Simon Fraser University
Econ 855 – Prof. Karaivanov

Notes on classical growth theory (optional read)

These notes provide a rough overview of "classical" growth theory. Historically, due mostly to data availability reasons economists have concentrated their attention to the growth patterns in the industrialized countries. Only recently data became available for a much larger set of countries and many economists realized that classical "institution-free" growth theories based on the Solow model are inadequate in explaining the variety and persistence in income inequality across countries. New theories of economic growth and development were subsequently advanced emphasizing the role of history dependence, contracts and institutions, asymmetric information, incomplete markets, etc.

1. Growth Facts for Industrialized Countries

- Kaldor - summarized several empirical regularities related to growth in industrial countries
- Kaldor's **stylized facts**
  - 1. output and capital per worker grow over time
  - 2. the rates of growth of output and capital per worker are similar, i.e. the ratio of capital to output stays relatively constant
  - 3. the return to capital (real interest rate) does not change much over time
  - 4. the labor and capital shares of GDP are almost constant over time
- other fact: convergence of GDP p.c. across regions within countries or across industrialized countries
- Unfortunately: no empirical regularities exist that apply to both developed and developing countries - this is where the "new theories of development" come into play (more on this later)

2. The Solow Model (a brief overview - you should be familiar with this model from Econ 807, 808)

- production function:
$$Y_t = (A_t L_t)^{\alpha} K_t^{1-\alpha}$$
where $\alpha \in (0,1)$ and A_t is labor productivity
- sources of growth - either the inputs, K_t, L_t or productivity, A_t must grow
- some (radical) assumptions:

- CRS production function (not by chance - crucial for generating the Kaldor facts)
- labor productivity grows exogenously: $A_t = (1+\mu)A_{t-1}$
- the population grows exogenously: $L_{t+1} = (1+\gamma)L_t$
- constant saving rate (independent of income, time): $I_t = sY_t$ where I_t is investment

- law of motion for capital (δ is the depreciation rate):

$$K_{t+1} = (1-\delta)K_t + I_t$$

- assume a competitive firm operates the production technology, the wage is w_t and the capital rental rate is r_t. Given the Cobb-Douglas production function fact 4 above follows immediately: labor share is $\frac{w_t L_t}{Y_t} = \alpha$; capital's share is: $\frac{r_t K_t}{Y_t} = 1-\alpha$ (derive this as an exercise)

- **dynamics** (easiest to do if express everything (i.e. divide by) in terms of per units of effective labor, $A_t L_t$) (why is this called effective labor?); use lowercase for normalized (divided) variables, e.g. $y_t = \frac{Y_t}{A_t L_t}$, etc.

- We have:

$$\begin{aligned} y_t &= k_t^{1-\alpha} \\ k_{t+1}(1+\mu)(1+\gamma) &= (1-\delta)k_t + i_t \\ i_t &= sy_t \end{aligned}$$

- putting the above together we can express k_{t+1} in terms of k_t (do the algebra as an exercise):

$$\frac{k_{t+1}}{k_t} = \frac{1-\delta+sk_t^{-\alpha}}{(1+\mu)(1+\gamma)} \tag{1}$$

- notice that the growth rate of capital per unit of effective labor, $\frac{k_{t+1}}{k_t}$ is inversely related to the current capital stock, i.e. when a country has a lower level of capital per unit of eff. labor its capital stock will grow faster. This implies that GDP across countries and regions will tend to converge over time (see more on this below)

- **steady state** - since the growth rate slows down as more k is accumulated eventually k_t converges to a steady state (growth in capital per eff. labor goes to zero). Notice that this not imply that GDP, Y_t does not grow!

- at the steady state: $k_t = k_{t-1} = k^*$, solve (1) for k^*, y^* :

$$k^* = (\frac{s}{\delta+\mu+\gamma+\mu\gamma})^{1/\alpha},\ y^* = (k^*)^{1-\alpha}$$

- notice that the steady state depends on:
  - the saving rate, s
  - the depreciation rate, δ
  - the technological and population growth rates, γ and μ
  - the importance of capital/labor in production, α
- steady state growth rate of capital/output:

$$\frac{K_{t+1}}{K_t} = \frac{k^*(1+\mu)A_t(1+\gamma)L_t}{k^* A_t L_t} = 1 + \gamma + \mu + \gamma\mu$$

- notice: the long run growth rate **does not depend** on the saving rate (why? think what is causing the growth)
- checking the remaining Kaldor facts - do as an exercise
- What does the Solow model imply:
  - the above stylized facts for industrialized countries
  - due to decreasing returns - economies grow faster at low levels of capital
  - differences in saving rates across countries can explain long term differences in levels but not in growth rates

Convergence

- the prediction of convergence is at the heart of the Solow model
- **unconditional convergence** - strongest prediction (easiest to refute)
  - suppose that all countries, in the long run, have no tendency to display differences in rate of technical progress, saving rates, population growth and capital depreciation
  - then the Solow model predicts that in all countries capital per unit of eff. labor should converge to the same value, *irrespective of the initial state (*e.g. their initial capital stock or income) - *history (initial conditions) does not matter!*
  - convergence should be indicated by strong negative relationship between growth rates of p.c. income and its initial value - *testable prediction*
  - *the evidence*: data problems (ideally need many countries over long period). Typically either many countries for short period or few countries for longer periods available

- few countries, long period: De Long (1988) - regress growth of p.c. income 1870-1979 on log of 1870 p.c. income (23 countries)- find very little if any convergence
- more countries, shorter periods: Ray (1998, p. 79), Barro (1990) - 1965-85 - no evidence of convergence, basically zero correlation between average p.c. growth and initial p.c. income

- **conditional convergence**
  - main weakness in unconditional convergence hypothesis: countries differ in their s, γ, μ, etc.
  - if countries differ in the above the Solow model predicts different steady states - no convergence in levels, but countries still do *converge to their own* steady states
  - we assume that technological know-how is the same for all countries, thus countries with similar population growths should grow at similar rates - *convergence in growth rates* - countries paths of growth should become parallel to each other
  - testing: much harder - the prediction is that countries that are further from their own steady states must grow faster - i.e. need to control for (condition on) *where the steady state is* (in unconditional convergence it is assumed all countries have the same steady state)
  - how to control for different steady states - need to add to a regression any factors that may affect them (variations in savings, population growth, etc.)
  - results: although the variables have the "right" signs - the actual cross-country differences in income are too large to be accounted for (more on this in topic 11 of the course);

- **criticisms to the Solow/convergence approach**
  - **evidence** - convergence implies that given certain parameters economies *inevitably* move towards some steady state due to the law of diminishing returns - not much evidence for this
  - **limited depth of analysis due to endogeneity** - even if the Solow model makes some sense of the data when assuming all its parameters can differ across countries it is not clear *why are they different* - aren't we explaining differences in incomes that we don't understand with differences in something else we don't understand? After all saving rates or fertility are not exogenous but potentially depend on income, growth, etc. Also, explaining underdevelopment as "this country is more corrupt; people like to save less; have different culture, etc." is flawed - these may be outcomes of underdevelopment, not reasons for it! A better and fuller growth theory is needed.

– technical progress is also not exogenous - basically (disregarding population growth) the Solow model asserts that all growth comes out of A_t - but which forces drive technical progress? How does it diffuse across countries? Why are there some countries which do not adopt readily available better technologies? A new theory of growth must answer these questions

– **policy bias -** the traditional theory stresses on factors as savings, population growth, level of corruption, etc. As we saw these may be symptoms rather than causes of underdevelopment; if they are indeed – policy effects may be unexpected; also it matters whether we would need *one-time vs. continuous* policies to fix those problems.