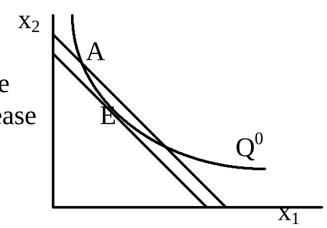
SIMON FRASER UNIVERSITY
Department of Economics

ECON 301: Intermediate Microeconomics. **Spring, 1998.**
Instructor: A. Appiah

Review Questions Based on Chapters 7 and 8

1. **If a firm is producing where $MP_1/w_1 > MP_2/w_2$, what can it do to reduce costs but maintain the same output?**

Ans:
$MP_1/w_1 > MP_2/w_2$. This implies that $MP_1/MP_2 > w_1/w_2$. The slope of the isoquant exceeds that of the isocost (at A). To reduce costs but maintain the same level of output, the firm should increase the use of input one and decrease the use of input 2 until the point of tangency is reached (E).



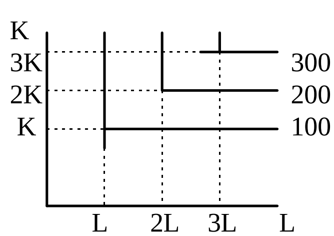
2. **In terms of "capital" and "labor", what does the law of diminishing returns state?**

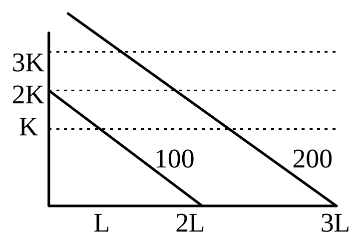
Ans: As more units of labor per unit of time is used to work a fixed quantity of capital, after a point the marginal returns to labor will *necessarily* decline.

With reference of stage II of production (a) why does the producer operate in stage II, (b) what factor combination (within stage II) will the producer actually use and (c) where will the producer operate if $P_{Labor} = 0$? If $P_{Labor} = P_{Capital}$?

Ans: (a) The producer will not operate in Stage I for labor (which coincides with Stage III for capital) because MP_K is negative. (b) Within Stage II (where both marginal products are positive), the producer will produce at the point where $MP_L/p_L = MP_K/p_K$. (c) If $p_L = 0$, the producer will want to produce at the point where the average efficiency of greatest, i.e., where AP_K is maximum and MP_L is zero. If $p_L = p_K$, the producer will produce at the point where $MP_L = MP_K$.

3. On one set of axes, draw three isoquants showing zero elasticity of factor substitution and constant returns to scale. On another set of axes, draw three isoquants showing infinite elasticity of factor substitution and constant returns to scale.





a. Zero elasticity of substitution and CRS. Production takes place with K/L = 1 regardless of relative factor prices. Thus, if relative factor prices change $\Delta(K/L) = 0$, and therefore, $(e\text{ subst})_{LK} = 0$.

b. Infinite elasticity of substitution and CRS. Slope of isoquants (MRTS) = 0 and $(e\text{ subst})_{LK} = \infty$.

4. Given the production function $Q = AL^{\alpha}K^{1-\alpha}$, find the elasticity of factor substitution. Show that the production function is homothetic. What is the equation of the expansion path? Discuss the significance of the intercept and slope of the expansion path.

Ans:

$MP_L = a(Q/L) = aAP_L$.

$MP_K = (1\text{-}a)(Q/K) = (1\text{-}a)AP_K$.

$$\Rightarrow MRTS_{LK} = \frac{MP_L}{MP_K} = \frac{a}{1-a}\frac{K}{L}$$

$$(e \text{ subst})_{LK} = \frac{d(K/L)/(K/L)}{d\left(MRTS_{LK}\right)/MRTS_{LK}} \Rightarrow \frac{d(K/L)/(K/L)}{d\left(\frac{a}{1-a}\frac{K}{L}\right)/\left(\frac{a}{1-a}\frac{K}{L}\right)} \Rightarrow \frac{d(K/L)/(a\ /1-a\ )}{(a\ /1-a\ )d(K/L)} = 1$$

A production function is homothetic if the output expansion path is a straight line through the origin. If a production function is homothetic, the MRTS (slope of the isoquants) is constant along the expansion path, i.e., the rate at which the firm substitutes one input for another does not change as the firms moves along its output expansion path.

The expansion path is determined from the equilibrium condition:

$MP_L/MP_K = p_L/p_K$.

$$\Rightarrow \frac{a}{1-a}\frac{K}{L} = \frac{P_L}{P_K}$$

$\Rightarrow K = \left(\frac{1-a}{a}\right)\frac{P_L}{P_K}L$ which is a straight line through the origin since the intercept is zero.

5. Explain what is meant by (a) constant returns to scale (b) increasing returns to scale (c) decreasing returns to scale. Explain briefly how each of these might arise.

Ans:

CRS: refers to the production situation where if all factors of production are increased in a given proportion, the output produced will increase in the same proportion. Similarly, if all inputs are reduced by a given proportion, output is reduced by the same proportion. If one of labor and one unit of capital will produce a given amount of output, then if we use two identical workers and two identical machines we will expect to produce twice the original output.

IRS: refers to the case where if all factors are increased in the same proportion, output increases by a greater proportion. Increasing returns to scale may occur because of productivity gains from greater division of labor and specialization.

DRS: refers to the case where if all factors are increased in the same proportion, output increases by a smaller proportion. Decreasing returns to scale may arise because as the scale of operation increases, communication difficulties and other organizational problems may make it more and more difficult for the entrepreneur to run his business effectively. Generally, it is believed that at a very small scale of operation, the firm faces increasing returns to scale. As the scale of operation rises, increasing returns to scale give way to constant returns to scale and eventually, to decreasing returns to scale.

6. **With respect to the production function in the Table below, (a) indicate the nature of returns to scale. Which of these points are on the same isoquant? (c). Is the law of diminishing returns operating?**

$3K$	80	120	150
$2K$	70	100	120
$1K$	50	70	80
	$1L$	$2L$	$3L$

Ans: Output increases in the same proportion as the increase in inputs. Therefore, we have CRS.
(i) The following input combinations lie on the 70 unit isoquant: (1,2) and (2,1).
(ii) The following input combinations lie on the 80 unit isoquant: (1,3) and (3,1).
(iv) The following input combinations lie on the 120 unit isoquant: (3,2) and (2,3).

The law of diminishing returns is a short-run concept. Maintaining K at say 1 unit and increasing labor from 1 to 2 and then to 3, the marginal product values are, respectively, 20 and 10.

6. What are some of the implicit costs incurred by an entrepreneur in running his firm? How are these costs estimated? What price does a firm pay to purchase or hire the inputs it does not own?

Ans: Implicit costs include transfer earnings of staff and other factors of production that the firm owns and uses in production.
For the inputs which the firm purchases or hires, it must pay a price at least equal to what these inputs could earn in their best alternative employment.

7. Give some examples of fixed and variable factors of production in the short run. (b) What is the relationship between the quantity of fixed inputs used and the short run level of output?

Ans: Fixed factors include payment for renting land, buildings, part of depreciation and maintenance expenditures, insurance and property taxes, some salaries which are fixed by contract.
Variable factors include raw materials, fuels, most type of labor, excise taxes, etc.
b. The quantity of fixed inputs used determine the size or the scale of the plant which the firm operates in the short run. Within the limits imposed by its scale of plant, the firm can vary it output in the short run by varying the quantity of variable inputs.

8. Assuming for simplicity that labor is the only variable input in the short run and that the price of labor is constant, explain the U-shaped of (a) the SAVC curve and (b) the SMC curve in terms of the shapes of the AP_L and MP_L curves, respectively.

Ans: When labor is the only variable input, $TVC = p_L L \Rightarrow AVC = \frac{TVC}{Q} = \frac{p_L L}{Q} \Rightarrow \frac{p_L}{Q/L} \Rightarrow \frac{p_L}{AP_L}$

where AP_L is the average product of labor. We know that with p_L constant, AP_L normally rises, reaches a maximum and then falls. It follows, therefore that AVC falls, reaches a minimum and then rises.
b. $MC = \frac{\Delta TVC}{\Delta Q} = \frac{\Delta(p_L L)}{\Delta Q} \Rightarrow \frac{\Delta(p_L L)}{\Delta Q} \Rightarrow \frac{p_L \Delta L}{\Delta Q} \Rightarrow \frac{p_L}{MP_L}$ where MP_L is the marginal product of labor. Use the same argument as (a) above.

9. What is the relationship between the long run and the short run? (b) How can the LAC be derived? What does it show? (c) Both the SAC and the LAC are U-shaped but the reasons for their shapes are different. What are these reasons?

Ans: a. The long run is also known as the planning period. All inputs can be varied to produce the anticipated output. In the short run, at least one of the inputs is fixed. The long run is made up of many short runs. The firm operates in the short run and plans for the long run.

b. The LAC curve is the envelope of all SAC curves. The LAC curve shows the minimum cost of producing each level of output.

c. The SAC curves are U-shaped because of the operation of the law of diminishing returns to the variable input. The LAC curve is U-shaped because of initially increasing returns to scale and subsequent constant returns to scale and, eventually, decreasing returns to scale.

10. State the relationship between production functions and cost curves. (b) Explain how we can derive TP, AP and MP curves for a factor of production from an isoquant diagram. (c) Explain how we can derive the TVC curve from a TP curve.

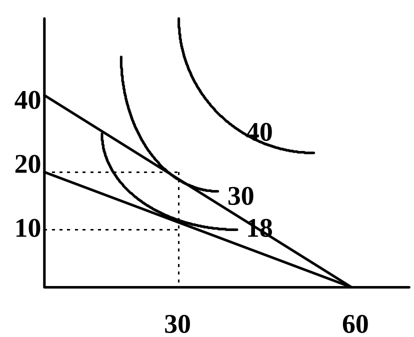
Ans: The production function of a firm, together with the prices that the firm must pay for its factors of production or inputs determine the firm's cost curves. (Try using graphs to do the derivations).

12. The production function provides estimates of maximum weekly output of a company producing embroidery shirts, given in combination of labor and capital.

Capital	Output					
60	12	28	32	40	48	50
50	18	30	40	48	52	48
40	18	30	40	48	48	40
30	14	32	36	40	40	30
20	8	24	30	32	30	24
10	2	8	18	24	24	18
Labor	10	20	30	40	50	60

If the total cost of producing shirts is $1200 per week, the wage rate is $20 per week, and then the rent on machinery is $60 per week, complete the following:
(a). Derive the isocost line in the form of a straight line relating K to L.
(b). Write down the actual isocost equation for this firm using the information provided. Draw this isocost. If the rental rate halves while the wage rate remains constant, draw the new isocost line.
(c). Draw the production isoquants at the levels of 18, 30 and 40 shirts per week and connect the points on each isoquant with a smooth curve. Read off from the graph the optimal number of shirts which could be produced when the wage rate is $20 per week and the rental rate on capital rate is $60 per week. What would be the optimum weekly output if the rental rate was halved?
(d). What are the values of the marginal rate of technical substitution at the optimal points noted in (c)?
(e). Compare the two optimum positions in terms of the capital and labor used. Does this help us understand the difficulties in using only labor productivity as a measure of efficiency?

Ans: a. Isocost line: $C = wL + rK \Rightarrow K = C/r - (w/r)L$
b. Actual equation for this firm: $K = 120/60 - (20/60)L \Rightarrow K = 20 - 1/3L$
c. If the rental rate halves, the vertical intercept is 40.



d. Slope at (30,10) = 1/3; Slope at (30, 20) = 2/3.
e. When we look at the two optimum positions on the 18- and 30-unit isoquants, it appears that the labor productivity has increased between these two points because with the same amount of labor (30), output has increased from 18 to 30. However, we can see that the increase in output is also due to employing more capital. That is, much of the apparent increase in output is attributable to an increased capital intensity of production.

13. Given that the total cost function of fertilizer company is in the form: TC = 300 + 50Q - 10Q^2 + Q^3 where Q = is in tons per hour of nitrates produced:
(a). Find the value of total fixed costs and expressions for the Average Total Cost, Total Variable Cost, Average Variable Cost and marginal cost functions.
(b). Show that the AVC curve is U-shaped and that the MC curve will intersect the AVC curve at the lowest point of the latter.
(c). What is the tonnage of fertilizer that should be produced to minimize short run AVC? What will be the AVC per ton at this output?

Ans: $FC = 300$; $ATC = 300/Q + 50 - 10Q + Q^2$; $TVC = 300 + 50Q - 10Q^2 + Q^3$; $AVC = 50 - 10Q + Q^2$; $MC = 50 - 20Q + 3Q^2$.

b. $\dfrac{dAVC}{dQ} = -10 + 2Q = 0$

$\Rightarrow Q = 5$. This is the minimum of *AVC*. Therefore, there is a turning point at Q = 5. To decide whether the turning point is a minimum. We have to investigate the second order condition, i.e.,

$\dfrac{d^2(AVC)}{dQ^2} = 2$. This is a positive number which confirms that it is a minimum point and that the curve is, in fact, U-shaped.

c. The quantity of fertilizer that minimizes AVC is 5 tonnes/hour which has already been calculated in (b) above. Substituting this value into the AVC function,
AVC = 50 - 50 + 25 = 25.