Simon Fraser University, Department of Economics, Econ 201, Prof. Karaivanov

MIDTERM EXAM #1 Answer key

I. TRUE or FALSE – no points will be given without a brief justification of your answer (6 pts each)

1. FALSE. The consumer can afford all bundles in her budget set, that is all quantities x_1, x_2 which satisfy $p_1x_1 + p_2x_2 \leq m$. The budget line connects only the bundles that exhaust the budget completely, but many others that do not (including say (0,0)) are affordable too.

2. FALSE. The slope of the indifference curve equals the rate at which a consumer is *willing* to exchange one good for the other. It's about the consumer's preferences. What the consumer *has* to give up is determined by the goods' prices that we assume are outside of the control of individual consumers.

3. TRUE. If both prices fall while income stays constant, the budget set of the consumer grows strictly larger as the budget line moves up and to the right. With preference monotonicity, this means that whatever was the previously optimal bundle, now bundles containing strictly more quantity of each good are affordable (draw a picture to see that clearly). Thus, the consumer indeed achieves higher utility.

4. TRUE. A Giffen good is a good for which a price fall implies a fall in demand. Suppose for instance p_1 goes down and both goods are Giffen. This means that, after the price change, $p_1x_1 + p_2x_2$ will be strictly lower than m (Why? since p_1, x_1, x_2 all go down and since we had $p_1x_1 + p_2x_2 = m$ before the price change). But choosing (x_1, x_2) with $p_1x_1 + p_2x_2 < m$ is not utility maximizing, so both goods cannot be Giffen.

5. FALSE. If a good is inferior we know that an increase in income would decrease its quantity demanded – thus, the Engel curve for such good should be downward sloping.

Problem 1 (40 pts)

(a) (4 pts) Indeed, as shown in class, the given preferences are a monotonic transformation of the Cobb-Douglas function $x_1^{1/3}x_2^{2/3}$ – to see that, take log– $\ln(x_1^{1/3}x_2^{2/3}) = \frac{1}{3}\ln x_1 + \frac{2}{3}\ln x_2$ and then multiply by 3. Both log and multiplying by a positive number are monotonic transformations (they preserve the preference ordering for any two bundles). So the power α is $\frac{1}{3}$.

(b) (10 pts) Proceed as in class. Jane's consumer problem is

$$\max_{x_1,x_2} \ \ln x_1 + 2\ln x_2$$
$$\text{s.t.} \ \ (1)x_1 + p_2x_2 = m$$

It is easiest to proceed by expressing $x_1 = m - p_2x_2$ and plugging into the utility function:

$$\max_{x_2} \ \ln(m - p_2x_2) + 2\ln x_2$$

take derivative and set to zero: $\frac{-p_2}{m-p_2x_2} + \frac{2}{x_2} = 0$. Cross-multiply and solve for x_2,

$$p_2x_2 = 2m - 2p_2x_2$$

and so

$$x_2^* = \frac{\frac{2}{3}m}{p_2}$$

Find x_1^* from the budget constraint,

$$x_1^* = m - p_2x_2^* = \frac{m}{3}$$

Jane has preferences represented by the utility function $u(x_1, x_2) = \ln x_1 + 2\ln x_2$ where x_1 and x_2 are the quantities of the only two goods she consumes. The goods' prices are $p_1 = 1$ and p_2 and Jane's income is m.

(c) (8 pts) Use the demand function for good 2, $x_2^*(p_1, p_2, m)$ to derive the Engel and demand curves. For the Engel curve, as shown in class, we need to express income m in terms of the quantity x_2^* from the demand function. We have $x_2^* = \frac{\frac{2}{3}m}{p_2}$ and so $m = \frac{3p_2x_2^*}{2}$ – this is a straight line slope $\frac{3p_2}{2}$ *[on a figure with x_2 on the horizontal axis and m on the vertical axis, the students should plot a straight line, with positive slope, passing through the origin].* Since x_2^* is increasing in income m the good is a normal good (higher income makes Jane consume more).

For the demand curve, express the price p_2 in terms of the quantity x_2^* from the demand function, to get $p_2 = \frac{2m}{3x_2^*}$ – this is a downward sloping convex line (as shown in class). Since an increase in the price leads to a decrease in the quantity of good 2, the good is ordinary.

(d) (4 pts) Note that these are preferences for perfect complements (the goods are consumed in fixed proportion), so the ICs are L-shaped lines. To find Ming's IC for $U = 3$, set $\min\{b, \frac{h}{3}\} = 3$ and think what combinations of b and h will result in utility (the value of the minimum) equal to 3. It is clear that any bundle with $b = 3$ or any bundle with $h = 9$ achieves that. Thus, the IC we are looking for is the set of all bundles with $b = 3$ or/and $h = 9$ (this is an L-shaped line with the right angle at $(3, 9)$.

(e) (8 pts) As in assignment 1, notice first that it is not optimal for Ming to choose any bundle which has $b > \frac{h}{3}$ or any bundle which has $b < \frac{h}{3}$. Basically any bundle which has one of b or $\frac{h}{3}$ strictly larger that the other does not yield any extra utility but costs extra money (that instead could be used to buy more b and h in the right proportion). Thus, at optimum we should have $b^* = \frac{h^*}{3}$. Use that in the budget constraint to find b^* and h^* :

$$2\frac{h^*}{3} + h^* = 60$$

so we obtain $h^* = 36$ and $b^* = \frac{h^*}{3} = 12$. If $m = 80$, the BL changes to $2\frac{h^*}{3} + h^* = 80$ and so $h^* = 48$ and $b^* = \frac{h^*}{3} = 16$.

(f) (6 pts) we still must have $b^* = \frac{h^*}{3}$. For any p_1 the budget constraint (now we write it in terms of b^*) now implies (note that $p_2 = 1$)

$$p_1 b^* + (1)3b^* = 60$$

and so $b^* = \frac{60}{3+p_1}$. This is Ming's demand function (quantity demanded as function of the good's price). To find the demand curve, express the price in terms of the quantity using the demand function. We have, $3b^* + p_1 b^* = 60$ and so

$$p_1 = \frac{60 - 3b^*}{b^*} = \frac{60}{b^*} - 3$$

This is a downward sloping and convex line. To plot it plug in a few values for b^* and make sure you put b^* on the horizontal axis and p_1 on the vertical axis. For example, the points $(1, 57)$, $(2, 27)$, $(5, 9)$ belong to Ming's demand curve.

Problem 2 (30 pts)

(a) (8 pts) This one is easy points. We are given that $5(10) + p_2(5) = 60$ and so $p_2 = \$2$. The BL is a straight line with intercepts m/p_1 and m/p_2 so it's the straight line between the points $(0, 30)$ and $(12, 0)$. The slope of the BL is $-\frac{p_1}{p_2} = -2.5$. Yes, assuming Edward's preferences are well-behaved (that is, have strictly convex indifference curves) we would have that his IC is tangent to the BL at his optimal bundle and so the IC's slope which equals his MRS or $-\frac{MU_1}{MU_2}$ should equal the slope of the BL, -2.5. This implies that, $\frac{MU_1}{MU_2} = 2.5$. If Edward's preferences are not well-behaved (e.g. perfect complements, then $\frac{MU_1}{MU_2}$ may not be defined). *[give full credit if the student ignored the latter case].*

(b) (8 pts) This is very similar to last year's midterm that you had for practice. The 20% proportional tax, $\tau = 0.2$ on subs implies that their price becomes $(1 + \tau)p_1 = \$6$. The 50-cent quantity subsidy on coffee implies that $p_2 = \$1.5$. Edward's new budget line has intercepts $\frac{60}{6}$ and $\frac{60}{1.5}$, or in other words, a straight line between the points $(0, 40)$ and $(10, 0)$. Yes, there are consumption bundles that Edward can afford now that he could not afford in part (a) (those at the top-left part of his new budget set, e.g. the bundle (0,40). Edward **cannot** afford his previous bundle $(10, 5)$ since it now costs $10(6) + 5(1.5) = 67.5$ which is more than his income of $60.

(c) (14 pts) This is harder. Just think what happens. Put subs on the horizontal axis. With coupon A, E's budget line looks the same as in part (a) up to $s = 2$, that is straight line between $(0, 30)$ and $(2, 25)$, but then at $s = 2$ he gets an extra coffee, so the BL jumps vertically up to $(2, 26)$. From then on, the usual prices apply, so the BL goes down at slope $-\frac{p_1}{p_2} = -2.5$. With his remaining $50, E. can buy maximum 10 more subs and so his budget line continues from the bundle $(2, 26)$ down to $(12, 1)$. The budget line then drops vertically to $(12, 0)$. Why? Because we don't allow Edward to sell the free coffee and buy more subs.

Now to coupon B. The logic is similar, but now the free subs shift the budget line horizontally. We start at $(0, 30)$ as in part (a), then when we reach 4 subs bought (bundle $(4, 20)$), the coupon kicks in and the BL extends horizontally to the bundle $(5, 20)$. From then on, the usual prices apply again (the BL is straight line with slope -2.5) until E. buys 4 more subs, which happens at bundle $(9, 10)$. The coupon kicks in again, so the BL extends horizontally to bundle $(10, 10)$. It goes down from there at slope -2.5 again, as E. buys 4 more subs, to reach bundle $(14, 0)$ at which point the coupon kicks in one last time (Edward has bought 12 subs in total and got 2 free), and the BL extends horizontally again to $(15, 0)$.

An example preferences when E. would prefer coupon A would be an IC touching at bundle $(2, 26)$ (see your graph above) – this bundle is not affordable with coupon B – think why. An example preference for which E. would prefer coupon B would be an IC touching at bundle (5,20) – this bundle is unaffordable with coupon A – think why.