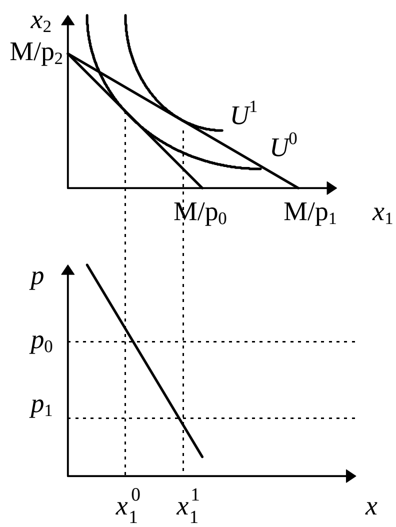
Suggested Answers to Midterm

Part I

1. The market price of any commodity reflects not the total value of commodity but the marginal value (the value of the last unit). The total value of water is very high but the marginal value of is very low. In contrast, the total value of diamonds is very low but the marginal value is very high. As such, water must sell for much less than diamonds.



b. Income, prices of all other goods and taste are held constant. (Any two of these will give you full marks).

3. The substitution effect of a price change for both goods is negative (negative relationship between price and quantity demanded). The substitution effect of a *price increase* is an unambiguous decrease in quantity demanded, and the substitution effect of a price reduction is an increase in quantity demanded. However, the income effect is opposite the substitution for both goods. For a giffen good, not only is the income effect opposite the substitution effect, but it also overwhelms the substitution effect. A giffen good, therefore, has to be inferior in the first place (since the income effect is negative), but an inferior good need not be giffen.

4. a. Diminishing marginal utility. b. Nonsatiation (More is preferred to less) c. Violates the assumption of nonsatiation.

5. Can't tell. Could be either. If demand is price elastic, then insulin can not be inferior.

6. Seat belts have costs as well as the benefits of reduced risk of injuries, and if motorists considered the marginal cost of purchasing and using seat belts greater than the marginal benefits, they would rational not to purchase them. This would be shown as a corner equilibrium. A Seat belt would be an inessential good to a motorist.

Part II

1.

a. Derive the demand functions using the consumer equilibrium conditions:

$\frac{MU_1}{MU_2} = \frac{p_1}{p_2}$; $M = p_1x_1 + p_2x_2$, or use the Lagrangean formula. The demand functions are

$$x_1^* = \frac{M}{2p_1} \; ; \; x_2^* = \frac{M}{2p_2}$$

b. $\frac{\partial x_1^*}{\partial p_1} = \frac{-M}{2}.\frac{1}{p_1^2} < 0$; $\frac{\partial x_2^*}{\partial p_2} = \frac{-M}{2}.\frac{1}{p_2^2} < 0$. These imply that the demand functions are downward sloping

c. Price elasticity for good 1: $e_1 = \frac{\partial x_1}{\partial p_1}.\frac{p_1}{x_1} \Rightarrow \left(\frac{-M}{2}.\frac{1}{p_1^2}\right)\left(p_1 \frac{2p_1}{M}\right)$ = - 1.

d. Equilibrium level of utility: $U^* = \left(x_1^*\right)\left(x_2^*\right) \Rightarrow \left(\frac{M}{2p_1}\right)\left(\frac{M}{2p_2}\right) \Rightarrow \frac{1}{4}.\frac{M^2}{p_1 p_2}$

e. Multiplying prices and income by the same constant should not change the demand functions.
$\frac{kM}{2kp_1} = \frac{M}{2p_1}$; and $\frac{kM}{2kp_2} = \frac{M}{2p_2}$.

f. The minimum expenditure possible to attain utility *U** is equal to total income.
$p_1 x_1^* + p_2 x_2^* \Rightarrow p_1\left(\frac{M}{2p_1}\right) + p_2\left(\frac{M}{2p_2}\right) = M$.

2. The demand functions of the Cobb-Douglas utility function are given by:

$$x_1 = \left(\frac{M}{10p_1}\right); \; x_2 = \left(\frac{9M}{10p_2}\right).$$

Thus, (x_1^*, x_2^*) = (10, 90), $U^0 = (10)^{0.1}(90)^{0.9} = 72.25$.

With the price system (2,1) i.e., an increase in the price of good 1,
$(\hat{x}_1, \hat{x}_2)$ = (5, 90) ; $U^1 = (5)^{0.1}(90)^{0.9} = 67.41$

<u>CV:</u> We find how much money would be necessary at the new prices (2,1) to make the consumer as well of as he was consuming bundle (10, 90). The utility of the optimal bundle at income *M* and prices (2,1) is given by $\left(\frac{M}{10p_1}\right)^{0.1}\left(\frac{9M}{10p_2}\right)^{0.9} = \left(\frac{M}{20}\right)^{0.1}\left(\frac{9M}{10}\right)^{0.9}$. Setting this equal to the utility of original bundle, we have
$\left(\frac{M}{20}\right)^{0.1}\left(\frac{9M}{10}\right)^{0.9}$ = 72.25. Solving for *M* gives *M* = 107.182. This means that the consumer will need 107.18 - 100 = $7.18 of additional income after the price change to make him as well off as he was before the price change.

<u>EV</u>: We find how much money would be necessary at the original prices (1,1) to make the consumer as well of as he would be consuming the new bundle (25, 50). The utility of the optimal bundle at income *M* and prices (1,1) is given by $\left(\frac{M}{10p_1}\right)^{0.1}\left(\frac{9M}{10p_2}\right)^{0.9}$. Setting this equal to the utility of new bundle (U^1) , we have

$\left(\frac{M}{10}\right)^{0.1}\left(\frac{9M}{10}\right)^{0.9}$ = 67.41. Solving for *M* gives *M* = 93.305. Thus, if the consumer had an income $93.30 at the original prices, he would just be as well off as he would be facing the new prices and having an income of $100. The EV is $100 **-** $93.30 = $6.7.

3.

a. Concave to the origin.

b. Risk averse.

c. $EU(W) = 2.0(30{,}000)^{1/2} + 0.8(100{,}000)^{1/2} = 287.6$.

Certainty equivalent:

$287.6 = W_{ce}^{1/2}$; $\Rightarrow$ $W_{ce} = 287.6^2 = 82{,}713.76$. Reservation price of insurance: $W_0 - W_{ce} = 100{,}000 - 82{,}713.76 = 17{,}286.2$.

d. Minimum supply price = *pL* = 0.2(70,000) = 14,000. Therefore, a mutually agreeable contract can be made between the price range 17,286 and 14,000.

4. An honest person receives a guaranteed income of $10,000. A criminal faces a gamble with an expected wealth of 0.25(23,000) + 0.75(1000) = 5,700. A criminal must be risk loving since a person who commits a crime prefers a gamble with a lower expected payoff (5,700) to a guaranteed income of 10,000. A criminal's utility function is therefore convex to the origin.

b. We can not deduce, unambiguously, the shape of an honest person's utility function. What we can say is that if an honest person is risk loving, he must be less risk loving than a criminal.

5. $MP = aAx_1^{a-1}\bar{x}_2^{\,b}$; $AP = Ax_1^{a-1}\bar{x}_2^{\,b}$;

b. $\frac{¶MP}{¶x_1} = (a-1)aAx_1^{a-2}\bar{x}_2^{\,b} < 0$ since $0 < a < 1$.

c. To find maximum AP, $\frac{¶AP}{¶x_1} = (a-1)Ax_1^{a-2}\bar{x}_2^{\,b} = 0$.

$\Rightarrow aAx_1^{a-2}\bar{x}_2^{\,b} = Ax_1^{a-2}\bar{x}_2^{\,b}$

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Multiplying through by x_1,

$aAx_1^{a-1}\bar{x}_2^{\,b} = Ax_1^{a-1}\bar{x}_2^{\,b}$

$\Rightarrow MP = AP$