a mechanism design problem, suppose the agent sends a message about k to the principal who offers him a contract conditional on the message which consists of transfer τ, recommended effort z, investment k', and future promised utility. Because of the dynamic adverse selection problem in k, following Fernandes and Phelan (2000) and Doepke and Townsend (2006),[1] the proper state variable in the recursive representation of this problem is not a scalar (as in the MH regime) but a promised utility *schedule*, $\mathbf{w} \equiv \{w(k_1), w(k_2), \dots, w(k_{\#K})\}$ belonging to some set of schedules $\mathbf{W}$ (to be determined), where k_1, k_2, etc., are the elements of the grid K.

The set of feasible promise schedules $\mathbf{W}$ is endogenously determined (not all promise-assets combinations are feasible) and must be iterated upon together with the value/policy functions (Abreu, Pierce, and Stacchetti (1990)). Specifically, using the incentive compatibility constraints, we restrict attention to nondecreasing promise vectors $\mathbf{w}(k)$. We "discretize" the set $\mathbf{W}$ by starting from a large set $\mathbf{W}_0$ consisting of linear functions $\mathbf{w}(k)$ with intercepts that take values on the grid $W = \{w_{\min}, w_2, \dots, w_{\max}\}$, with $w_{\min}$ and $w_{\max}$ defined in our discussion of the MH regime, and a discrete set of nonnegative slopes. We initially iterate on the unobserved investment (UI) dynamic program using value function iteration, that is, we iterate over the promise set $\mathbf{W}$ together with the value function V, dropping all infeasible vectors $\mathbf{w}$ at each iteration and thus "shrinking" $\mathbf{W}$ as a result. Once we have successively eliminated all elements $\mathbf{w}$ in $\mathbf{W}$ for which the respective linear programs have no feasible solution, that is, once we have converged to a self-generating feasible promise set $\mathbf{W}^*$, we switch to (much faster) policy function iteration and continue iterating on the Bellman equation until convergence. We also verified our results against proceeding with value function iteration all the way.

The computational method we use to solve for the optimal contract in this UI regime requires separability in consumption and leisure, $U(c, z) =$

[1] All references not listed at the back are in the main paper.

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$u(c) - d(z)$. Note, this was not needed for the MH, FI, or the exogenously incomplete regimes. The separability allows us to split each time period into two subperiods and use dynamic programming within the time periods. This helps keep dimensionality in check, since the resulting subproblems are of much lower dimension. The first subperiod includes the announcement of k by the agent, the principal's effort recommendation z, the agent's actual effort supply, and the realization of the output q. The second subperiod includes the transfer, the investment recommendation, and the agent's consumption and actual investment decisions. To tie the two subperiods together, we introduce the extra variables, $\mathbf{w}_m$, that we call "interim promised utility"—a representation of the agent's expected utility from the end of subperiod 1 (i.e., from the middle of the period) onward. The interim promised utility is a *schedule* (vector), $\mathbf{w}_m = \{w_m(k_1), w_m(k_2), \dots\} \in \mathbf{W}_m$, similar to $\mathbf{w}$. Like $\mathbf{W}$, the set $\mathbf{W}_m$ is endogenously determined along the value function iteration.

The first subperiod problem for computing the optimal contract with an agent who has announced k and has been promised $\mathbf{w}$ is:

PROGRAM UI1:

$$V(k, \mathbf{w}) = \max_{\{\pi(q,z,\mathbf{w}_m|k,\mathbf{w})\}} \sum_{q,z,\mathbf{w}_m} \pi(q, z, \mathbf{w}_m|k, \mathbf{w})\big[q + V_m(k, \mathbf{w}_m)\big]. \tag{S-1}$$

The choice variables are the probabilities over allocations $(q, z, \mathbf{w}_m) \in Q \times Z \times \mathbf{W}_m$. The function $V_m(k, \mathbf{w}_m)$ is defined in the second subperiod problem (see Program UI2 below). The maximization in (S-1) is subject to the following constraints. First, the optimal contract must deliver the promised utility on the equilibrium path, $w(k)$:

$$\sum_{q,z,\mathbf{w}_m} \pi(q, z, \mathbf{w}_m|k, \mathbf{w})\big[-d(z) + w_m(k)\big] = w(k). \tag{S-2}$$

The utility from consumption and discounted future utility are incorporated in w_m. Second, as in the MH regime, the optimal contract must satisfy incentive compatibility in effort. That is, $\forall(\bar{z}, \hat{z}) \in Z \times Z$:

$$\begin{aligned}\sum_{q,\mathbf{w}_m} \pi(q, \bar{z}, \mathbf{w}_m|k, \mathbf{w})\big[-d(\bar{z}) + w_m(k)\big] \\ \geq \sum_{q,\mathbf{w}_m} \pi(q, \bar{z}, \mathbf{w}_m|k, \mathbf{w}) \frac{P(q|\hat{z}, k)}{P(q|\bar{z}, k)}\big[-d(\hat{z}) + w_m(k)\big].\end{aligned} \tag{S-3}$$

Third, since the state k is private information, the agent needs incentives to reveal it truthfully. On top of that, the agent can presumably consider joint deviations in his announcement, k, and his effort choice, z. To prevent such joint deviations, truth-telling must be ensured to hold regardless of whether the agent decides to follow the effort recommendation, z, or considers a deviation to another effort level $\delta(z) \in Z$, where $\delta(z)$ denotes all possible mappings from recommended to actual effort, that is, from the set Z to itself. Such behavior is ruled out by imposing the following "truth-telling" constraints, which must hold for all $\hat{k} \neq k$ and $\delta(z)$:

$$w(\hat{k}) \geq \sum_{q,z,\mathbf{w}_m} \pi(q, z, \mathbf{w}_m | k, \mathbf{w}) \frac{P(q|\delta(z), \hat{k})}{P(q|z, k)} \bigl[-d\bigl(\delta(z)\bigr) + w_m(\hat{k})\bigr]. \tag{S-4}$$

In words, an agent who actually has $\hat{k}$ but considers announcing k, triggering $\pi(\cdot|k, \mathbf{w})$, should find any such deviation unattractive. There are $(\#K - 1)\#Z^{\#Z}$ such constraints in total. Finally, the contract must satisfy the already familiar technological consistency, adding-up, and nonnegativity constraints for the probabilities $\pi(q, z, \mathbf{w}_m | k, \mathbf{w})$.

To solve Program UI1, we first need to compute the principal's "interim value function" $V_m(k, \mathbf{w}_m)$. The state variables are the schedule, $\mathbf{w}_m$, of interim utilities for each $k \in K$ and the agent's actual announcement k. Constraints will introduce truth-telling and obedience in the second-stage program. We need to ensure that, when deciding on k', the agent cannot obtain more than his interim utility, $w_m(k)$ for any announcement k.

PROGRAM UI2:

$$V_m(k, \mathbf{w}_m) = \max_{\{\pi(\tau,k',\mathbf{w}'|k,\mathbf{w}_m)\},\{v(\hat{k},k',\tau|k,\mathbf{w}_m)\}} \sum_{\tau,k',\mathbf{w}'} \pi\bigl(\tau, k', \mathbf{w}' | k, \mathbf{w}_m\bigr) \times \bigl[-\tau + (1/R)V\bigl(k', \mathbf{w}'\bigr)\bigr]. \tag{S-5}$$

Note that, in addition to the allocation lotteries, $\pi(\tau, k', \mathbf{w}'|k, \mathbf{w}_m)$, we introduce additional choice variables, $v(\hat{k}, \tau, k'|k, \mathbf{w}_m)$, that we refer to as "utility bounds" (see Prescott (2003) for details). These bounds specify the maximum expected utility that an agent who is actually at $\hat{k}$ receiving transfer τ and an investment recommendation k' could obtain by reporting k and doing $\hat{k}'$. This translates into the constraint

$$\sum_{\mathbf{w}'} \pi\bigl(\tau, k', \mathbf{w}' | k, \mathbf{w}_m\bigr)\bigl[u\bigl(\tau + (1-\delta)\hat{k} - \hat{k}'\bigr) + \beta w'\bigl(\hat{k}'\bigr)\bigr] \leq v\bigl(\hat{k}, \tau, k' | k, \mathbf{w}_m\bigr), \tag{S-6}$$

which must hold for all possible combinations τ, k', $\hat{k} \neq k$, and $\hat{k}' \neq k'$. To ensure truth-telling, the interim utility $w_m(\hat{k})$ that the agent obtains in the second subperiod by reporting k when the true state is $\hat{k}$, must satisfy, for all k, $\hat{k}$,

$$\sum_{\tau,k'} v(\hat{k}, \tau, k'|k, \mathbf{w}_m) \leq w_m(\hat{k}). \tag{S-7}$$

The two sets of constraints, (S-6) and (S-7), rule out any joint deviations in the report k and the action k'. Finally, by definition, the interim utility must satisfy

$$w_m(k) = \sum_{\tau,k',\mathbf{w}'} \pi(\tau, k', \mathbf{w}'|k, \mathbf{w}_m)\big[u(\tau + (1-\delta)k - k') + \beta w'(k')\big], \tag{S-8}$$

and the probabilities $\pi(\tau, k', \mathbf{w}'|k, \mathbf{w}_m)$ must satisfy nonnegativity and adding-up.

S-2. HIDDEN OUTPUT

In this model, we allow output, q, to be unobservable to the financial intermediary, similarly to Townsend (1982) or Thomas and Worrall (1990). Assume effort, z, is contractible, so there is no problem with joint deviations. We have

$$V(k, w) = \max_{\{\pi(\tau,q,z,k',w'|k,w)\}} \sum_{\tau,q,z,k',w'} \pi(\tau, q, z, k', w'|k, w) \times \big[-\tau + (1/R)V(k', w')\big],$$

subject to the promise keeping constraint

$$\sum_{\tau,q,z,k',w'} \pi(\tau, q, z, k', w'|k, w) \times \big[U(q + \tau + (1-\delta)k - k', z) + \beta w'\big] = w$$

and the truth-telling constraints (true output is $\bar{q}$, but the agent considers announcing $\hat{q}$), $\forall(\bar{z}, \bar{q}, \hat{q} \neq \bar{q} \in Z \times Q \times Q)$:

$$\sum_{\tau,k',w'} \pi(\tau, \bar{q}, \bar{z}, k', w'|k, w)\big[U(\bar{q} + \tau + (1-\delta)k - k', \bar{z}) + \beta w'\big] \geq \sum_{\tau,k',w'} \pi(\tau, \hat{q}, \bar{z}, k', w'|k, w)\big[U(\bar{q} + \tau + (1-\delta)k - k', \bar{z}) + \beta w'\big], \tag{S-9}$$

subject to the technological consistency and adding-up constraints:

$$\sum_{\tau,k',w'} \pi\big(\tau,\bar{q},\bar{z},k',w'|k,w\big) = P(\bar{q}|\bar{z},k) \sum_{\tau,q,k',w'} \pi\big(\tau,q,\bar{z},k',w'|k,w\big) \quad \text{for all } (\bar{q},\bar{z}) \in Q \times Z$$

and

$$\sum_{\tau,q,z,k',w'} \pi\big(\tau,q,z,k',w'|k,w\big) = 1,$$

as well as nonnegativity: $\pi(\tau,q,z,k',w'|k,w) \geq 0$ for all $(\tau,q,z,k',w') \in T \times Q \times Z \times K \times W$.

An important issue to take into account in numerically solving this model is that, due to the truth-telling constraints (S-9) (w' appears on both sides combined with different q), the feasible set of promises W has to be determined (iterated over) endogenously within the solution process. This, and the fact that there are more constraints (see Table I), make this regime much harder computationally (up to ten times slower per evaluation) than the MH regime, even though they all have the same total number of variables.

S-3. EMPIRICAL RESULTS

Table S.I reports results from using our methods described in Sections 3 and 4 of the paper to compute and estimate two additional financial/information regimes—a moral hazard regime with unobserved investment (UI) and a hidden output regime (HO); see Sections 1 and 2 for detailed descriptions. These estimation runs demonstrate the generality of our approach and its ability to accommodate variants of information constraints outside our six baseline regimes, including those in the literature reviewed in the Introduction. Because of high computational time and memory requirements (computing over 100,000 linear programs per iteration), we are unable to estimate the UI regime with the baseline grids used in Tables V–VIII and instead use a coarse, three-point grid specification (the UI results in Table S.I should be read together with line 5.9 in Table IX).[2] We use the parametric production function specification to be able to compute the HO regime (see the footnote in Table S.I).

[2]Currently, a single evaluation of the UI regime likelihood function with our baseline grids takes about 45 minutes (as opposed to 7–9 sec for the MH regime). Over 1,500 such evaluations (47 days) are typically required to find the MLE estimates for a single estimation run. We are working on a parallel computing version of our estimation algorithm as well as optimizations based on the NPL approach (Aguirregabiria and Mira (2002), Kasahara and Shimotsu (2011)).

TABLE S.I

MODEL COMPARISONS—VUONG TEST RESULTS[a]

	v MH	v FI	v B	v S	v A	v LC	Best Fit
Rural data, 1999 (c, q, i, k)							
Hidden output (HO)	tie	tie	B***	S***	A***	LC**	B, S
Unobserved investment (UI)	UI***	UI***	B***	S***	tie	UI***	B
Urban data, 2004 (c, q, i, k)							
Hidden output (HO)	HO***	HO***	HO***	HO***	HO***	HO***	HO
Unobserved investment (UI)	UI***	UI***	UI***	UI***	UI***	LC*	LC

[a]For computational reasons, the HO model is estimated using the parametric production function (read together with Table VIII in the paper) and the UI model is computed with coarser grids (read with line 5.9 in Table IX in the paper).

In the rural sample, using 1999 (c, q, i, k) data, the UI regime outperforms the other mechanism design regimes (MH, FI, and LC) in likelihood. However, the UI and HO regimes achieve worse fit with the rural data compared to the S and B regimes, so our overall conclusions from the baseline runs stand. Kinnan (2011) did find evidence in favor of hidden income in a related data set, but she used a Euler-equation metric with consumption and lagged income data alone. In our urban sample, using 2004 (c, q, i, k) data, the LC regime achieves the highest likelihood in the coarse-grids specification, followed closely by UI (see the last row of Table S.I), while the HO regime fits best, followed by MH, in the parametric production function specification (the penultimate row in Table S.I). We also did a run with (c, q) data alone (not reported in the table) in which MH achieved higher likelihood than HO.

ADDITIONAL REFERENCES

AGUIRREGABIRIA, V., AND P. MIRA (2002): "Swapping the Nested Fixed Point Algorithm: A Class of Estimators for Discrete Markov Decision Models," *Econometrica*, 70 (4), 1519–1543. [5]

KASAHARA, H., AND K. SHIMOTSU (2011): "Sequential Estimation of Dynamic Programming Models With Unobserved Heterogeneity," Working Paper. [5]

PRESCOTT, E. S. (2003): "Communication in Models With Private Information: Theory and Computation," *Geneva Papers on Risk and Insurance Theory*, 28 (2), 105–130. [3]

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