Econ 301E Spring 1998.

Answers to Assignment due March 12

1. a. It violates the assumption of diminishing marginal rate of technical substitution. In fact, the Marginal Rate of Technical Substitution undefined at the kink. This kind of technology does not permit the substitution of one input for another and therefore has zero elasticity of substitution.
b. No. Could produce the same output level (10 units) with fewer units of *K* (at the kink on ray 3) or a fewer units of *L* (at the kink on ray 1). In both cases, the cost of production is lower than it is at point A.
c. Rays 1 and 3.

2. Assume that the speed of each truck is *s* miles per hour. Then it takes 1/*s* hours to move 1 mile. Labor costs per mile is \$*w/s* and fuel cost per mile is \$*p*(*A* + *Bs*). With an unlimited number of trucks available, the firm's output is unlimited and its total variable cost function is

$$\left[\frac{\overline{w}}{s} + p(A+Bs)\right]m$$

where *m* is the number of miles moved.

b. If there is only one truck, and it can be driven for a maximum of 10 hours per day at s miles per hour, then the firm's total variable cost is

$$Min\left\{\left[\frac{\overline{w}}{s} + p(A+Bs)\right]m, \left[\frac{\overline{w}}{s} + p(A+Bs)\right]10s\right\}$$

3. a. IRS b. CRS c. CRS

4. We assume that the production of lawn-mowing services is characterized by CRS and fixed proportions (Leontieff technology). The small mower produces 1 unit (10,000 square feet of lawn) by combining labor hours, gasoline and lawn mower hours in the ratio of 1: 1/3: 1. Similarly, the large mower produces 3 units by combining labor hours, gasoline and lawn mower hours in the ratio of 1:1:1. Let
   - z_1 = hours of labor ; w_1 = unit price of labor;
   - z_2 = gallons of gasoline; w_2 = unit price of gasoline;
   - z_3 = number of hours the small mower is used; w_3 = rental rate for small mower/hour;
   - z_4 = number of hours the large mower is used; w_4 = rental rate for large mower/hour;

The production functions are, respectively,

$$Q_s = Min(z_1, 3z_2, z_3)$$
$$Q_l = Min(z_1, z_2, z_4)$$

b. The minimum cost of producing one unit of output with the small mower is $\$w_1 + \$w_2/3 + \$w_3$. Therefore, the total cost function for the small mower is

$$C_s(w_1, w_2, w_3, Q) = (\$w_1 + \$w_2/3 + \$w_3)Q$$

Similarly, the minimum cost of producing one unit of output with the large mower is $\$w_1/3 + \$w_2/3 + \$w_4/3$. Therefore, the total cost function for the large mower is

$$C_l(w_1, w_2, w_4, Q) = (\$w_1/3 + \$w_2/3 + \$w_4/3)Q.$$

Note that the small mower will be cheaper than the large one if $C_s(Q) < C_l(Q)$.

$$\Rightarrow 2w_1 < w_4 - 3w_3.$$

Why is this result independent of the price of gasoline?