Heterogeneity and Evolution of Expectations in a Model of Currency Crisis

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Abstract

A model of a portfolio allocation between mature and emerging markets is simulated with heterogeneous expectations, imitation, and experimentation. Solutions produce periodic crises. The predictions of the model are compared to a representative-agent, rational expectations model with multiple equilibria.

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1. Introduction

The recent occurrence of currency crises in emerging market countries has inspired widespread interest in whether currency crises were the result of fundamental policy failures or structural problems in the countries concerned, or whether instead the attacks on specific countries were essentially arbitrary, stimulated by changes in market sentiment and perhaps subject to contagion from other countries suffering crises. As a result of this interest, numerous models have been presented to account for the 1994-95 Mexican crisis, the 1997-98 Asian crises, and the after-effects of the Russian default; some of these models, following Obstfeld (1986, 1994) have allowed for the possibility of self-fulfilling speculative attacks, in which the deterioration of a country's fundamentals is not the sole reason for the crisis (Cole and Kehoe, 1996; Sachs, Tornell, and Velasco, 1996; Radelet and Sachs, 1998; Chang and Velasco, 1998). Other authors have also ascribed contagion to jumps between multiple equilibria (Krugman, 1999; Masson 1999). In these models, rational expectations are consistent with more than one solution for asset prices and other financial and real variables. For instance, devaluation probabilities and interest rates can reflect confidence in the prospects for a country, and this will be self-validating in that low interest rates (or inflation) make the authorities' policy trade-offs more favorable. In contrast, there is another equilibrium in which lack of confidence makes a devaluation (or more generally a more expansionary monetary or fiscal policy) more likely. However, typically such models have little, if anything, to say about how investors coordinate on one or another of the equilibria. In fact, they typically assume that investors all act in the same way, with their actions dictated by an extraneous, sunspot variable.

Another important feature of emerging financial markets has been surges of capital inflows that seem to plant the seeds for the subsequent crises. Dooley (1999) has rightly argued that theories of currency crises should also shed light on the boom periods. In his model, rational investors put their money in emerging markets to exploit government guarantees; when the resources available to honor those guarantees run out, investors run for the exits, provoking a crisis.

There are two general approaches that have been used in macroeconomics, and financial markets in particular, to explain how the formation of expectations might evolve, which put some structure on the coordination of expectations. One set of models has examined why investors may imitate others and be subject to fads and bandwagon effects which may result in what appear to be arbitrary swings in sentiment (Banerjee, 1992; Bikchandani et al., 1992; Caplin and Leahy, 1994; Lee, 1997; Chari and Kehoe, 1998). These models, which are based on imperfect and asymmetric information, emphasize the heterogeneity of investors, who receive a private signal but need to infer whether to give weight to that information or rather imitate others. Depending on the sequence of signals received, the market price can take values that are arbitrary. Moreover, a new signal that tips the balance of sentiment from optimism to pessimism can provoke a "cascade" or "avalanche" of sell orders and a large change in price. Calvo and Mendoza (1996) present a model of herding behavior by international investors that is applied to the Mexico crisis; Calvo (1998) uses a model of informed and uninformed investors to try to understand contagion from Russia.

These models emphasize the heterogeneity of expectations and the arbitrariness of the resulting asset prices, which result from a sequence of generally unobservable, individual shocks. While multiple equilibria models concentrate on macroeconomic (and financial) interactions, herding models focus almost exclusively on the interactions in the formation of expectations. Thus, there is a need to study more general models which consider both sets of interactions. Moreover, there are still theoretical puzzles relating to the types of models that allow for multiple equilibria. Morris and Shin (1998) show that solutions to a simple speculative attack model may be very sensitive to the information structure, so that if common knowledge about the distribution of the (private) shock received by all agents does not exist, then the possibility of multiple equilibria disappears. However, the applicability of this result to more general models is unclear, justifying further experimentation with models with macroeconomic linkages and heterogeneous expectations.

Another strand of literature considers alternatives to rational expectations that involve modeling the process of expectations formation. In particular, there is an extensive literature on learning models and bounded rationality in macroeconomics (see Sargent 1993 for a partial survey). These types of models have proven very useful in studying the out-of-equilibrium dynamics and have generated interesting behavior that traditional analysis and models have not been able to address, for example, persistent fluctuations of asset prices (LeBaron et al., 1999; Brock and Hommes, 1997), exchange rates (Arifovic, 1996), recurrent inflations (Cho and Sargent, 1999; Sargent, 1999), and recurrent hyperinflations (Marcet and Nicolini, 1997). Lettau and Uhlig (1999) show that learning may not lead households to reject "rule of thumb" saving behavior in favor of the optimal, dynamic programming solution, thus helping to explain the excess sensitivity of consumption to income. Kasa (1999) presents a version of Obstfeld's (1997) "escape clause" model in which agents learn about the government's decision rule using a stochastic approximation algorithm; the dynamics are characterized by recurrent episodes of currency crisis.

However, this strand of literature has not to date been applied to issues relating to the boom and bust cycles of capital flows to emerging markets, which is the subject of this paper. We embed a model of portfolio selection and expectations formation that involves both imitation and experimentation in a very simple balance of payments model of speculative crises, detailed in Masson (1999). This approach to modeling expectations, which embodies trial-and-error learning through investors' interaction and through occasional experimentation, has some important advantages relative to other models of bounded rationality. First, it provides a convenient framework to study the impact of heterogeneous beliefs. Second, the information requirements on economic agents are minimal. Finally, the predictions of models that employ evolutionary adaptation (similar to the evolutionary algorithm we use in this paper) have proven very successful in capturing the behavior observed in laboratory experiments with human subjects (e.g. Arifovic, 1994, 1996).[1]

Investors are each assumed to be risk neutral, and to choose between putting all their money either into U.S. assets or into emerging market assets, the latter subject to possible devaluation (or default). Risk neutrality permits highlighting the role of the formation of expectations by heterogeneous agents in determining period-by-period the volume of capital flows to emerging markets, while making their long-run equilibrium level arbitrary. While an extreme assumption, it seems to square with boom phases when obvious risk factors seemed to have little role in discouraging inflows. In fact, in all three crises–tequila, Asia, and Russia–a common and striking feature was the neglect of obvious risk factors in the priod leading up to the crises. The model, as we shall see, also predicts sharp reversals, which square with the crises we have seen, despite no assumption that attitudes toward risk changed.

In general, the dynamics that result from our model are too complex to be completely characterized analytically, and we present some simulations below that illustrate some of the properties, as well as deriving a few analytical results from a simplified version of the model. We explore the interaction between the macroeconomic causes of balance of payments crises and the shifts in expectations that correspond to imitation and experimentation. We are able to illustrate the abrupt

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[1] For a survey of the applications of evolutionary models in macroeconomics see Arifovic (1999).

shift in market sentiment, triggering shifts in the holdings by investors of emerging market debt, that seems to correspond to the swings between booms and busts in capital flows to emerging markets. In the model, it is the decline in the premium of emerging market interest rates over those in developed countries (the boom phase) that eventually leads to a reversal in sentiment. At first, inflows lead to a favorable balance of payments position, and the absence of a devaluation means that emerging market investors make excess returns, leading other investors to imitate them. However, there is a limit to this process, since there is a limit to the funds available for investment into emerging markets. Success in attracting inflows makes countries especially vulnerable, since the resulting high debt requires for its servicing continually larger inflows. At some point, too many investors already have placed their wealth in emerging market assets for their fraction to increase further, and this fraction can only stabilize or fall, leading to a drying up of capital inflows. If the stock of debt on which interest needs to be paid is high enough, and reserves are low enough, this leads to a crisis. As has been the case for a number of the countries involved in the crises cited above, in our model these shifts in sentiment trigger occasional devaluations or defaults. The analytics of the simplified model with a continuum of agents but without shocks imply that expectations formation produces regular cycles of booms and busts of capital flows.

While these results do not constitute a new theory of speculative attacks or of expectations formation, they make clearer the role of heterogeneity in making determinate the amount of investment in emerging markets. When the degree of consensus shifts, it can cause large shifts in amounts invested, substantial changes in interest rates, and increased vulnerability to crisis (the country is assumed subjected to shocks to its trade balance as well). And the shifts in consensus result in the model from the intrinsic dynamics of an unchanged and plausible process of expectations formation.

The plan of the paper is as follows: Section 2 gives the model of the individual investor's portfolio decisions, the market interest rate, and the authorities' decision to devalue. Section 3 details the formation of expectations, Section 4 presents the simulation results, Section 5 gives the analytics of a simplified model, while Section 6 concludes.

2. Equations of the model

The model describes the behavior of risk neutral investors who decide to put their wealth either in an emerging market country or the United States, and the behavior of an emerging market central bank which defends a currency peg using its foreign exchange reserves until those reserves reach some minimum threshold value. The model is extremely simple, but is designed to highlight: i) the possibility of long-run indeterminacy in asset holdings, at least within some range; ii) the heterogeneity of expectations; and iii) the idea that investment decisions depend on an investor's degree of optimism or pessimism *relative to the average expectation.*

The U.S. asset is riskless, and pays a known rate r^*, while the emerging market asset is subject to devaluation (or default) risk. There are n investors, each with beginning-of-period wealth $\overline{W}$, who form expectations of the devaluation probability π_t^i. We assume that the expected devaluation size is equal across investors and constant over time. Denote this amount by δ^e. Since investors are risk neutral, they will be indifferent between investing in the two assets when their ex ante returns are equal, and choose between putting all their beginning-of-period wealth into the safe foreign asset, at rate r^*, or into emerging market claims, at rate r_t, depending on which expected return is greater. These short-term assets are best thought of as bank deposits, with the bank setting the rate. The supply of both U.S. and emerging market debt is thus infinitely elastic at these rates, and the demand for emerging market deposits determines the capital flow to emerging markets. Short selling of either asset is ruled out; neither portfolio proportion can be negative.[2] If λ_t^i is the share of i's wealth in emerging market debt, then

$\lambda_t^i = 0$ or 1 as $(1 + r^*) >$ or $< (1 + r_t)/(1 + \pi_t^i \delta^e)$.

So at any period t, the amount of emerging market deposits held by all foreign investors is

$$D_t = \sum_{i=1}^{n} \lambda_t^i \overline{W}. \qquad (2.1)$$

Emerging market banks set the interest rate on bank deposits to reflect market expectations of the return on emerging market debt. We assume that banks do not form expectations of devaluation themselves; they just use the average of all investors' expectations as a measure of the expected value of devaluation. Thus,

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[2] Similar properties can be obtained if borrowing is allowed, but there are limits on leverage (such as a minimum capital requirement).

the interest rate on emerging market deposits r_t is set equal to the U.S. rate plus a weighted average of the expected rate of devaluation. This equation, which is analogous to an interest parity condition, can be written

$$r_t = (1 + r^*) \prod_{i=1}^{n} (1 + \pi_t^i \delta^e)^{1/n} - 1. \tag{2.2}$$

With different expectations, expected returns will be equalized only for the marginal investor whose expectation equals the average expectation. Each individual investor will make her investment choice on the basis of a comparison with the average expectation embodied in the interest rate. If more optimistic on emerging markets, in the sense of estimating a lower probability of devaluation than the average, then she will put all her wealth into emerging market debt; otherwise, she will put it all into U.S. assets. In this model, investor heterogeneity is key to determining the amount of emerging market assets held. In a representative agent model of risk neutral investors, there would be no way of determining (without further assumptions) the portfolio proportions allocated to the two assets.

Turning to the macroeconomic equilibrium, the balance of payments identity relates the change in reserves to the trade balance plus the purchase of new debt by investors minus the principal and interest on maturing debt, assuming that there has been no devaluation or default:

$$R_t = R_{t-1} + T_t + D_t - (1 + r_{t-1}) D_{t-1}. \tag{2.3}$$

The trade balance is assumed to be described by an AR(1) process:

$$T_t = \alpha + \beta T_{t-1} + \varepsilon_t, \tag{2.4}$$

where ε is assumed to be normally distributed with mean zero and variance σ^2.

Provided that R_t is above some threshold level (which we assume without loss of generality to be zero), there is no devaluation at t, i.e. $\delta_t = 0$ (absence of superscript indicates that this is the realized value of depreciation, not its expectation). However, if reserves would otherwise be negative, there is a devaluation or default which reduces the amount that will be repaid on borrowing undertaken at t. That is, the ex post return for the lender will be $(1 + r_t)/(1 + \delta_t)$, where the amount of the devaluation is equal to the shortfall in the balance of payments that would have pushed R_t negative, divided by D_t:

$$(1+\delta_t) = [(1+r_{t-1})D_{t-1} - R_{t-1} - T_t - D_t]/D_t \tag{2.5}$$

Though the devaluation/default reduces the amount owed at $t+1$, not t, we assume that in this case balance of payments arrears are accumulated within the period such that reserves at t do not go negative but instead equal zero.

What happens if all investors form identical expectations? Suppose that for some reason expectations converge on some value π, and that the fraction of wealth allocated to emerging markets takes on some value λ which is constant from period to period. What is the value for the interest rate, r, that is consistent with a rational expectation for the devaluation probability? Recall that a devaluation occurs when reserves fall to a threshold value, which we will assume to be zero. Thus, $\pi_t = \Pr_t(R_{t+1} < 0|\text{no devaluation})$ Now, since in this case

$D_t = D_{t-1} = n\lambda\overline{W}$,

$$R_{t+1} = R_t + T_{t+1} - r_t n\lambda\overline{W} \tag{2.6}$$

and so

$$\pi_t = \Pr_t(\varepsilon_{t+1} < (r^* + \pi_t\delta^e)n\lambda\overline{W} - (R_t + \alpha + \beta T_t)) \tag{2.7}$$

This latter equation determines the rational expectation for the devaluation probability, and an interesting question is whether the boundedly rational agents' expectations would converge to it. It should be noted that both the right hand and left hand sides of the above equation depend positively on π, and under certain conditions it can have multiple solutions, as discussed in Masson (1999). This suggests that it is unlikely that all agents would permanently converge to the same estimate of π_t. However, the indeterminacy in the model of the average portfolio proportion is quite independent of the existence of such multiple equilibria. Indeed, even if there is a single rational expectations equilibrium, there can be alternation of booms and busts in lending to emerging market countries that resemble arbitrary shifts between multiple equilibria.

3. Evolution of Expectations

We now consider a learning model where agents are boundedly rational and acquire the experience and knowledge needed to improve their performance over time. We impose very weak requirements on agents' computational abilities. The learning algorithm describes imitation-based adaptation of the agents' expectational rules. Investors consider their own and the success of other investors and try to imitate those rules yielding above-average returns. In addition, they occasionally experiment with new expectational rules. The adaptive algorithm may be interpreted as a stochastic version of replicator dynamics (Taylor and Jonker, 1978).

Our economy consists of a population of n investors who make portfolio decisions, i.e., how much of their wealth W to invest in emerging market deposits. The rest of their wealth investors invest at a world rate of return that prevails in developed markets.[3]

At each t, investor i, $i \in [1, \ldots, n]$, is characterized by her expectation of how likely devaluation in the following period is, that is the probability of devaluation, denoted by π_t^i. The range of values of π_t^i is between 0 and π_{max}.[4]

At the beginning of each t, r_t is calculated using the values of individual π_t^i's. We also add a constant risk premium ρ to the return on emerging market deposits, in order to help in calibrating with real-world data. With this modification, the value of r_t is now given by:

$$r_t = (1 + r^*)(1 + \rho) \prod_{i=1}^{n} (1 + \pi_t^i \delta^e)^{\frac{1}{n}} - 1 \tag{3.1}$$

Then investor i's rule for making a portfolio decision is given by: $\lambda_t^i = 0$ or 1 as $(1 + r^*) >$ or $< (1 + r_t)/[(1 + \rho)(1 + \pi_t^i \delta^e)]$.

Based on the value of r_t and on their individual expectations about the devaluation probability, investors make their decision using the rule described in the previous section. These individual decisions determine the amount invested in emerging market deposits (equation 2.1). Then the level of reserves at t, R_t is computed, and compared to zero. If reserves would otherwise go negative, the amount of devaluation is determined by equation (2.5).

Realized rates of return determine measures of performance of the expectations used at time t that we call *fitness* values. A fitness value for investor i, μ_t^i, is equal

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[3] In our initial set-up, individual investors' wealth evolved over time. However, wealth accumulation did not affect the main features of the dynamics. In order to simplify the model and make it comparable to our analytical results, we assume that each investor is endowed with the same constant amount of wealth at the beginning of each period.

[4] The choice of a maximum value for π was made to facilitate convergence of the simulations. The value chosen, a 10 percent probability in the coming month, corresponds to a large annualized devaluation probability, and is thus not very constraining.

to:

$$\mu_t^i = (1 + r_t)/(1 + \delta_t) - 1 \tag{3.2}$$

if investor i invested her wealth in the emerging market and to

$$\mu_t^i = r^* \tag{3.3}$$

if she invested in the US market. In the case that due to devaluation ($\delta_t > r_t$) the fitness of an expectation takes a negative value, it is truncated to zero. Thus all the expectations that resulted in $\lambda_t^i = 1$ receive the same fitness value even though they may have different values of π_t^i. Similarly, all those that resulted in $\lambda_t^i = 0$ receive the same fitness value even though they may have different π_t^i's. Investors update their expectations of the devaluation probability π_t^i at the end of each period by imitating rules that have proven to be relatively successful and by occasional experimentation with new expectational rules.

Imitation

At the beginning of each period t, investor i, $i \in [1, \ldots, n]$ compares her expectational rule to a rule of a randomly selected investor j. The probability, Pr_t^j, that an expectational rule j is selected for comparison is equal to the expectational rule's relative fitness:

$$Pr_t^j = \frac{\mu_t^j}{\sum_{i=1}^n \mu_t^i}. \tag{3.4}$$

We can think of the selection of an expectation j as resulting from a spin of a roulette wheel where each expectation is assigned a slot proportionate to its relative fitness (proportional selection). Rules that performed better get larger slots than rules that did worse in the previous period, and thus well-performing rules have higher probability of being selected. (Rules are selected with replacement.) Once j is selected, investor i compares the fitness of her own expectational rule to the fitness of investor j's expectational rule. If the fitness of her own rule is equal or higher, she keeps her own rule. Otherwise, investor i imitates (adopts) the expectational rule of investor j. Denote the probability of devaluation that is assigned to investor i by $\pi_{t+1}^{i,m}$. If $\mu_t^i \geq \mu_t^j$ then $\pi_{t+1}^{i,m} = \pi_t^i$. On the other hand, if $\mu_t^i < \mu_t^j$ then $\pi_{t+1}^{i,m} = \pi_t^j$.

Note that in case of devaluation, if $\delta_t > r_t$, expectational rules of the investors who invested in the emerging market yield a negative return, which is truncated to zero. Thus expectations of all investors who invested in the emerging market will receive fitness equal to 0 and will not be imitated. Only the expectations of those investors who invested in the US market receive positive, equal probabilities of being selected.

Thus the imitation operator M is applied to the population of expectations of period t. Denote the resulting population by $\Pi_{t+1}^{m} = ME_t = [\Pi_t^m]$ where Π_t^m is a vector $[\pi_t^{1,m}, \ldots, \pi_t^{n,m}]$. Imitation alone represents a type of herd behavior in that on average, over time, well-performing expectations will be imitated (followed) by a larger number of investors and on average, investors will encounter better-performing expectations more frequently.

Experimentation

Each investor $i \in [1, ..., n]$ can experiment with her expectational rule. Experimentation takes place with probability p_{ex}. First, a random number is drawn in the interval $[0, 1]$ from a uniform distribution. If it is less than p_{ex}, the investor experiments with the expected probability of devaluation: a random number is drawn from the interval $[0, \pi_{max}]$ and that number, $\pi_{t+1}^{i,ex}$, replaces $\pi_{t+1}^{i,m}$. Otherwise, there is no experimentation and investor i keeps $\pi_{t+1}^{i,m}$. The application of the experimentation operator X on the population Π_{t+1}^{m} results in the population of expectations, $\Pi_{t+1} = X\Pi_{t+1}^{m}$ that are used by investors at time period $t + 1$.

The above describes how the population of investors interacts. If investors are not able to gather enough information to form reliable estimates of the future behavior of the markets, and based on that determine their optimal behavior, imitation of previously successful strategies seems a plausible behavioral assumption. This type of behavior is explicitly modeled in our framework using proportional selection such that expectational rules that yielded an above-average payoff tend to be used by more investors in the following period. Experimentation incorporates innovations by investors, done either on purpose or by chance.

4. Simulation Results

In order to make the simulations as realistic as possible, starting values and parameters were chosen from a real-world case; in particular, initial values for external debt, reserves, and the trade balance were taken to be those prevailing in Argentina at the end of 1996, and the process driving the trade balance was estimated by a regression using historical data for that country. These values were all taken from Table 1 of Masson (1999), which also describes in more detail the balance of payments, currency crisis model. The model there is calibrated to annual data; however, our interest here is in higher-frequency observations, so we calibrate interest rates to monthly. All variables are expressed as ratios to

GDP, so the relevant interest rates are actually the difference between a nominal interest rate and the rate of growth of nominal GDP. Thus for r^*, the U.S. interest rate, we use $(0.05 - 0.03)/12$, or 0.0018333. In order to square with real world observations, in our simulations we also include a risk premium ρ of 0.0083 (annual rate of 10 percent, which roughly corresponds to the spread of Argentina's dollar bonds over U.S. treasuries).

Initial values of variables are given by:
$\sigma = 0.0212$, $D_1 = 34.4$, $R_1 = 6.1$, $T_1 = -0.3$, total wealth $\overline{W} = 68.8$ (this figure was arbitrarily chosen to be twice D_1)[5]. All these values are percentages of GDP. The evolution of the trade deficit over time is given by the following estimated historical relationship:

$$T_{t+1} = 0.006743 + 0.6167 T_t + \epsilon_{t+1}$$

where $\epsilon_{t+1} \sim N(0, 0.0212^2)$.

We use values of p_{ex} equal to either 0.33 or 0.033. The number of investors, n, is equal to 100. The value of π_{max} is 0.1. For the devaluation size, we use $\delta^e = 1$, that is a devaluation of 50 per cent.[6] Actual devaluation size is endogenous, however, and is determined by equation (2.5).

Our simulations exhibit interesting behavior with recurrent devaluations: extended periods of $\delta_t = 0$ are followed by instances of devaluation, $\delta_t > 0$, which take place over several periods. Qualitative features of the dynamics are the same for all the different cases that we simulated. During the periods when $\delta_t = 0$, $\bar{\pi}_t$, and consequently r_t, are decreasing while $\bar{\lambda}_t$ and R_t are increasing. On the other hand, during the periods of $\delta_t > 0$, $\bar{\pi}_t$, and r_t are increasing, R_t is equal to zero and $\bar{\lambda}_t$ is decreasing.

Devaluations are triggered by a reversal in the general pattern of falling values of $\bar{\pi}_t$ and rising $\bar{\lambda}_t$ – that is, they reflect a change in market sentiment as embodied in the distribution of devaluation expectations. Once the first devaluation takes place, devaluations in subsequent periods result from a general pattern of rising values of $\bar{\pi}_t$ and falling $\bar{\lambda}_t$. This process can be halted only by a reversal in the pattern of falling values of $\bar{\lambda}_t$.

A lower value of p_{ex} results in more frequent devaluations that last for fewer periods. Table 1 presents the average number of occurrences of devaluations

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[5] Note that here $\overline{W}$ is total wealth, not the wealth of an individual investor.

[6] Note that since the value of δ^e only enters into the formula for computing r_t, its values just affect the average level of r_t.

(c_1) and average duration of each occurrence of devaluation (c_2) for two different values of p_{ex}. The numbers represent averages over 5 simulations (using different seed values for a random number generator). Each simulation was conducted for $10,000$ months, and each month a new shock was generated for the trade balance and new draws for the other random variables.

Table 1

Average frequency and duration (in months) of devaluation

$\delta^e = 1$		
	c_1	c_2
$p_{ex} = 0.33$	$680/10,000$	3.79
$p_{ex} = 0.033$	$725/10,000$	1.31

4.1. Understanding the simulations

Analysis of the limiting case with an infinite number of agents helps to shed some light on the simulations.[7] In this case, there is a continuum of values for the devaluation probability, and the population of investors is identical to the probability density. What is interesting is that the form of the distribution of devaluation expectations is crucial for the results, and the dynamics of the evolution of expectations produce predictable changes in the shape of the distribution.

The key to understanding the results comes from recognizing that whether $\overline{\lambda}_t$ is greater or less than one-half depends on whether the distribution of expectations is skewed to the left or the right. If to the right, there is more probability mass to the left of the mean expectation, so $\overline{\lambda}_t > 0.5$, and conversely (see Figure 1). This is exactly what we observe in our simulations of the model with a finite number of investors. Moreover, the skewness shifts over time as expectations evolve. Suppose that there is a sequence of periods of no devaluation (so that $\delta_t > 0$). This will typically involve a period where the distribution is skewed to the right, so that $\overline{\lambda}_t > 0.5$. Absence of devaluation yields a higher fitness value to those expectations to the left of the mean (i.e. those more optimistic on emerging markets); it will therefore raise the probability mass on the left side of the distribution, and lower that on the right. But as this process continues,

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[7] We abstract from the balance of payments dynamics (and from shocks to T_t) and assume a simplified version of imitation, in which each investor chooses randomly her rule from the same pool of rules (with probability determined by fitness values), rather than comparing her rule to a randomly selected rule. Derivations are presented in the Appendix.

eventually the positive skewness declines or disappears, making $\overline{\lambda}_t$ decline and increasing $\bar{\pi}_t$. The Appendix shows that if $\bar{\pi}_t$ gets too low, this leads to a reversal, with $\overline{\pi}_{t+1} - \overline{\pi}_t > 0$, despite no devaluation having occurred. This will happen when $\bar{\pi}_t$ gets into the range of values $[0, \pi_{max}/2]$. In the simulations of the model with n investors, it is the reversal of $\bar{\pi}_t$ that can trigger a crisis with several periods of devaluation. Conversely, periods of devaluation involve negative skewness, but repeated devaluations lower the probability mass to the left of the distribution and increase it to the right, eventually *lowering* $\bar{\pi}_t$ and raising $\overline{\lambda}_t$. The condition that we derive shows that, following a devaluation, $\overline{\pi}_{t+1} - \overline{\pi}_t > 0$ if and only if $\overline{\pi}_t < \frac{\pi_{\max}}{1+p_{ex}}$. This condition is normally satisfied in the simulations during periods of devaluation. Thus, in the normal case, a devaluation would increase the estimated probability of devaluation next period. However, if devaluations continue for several periods, $\bar{\pi}_t$ can rise to a high enough level that the inequality is not satisfied, triggering a reversal.

These analytical results are borne out by the simulations. Figure 2 (which displays a simulation that assumes $p_{ex} = 0.33$) shows that the dynamics are characterized by sequences of decreasing values of $\bar{\pi}_t$ during which waves of optimism induce many investors to put their wealth into the emerging market, and by sequences of increasing values of $\bar{\pi}_t$ during which waves of pessimism make many investors pull out of the emerging market and invest in the US market.

Consider the periods during which $\delta_t = 0$. During these periods, $\bar{\pi}_t$ is generally decreasing. The expectational rules with relatively low values of π_t^i have higher fitness values than those with relatively high values of π_t^i and thus evolutionary selection will favor the low π_t^i rules over the high π_t^i rules. In addition, among the new rules generated via experimentation, the evolutionary selection will favor those with low values of π_t^i. This pushes the value of $\bar{\pi}_t$ down. In order for an investor to keep investing in the emerging market, her expectational rule has to remain below $\bar{\pi}_t$. Overall, evolutionary pressure works towards lowering the values of $\bar{\pi}_t$ and π_t^i's. Eventually, once $\bar{\pi}_t$ reaches sufficiently low values, a reversal occurs and the value of $\bar{\pi}_t$ increases, and the simulations confirm that this reversal occurs in the model with a finite number of investors as well. The increase in $\bar{\pi}_t$ will result in a decrease in $\overline{\lambda_t}$ and an increase in r_t.

At this point, two things can happen and which of them occurs depends on the level of reserves held by the emerging market central bank. On the one hand, if R_t is low enough at the point when the reversal takes place, a decrease in $\bar{\lambda}_t$ may result in a withdrawal of deposits sufficient to trigger a devaluation (or default). If this happens, a sequence of periods during which $\delta_t > 0$ starts, leading to a sequence of increasing values of $\bar{\pi}_t$ and decreasing values of $\bar{\lambda}_t$.

On the other hand, at the moment when the reversal takes place, the level of reserves may be high enough so that even though there is a decrease in $\bar{\lambda}_t$ and hence in the amount of deposits, there is no need for devaluation. If δ_t remains equal to 0 at the time period when the reversal of $\bar{\pi}_t$ occurs, the expectations of the investors who invested in the emerging market will be vindicated, and the process of declining $\bar{\pi}_t$ will be set off again. However, another reversal will eventually take place once again, and when that is coupled with a low level of reserves, a devaluation is triggered.

Next, consider what happens during a devaluation. The values of $\bar{\pi}_t$ are increasing and of $\bar{\lambda}_t$ are decreasing. Decreases in $\bar{\lambda}_t$ result in further depletion of reserves, and further devaluation. But a reversal of the direction of $\bar{\pi}_t$ movement will occur again once $\bar{\pi}_t$ exceeds $\pi_{max}/(1+p_{ex})$. This will result in a (usually small) increase in $\bar{\lambda}_t$ that is sufficient to set $\delta_t = 0$.

4.2. An episode of devaluation

In the actual simulations, it is interesting to look at the behavior of $\bar{\pi}_t$, $\bar{\lambda}_t$ and r_t around the time of a devaluation. Thus we take a particular instance of devaluation, between $t = 66$ and $t = 77$. Figure 3 presents the behavior of $\bar{\pi}_t$, $\bar{\lambda}_t$, R_t/D_t and r_t during this interval, and Figure 4 the distribution of the values of π_t^i for the n investors in our population.

Between $t = 66$ and $t = 71$, there are increases and decreases in $\bar{\pi}_t$, but overall it takes low values, between 0.0151 and 0.0236. Figure 4 shows that for observations 66-71, during which $\delta_t = 0$, most of the mass of the distribution is concentrated below the mean value, $\bar{\pi}_t$, leading to $\bar{\lambda}_t >> 0.5$. Initially, at $t = 67$, $\bar{\pi}_t$ (and hence r_t) increases. This causes R_t to go down, but it decreases from a sufficiently high level, so that the combination of a decrease in $\bar{\lambda}_t$, which implies a withdrawal of deposits, and of an increase in r_t, which implies higher interest payments, does not result in devaluation, just a smaller level of R_t. (From Figure 3, it is clear that R_t/D_t decreases during these periods.) Then, at $t = 68$, $\bar{\pi}_t$ decreases again, and the reversed process takes place. However, subsequent higher values of $\bar{\pi}_t$, at $t = 70$ and 71, bring R_t to a lower level.

At $t = 72$, $\bar{\pi}_t$ increases again and this time, reserves are so low that devaluation has to take place. Thus, $\delta_t > 0$ first occurs at $t = 72$. Even though it is a small devaluation of only 0.04, it triggers large changes in expectations. The average value of π_t goes from 0.0269 in $t = 72$ to 0.0581 in $t = 73$, the first period following the initial devaluation. This huge increase in $\bar{\pi}_t$ results in a decrease in $\bar{\lambda}_t$. In fact, a sequence of repeated devaluations is set off with increasing values of $\bar{\pi}_t$ and decreasing values of $\bar{\lambda}_t$. In Figure 4, observations 72-75 show that, as devaluation wipes out positive returns in emerging markets, the distribution starts shifting towards the right and at $t = 74$ and 75, most of the mass has now shifted to the right of $\bar{\pi}_t$.

Note that at the time when the devaluation process starts, the value of $\bar{\pi}_t$, 0.0269, is less than $\pi_{max}/(1+p_{ex})$ and thus it is in the region where (for the limiting case) the value of $\bar{\pi}_t$ is rising. This is confirmed in our simulations; the value of $\bar{\pi}_t$ is rising during the periods when $\delta_t > 0$.

Once $\bar{\pi}_t$ reaches a high enough value, the reversal occurs and its value decreases, i.e. $\bar{\pi}_{76} = 0.0700 < \bar{\pi}_{75} = 0.07440$. According to the calculation using our condition under which this can happen in the limiting case, $\bar{\pi}_t$ has to exceed 0.0750. The actual value of $\bar{\pi}_{75}$ is little bit lower than that, but the reason is that the derived condition is based on the assumption of infinite number of investors, while the number of investors in our simulations is equal to 100. This decrease in $\bar{\pi}_t$ results in $\bar{\lambda}_{76} = 0.29 > \bar{\lambda}_{75} = 0.24$, only a slightly higher value for $\bar{\lambda}_t$, but sufficient to result in an inflow of deposits that avoids further devaluation ($\delta_{76} = 0$). Once $\delta_t = 0$, the variables start moving in the opposite direction (decrease in $\bar{\pi}_t$ and increase in $\bar{\lambda}_t$). Looking at Figure 4 again reveals that at $t = 77$, one period after δ_t has again become equal to 0, the distribution has started shifting to the left and will continue in that direction until again most of the mass is to the left of $\bar{\pi}_t$.

4.3. Discussion

To sum up, in any given period, as long as $\pi_t^i \leq \bar{\pi}_t$, investor i will invest in the emerging market and as long as $\delta_t = 0$ will earn the return $r_t > r^*$. Thus, any individual investor has to be more optimistic than the average investor in order to invest in the emerging market and earn a higher return than on US investments. The evolutionary dynamics in the absence of devaluation drive the value of $\bar{\pi}_t$ down, and increase the proportion of investors in emerging markets. However, increasing optimism eventually comes to an end, once $\bar{\pi}_t$ becomes small enough to enter into the region where a reversal of its decline occurs. The reversal occurs because there is a limit to the number of investors, so that increasing optimism cannot continue indefinitely. Similarly, once $\bar{\pi}_t$ starts increasing, investors become more and more pessimistic about emerging markets. They have to be more pessimistic than the average investor to invest in an asset with a rate of return of only r^*. Excessive optimism is then followed by excessive pessimism, which also contains the seeds of its own reversal. This type of behavior seems to correspond to what is actually observed in emerging financial markets.

The model's balance of payment equation reflects the fact that the emerging market economy is dependent on a continuation of foreign investment inflows. Their reversal (a "sudden stop" as in Calvo and Reinhart 1999) triggers a currency crisis and devaluation. This, coupled with the evolution of heterogeneous beliefs produces the dynamics, which are in fact mainly driven by evolution of expectations (not by shocks to the trade balance). Even though we do not observe convergence of expectations to a particular equilibrium value, or explicit coordination, the evolution of expectations does generate recurrent waves of optimism and pessimism. The periods of time during which optimism prevails and a large fraction of deposits is invested in the emerging market last on average between 10 and 20 months (since our interest rates are calibrated to monthly frequency). This can be compared to stylized facts concerning the actual length of boom periods. According to Klein and Marion (1994), for instance, historically the average length of an exchange rate peg has been 10 months.

The observed dynamics look very much like the dynamics of currency crisis observed in actual markets. Usually there is no apparent reason for a sudden shift in investors' expectations and a withdrawal of deposits from the emerging market. An analysis of crises versus non-crisis periods fails to identify significantly worse macroeconomic fundamentals in the former (Eichengreen, Rose, and Wyplosz 1996).

It is worthwhile to point out the difference between the results of our model and models of speculative attacks where a currency crisis can take place due to the existence of sunspot equilibria (e.g. Jeanne and Masson, 1999). While sunspot models show the potential for currency crisis, they do not explain how investors coordinate on a currency crisis path or the timing of shifts between equilibria. In such models, recurrent crises and booms occur because of unexplained jumps between equilibria. In the model of this paper, the alternation occurs because of the interaction of myopic, or "boundedly rational," investors.

Our model hypothesizes a simple mechanism for forming expectations, depending on both imitation and experimentation, that can produce the symptoms of herd behavior, namely sharp shifts in the expectations held by a number of investors triggered by little or no new information. Thus "information cascades" or "avalanches" can occur here, but not because there is a private signal which an agent is led to ignore as a result of observing the behavior of others, as in most models of herd behavior. Instead, it is the success of each investment strategy (based on the value assigned to the expectation of devaluation) that influences the choice of expectation next period, and hence the decision to invest or not in the emerging market. But this success (leading to greater capital inflows) ultimately leads to too much debt, which can only be serviced if more and more investors are attracted to the emerging market–a classical Ponzi scheme.

5. Analytics of a simplified model

In order go further in analyzing the dynamics driving the simulations reported above we simplify the model, while retaining its essential features. In particular, let us assume that there are an infinity of agents, so that the sample frequencies converge to the probability density functions, but that each chooses between two discrete values for expectations of a devaluation in the following period, namely π^l and π^h, corresponding to a low and high expected probability of devaluation, respectively. In these circumstances, all those with expectations π^l invest their wealth in emerging markets, since their expectation is more optimistic than the market expectation, while those with expectation π^h invest in the safe U.S. asset. The market expectation for a devaluation depends on the relative size of the two groups of investors, and the dynamics of the model can be summarized in terms of the proportion of investors with the low expectations, $\overline{\lambda}_t$, and the total stock of wealth held by all investors, $\overline{W}$, which is assumed constant. We linearize the expression for the equilibrium interest rate. Thus,

$$\bar{\pi}_t = \overline{\lambda}_t \pi^l + (1 - \overline{\lambda}_t)\pi^h \tag{5.1}$$

$$r_t = r^* + \bar{\pi}_t \delta^e \tag{5.2}$$

and

$$D_t = \overline{\lambda}_t \overline{W}$$

A devaluation occurs at $t+1$ if, in the absence of a devaluation, reserves R_{t+1} would have gone negative:

$$R_{t+1} = R_t - r_t D_t + T_{t+1} + D_{t+1} - D_t < 0$$

i.e.

$$\Delta\overline{\lambda}_{t+1} < \{r^* + [\overline{\lambda}_t\pi^l + (1-\overline{\lambda}_t)\pi^h]\delta^e\}\overline{\lambda}_t - (T_{t+1} + R_t)/\overline{W} \qquad (5.3)$$

As in the simulations, the investors change their expectations on the basis of the previous period's outcomes, as well as experimentation that involves a random choice of one or the other devaluation expectations. We use a slightly simplified version of the imitation operator, in which each investor becomes a blank slate at the beginning of t and she adopts a rule selected by a spin of a roulette wheel. Again, rules are selected with replacement and each rule's slot on a roulette wheel is determined by its relative fitness. A rule that is selected is assigned to investor i at time period $t+1$. In particular, in the absence of experimentation, the weight (or probability density $f_{t+1}()$) on each of the two values of the expectations is equal to their relative return, which in the absence of devaluation, yields:

$$f_{t+1}(\pi^l) = \frac{r_t}{r_t + r^*} \qquad f_{t+1}(\pi^h) = \frac{r^*}{r_t + r^*} \qquad (5.4)$$

while experimentation occurs with probability p_{ex}, and involves a 50-50 chance of choosing π^l or π^h. As a result, in the absence of devaluation,

$$\overline{\lambda}_{t+1} = (1-p_{ex})\frac{r_t}{r_t + r^*} + p_{ex}/2 > 1/2 \qquad (5.5)$$

If there is a devaluation at t, it is assumed here that it is large enough to wipe out the interest return on the emerging market deposits. Hence, as described above, the return is truncated to zero, implying in this case that

$$f_{t+1}(\pi^l) = 0, \qquad f_{t+1}(\pi^h) = 1 \qquad (5.6)$$

so

$$\overline{\lambda}_{t+1} = p_{ex}/2 < 1/2 \qquad (5.7)$$

that is, only those investors who experiment will have a (50-50) chance of picking π^l.

It is possible to characterize the dynamics in the two cases when there is no devaluation or when a devaluation occurs.

No devaluation at t.

Substituting equations (5.1) and (5.2) into equation (5.5), when there is no devaluation we have

$$\Delta\overline{\lambda}_{t+1} = \frac{r^*(1-2\overline{\lambda}_t) + \{(1-p_{ex}/2)\pi^h + \overline{\lambda}_t[(1-p_{ex}/2)(\pi^l-\pi^h) - \pi^h] - \overline{\lambda}_t^2(\pi^l-\pi^h)\}\delta^e}{2r^* + [\overline{\lambda}_t(\pi^l-\pi^h) + \pi^h]\delta^e} \tag{5.8}$$

The numerator of the right-hand-side of the equation, which we can call $g(\overline{\lambda}_t)$ is non-linear in $\overline{\lambda}_t$, but it can be shown that there is one value λ^* in the interval $(0.5, 1)$ for which it is equal to zero. Indeed, it can be shown that

$$g'(\overline{\lambda}_t) < -1 \qquad g''(\overline{\lambda}_t) < 0$$

while

$$g(1/2) = (1-p_{ex})(\pi^l + \pi^h)\delta^e/4 > 0, \qquad g(1) = -(r^* + \pi^l\delta^e p_{ex}/2) < 0$$

Thus, at some point the proportion of those with optimistic emerging market expectations becomes so high that it triggers a decline, leading to a capital outflow. Figure 5 shows the dynamics for a particular set of parameter values, initial reserves, and the trade balance[8]. Equation (5.8) gives a vertical line $\overline{\lambda}_t = \lambda^*$ in $(\overline{\lambda}_t, R_t)$ space. Starting from some positive level of reserves and a low value for $\overline{\lambda}_t$, both reserves and $\overline{\lambda}_t$ will increase until the vertical line is attained; then $\overline{\lambda}_t$ stabilizes at λ^*, while R_t falls (provided the trade balance is not in too large a surplus, and is exceeded by the interest service on outstanding debt[9]). Reserves decline continuously until the devaluation boundary, equation (5.3), is reached (showed as a dashed curve). At this point, $\overline{\lambda}_t$ declines to $p_{ex}/2$. Provided the trade balance is in sufficient surplus[10], $\overline{\lambda}_t$ and R_t both increase, until the vertical line is once again reached, repeating now the same cycle indefinitely. Thus, whatever the initial values of reserves and the proportion of investors in emerging markets, after the first devaluation, $(\overline{\lambda}_t, R_t) = (p_{ex}/2, 0)$, and hence subsequent values are identical and are independent of the initial conditions. In the simulations, this means that there are 30 months between periods of devaluation, which last 2 months. Of course, in the stochastic simulations, T_t is not a fixed number, so that there is some variation in the trajectories.

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[8] The values were chosen to be roughly consistent with those described in section 4; here, balance of payments variables are divided by total wealth, and the US interest rate is equivalent to what was earlier $r^*+\rho$: $T_t = 0.02, R_1 = 0.10, r^* = .01, p_{ex} = 0.33, \pi^l = 0.02, \pi^h = 0.1, \delta^e = 1$.

[9] The condition is $T_{t+1}/W < r_t\lambda^*$.

[10] The condition is $T_{t+1}/W > r_t p_{ex}/2$.

6. Conclusions

Models with multiple equilibria, like the basic balance of payments model which is the starting point for that used in this paper, typically do not explain how investors coordinate on one or another of the equilibria. Instead, a Markov process is generally assumed to predict the transition between them; it is the jump from one equilibrium to another that produces the regime shifts that help explain the volatility in financial markets. However, why investors should all change their expectations in such a way as to produce discontinuous jumps is not explained.

Alternative theories of expectations suggest why such discontinuous jumps might occur. One strand of literature draws on asymmetric information to explain the possibility of imitation and herd behavior, but typically does not model their dynamics. In this paper, we draw on the literature on bounded rationality to model the dynamics of expectations formation, and we embed such behavior in a dynamic balance of payments model to see if such expectations formation can also give the sharp movements in expectations that might produce boom and bust cycles in lending to emerging financial markets, and recurrent currency crises.

Our simulations show that such behavior is possible with learning models that incorporate imitation and experimentation in the formation of expectations. Thus, the simulations seem to square with the facts of lending to emerging markets, that is an alternation of strong inflows, followed by reversals and currency crisis. At the same time, the model does not assume that all investors are the same, or coordinate explicitly on one or another equilibrium.

7. Appendix: The dynamic behavior of $\overline{\pi}_t$when there is a continuum of devaluation probabilities

7.1. Understanding the model

In deriving the properties of the model, it is convenient to make some further simplifications.

In this appendix we derive properties of the model in which the number of investors n goes to infinity, so that we can work with a continuum of investors and a continuous density function over the expected probability of devaluation. We can identify investors with the value of their expectation in any time period. We also work with the log-linearized approximation of the model, using the approximation $\log(1 + r) = r$.

Under these assumptions, risk neutral investors set the interest rate to equal the foreign rate plus the average expected probability of devaluation $\overline{\pi}_t$, times the fixed expected devaluation size δ^e:

$$r_t = r^* + \overline{\pi}_t \; \delta^e \tag{7.1}$$

Let $f_t(\pi)$ be the density function for the estimates of π, then

$$\overline{\pi}_t = \int_0^1 f_t(\pi)\pi d\pi \tag{7.2}$$

$$\overline{\lambda}_t = \int_0^{\overline{\pi}_t} f_t(\pi) d\pi \tag{7.3}$$

and (where total wealth is $\overline{W}$)

$$D_t = \overline{\lambda}_t \overline{W} \tag{7.4}$$

Assuming heterogeneity of expectations, whether $\overline{\lambda}_t$ is greater or less than one-half depends on whether the distribution of expectations is skewed to the left or the right (see Figure 1). If to the right, there is more probability mass to the left of the mean expectation, so $\overline{\lambda}_t > 0.5$, and conversely.

7.2. No devaluation

The probability density function of the estimated probability of devaluation at $t + 1$ is given by $f_{t+1}(\pi)$, assuming no devaluation occurred at t:

$$f_{t+1}(\pi|\pi < \overline{\pi}_t\ ) = (1-p_{ex})\frac{r^* + \overline{\pi}_t\delta}{r^*\pi_{\max} + (\overline{\pi}_t)^2\delta^e} + p_{ex}/\pi_{\max}$$

$$f_{t+1}(\pi|\pi \geq \overline{\pi}_t\ ) = (1-p_{ex})\frac{r^*}{r^*\pi_{\max} + (\overline{\pi}_t)^2\delta^e} + p_{ex}/\pi_{\max}$$

$$\overline{\pi}_{t+1} = \int_0^{\overline{\pi}_t} \pi f_{t+1}(\pi)d\pi = \tfrac{1}{2(r^*\pi_{\max}+\overline{\pi}_t^2\delta^e)}\{r^*\pi_{\max}^2 + \overline{\pi}_t^2\delta^e[(1-p_{ex})\overline{\pi}_t + p_{ex}\pi_{\max}]\}$$

So

$$\overline{\pi}_{t+1} - \overline{\pi}_t = \tfrac{1}{2(r^*\pi_{\max}+\overline{\pi}_t^2\delta^e)}\{r^*\pi_{\max}(\pi_{\max} - 2\overline{\pi}_t) + \overline{\pi}_t^2\delta^e[p_{ex}\pi_{\max} - (1+p_{ex})\overline{\pi}_t]\}$$

The change in $\overline{\pi}_{t+1}$will depend on the sign of the cubic in $\overline{\pi}_t$ within $\{\}$, which we can call $g(\overline{\pi}_t)$. It can be seen from an expansion of $g(\overline{\pi}_t)$ that there are 3 sign reversals, implying that there are three positive real roots to $g(\overline{\pi}_t) = 0$. At least one is in the interval $[0,\pi_{\max}/2]$, since $g(0) > 0$ while $g(\pi_{\max}/2) < 0$. Thus, if $\overline{\pi}_t$ gets too low, this leads to a reversal, with $\overline{\pi}_{t+1} - \overline{\pi}_t > 0$, despite no devaluation having occurred.

7.3. Devaluation at t

If a devaluation occurred at t, then assuming that the devaluation δ_t is greater than $r^* + \overline{\pi}_t\delta^e$ the density function will be:

$$f_{t+1}(\pi|\pi < \overline{\pi}_t\ ) = p_{ex}/\pi_{\max}$$

$$f_{t+1}(\pi|\pi \geq \overline{\pi}_t\ ) = (1-p_{ex})\frac{1}{\pi_{\max} - \overline{\pi}_t} + p_{ex}/\pi_{\max}$$

$$\overline{\pi}_{t+1} = \pi_{\max}/2 + (1-p_{ex})\overline{\pi}_t/2$$

So $\overline{\pi}_{t+1} - \overline{\pi}_t > 0$ if and only if $\overline{\pi}_t < \frac{\pi_{\max}}{1+p_{ex}}$ which is normally satisfied in the simulations. Thus, in the normal case, a devaluation would increase the estimated probability of devaluation next period. However, if devaluations continue for several periods, $\bar{\pi}_t$ can rise to a high enough level that the inequality is not satisfied, triggering a reversal.

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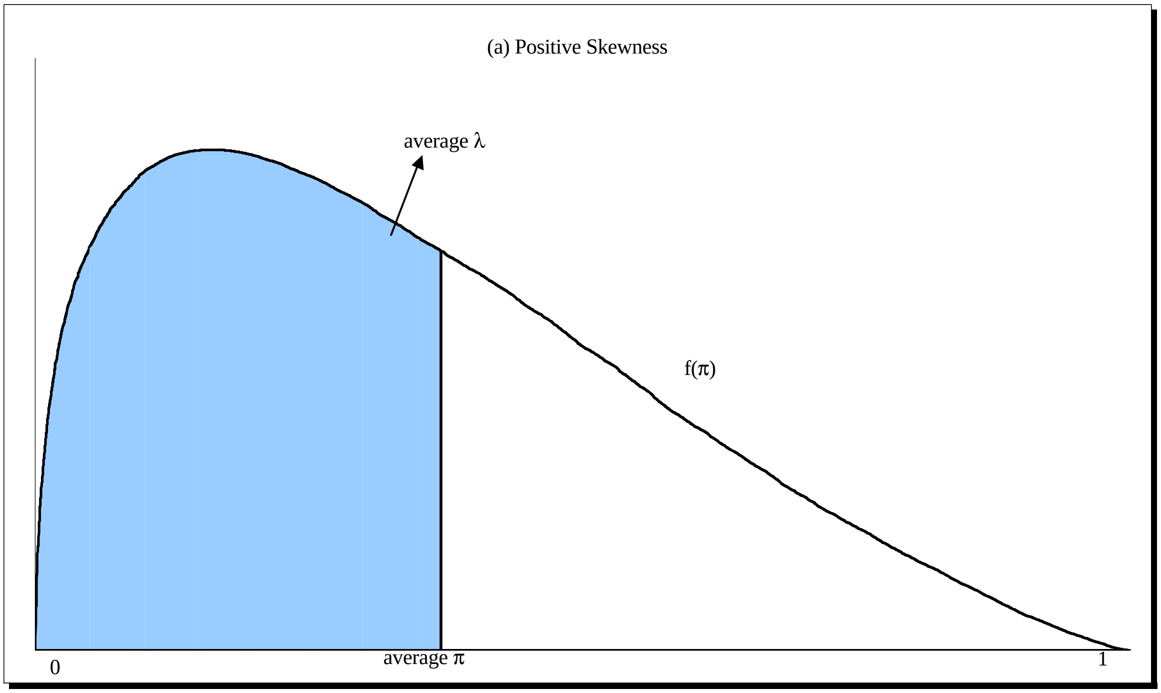
Figure 1. Portfolio Shares (average λ) and the Distribution of Devaluation Expectations (π)


(a) Positive Skewness

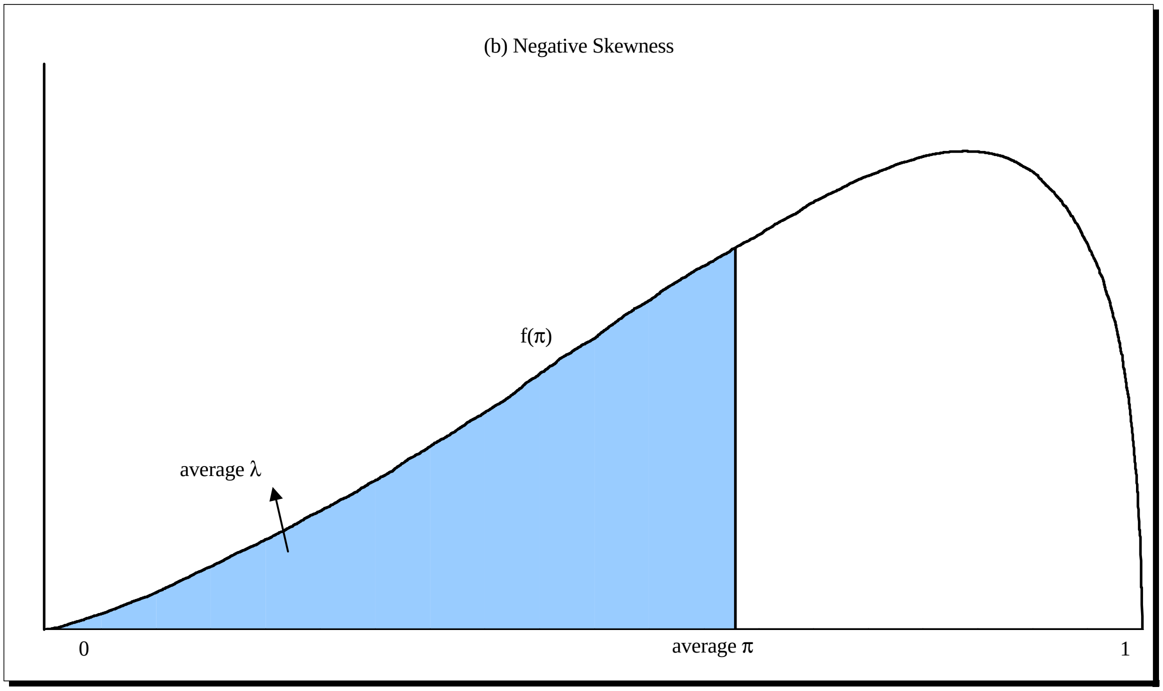
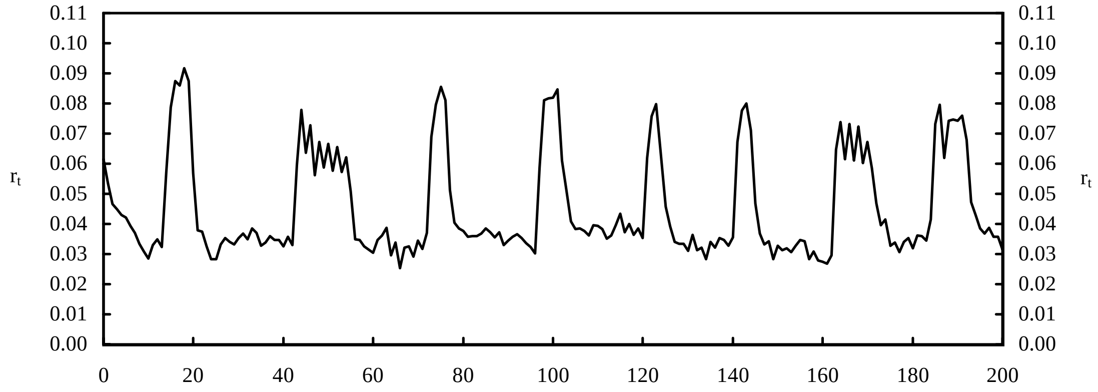
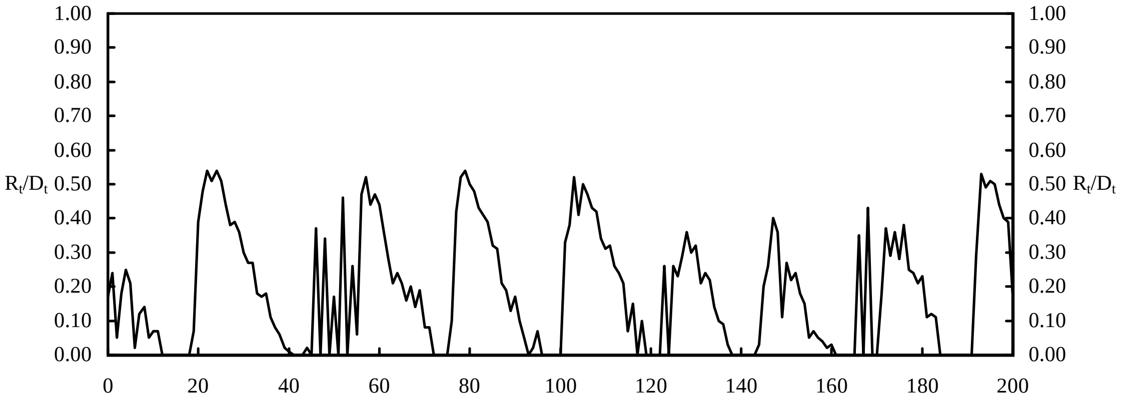

(b) Negative Skewness

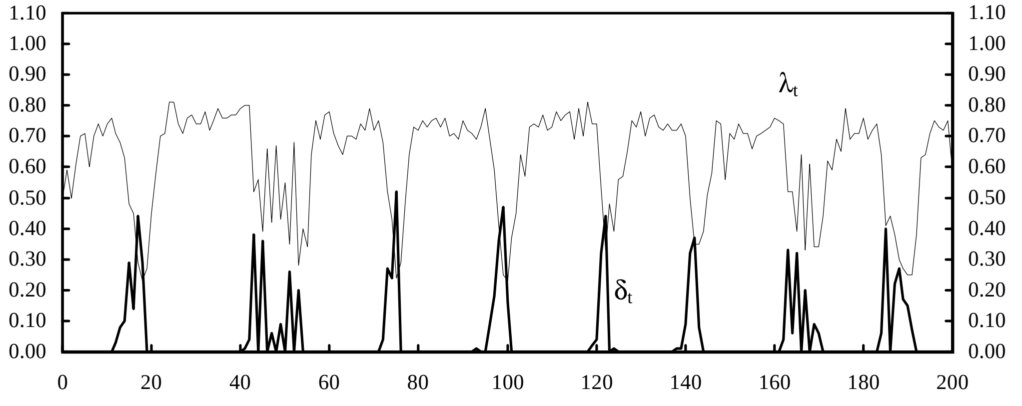
Figure 2. Simulation With Fixed δ^e, $p_{ex} = 0.33$



Panel A: Interest Rate



Panel B: Reserve to Debt Ratio



Panel C: Devaluation and Average Portfolio Share

Figure 3. Process of Devaluation
(With Fixed δ^e, $p_{ex} = 0.33$)

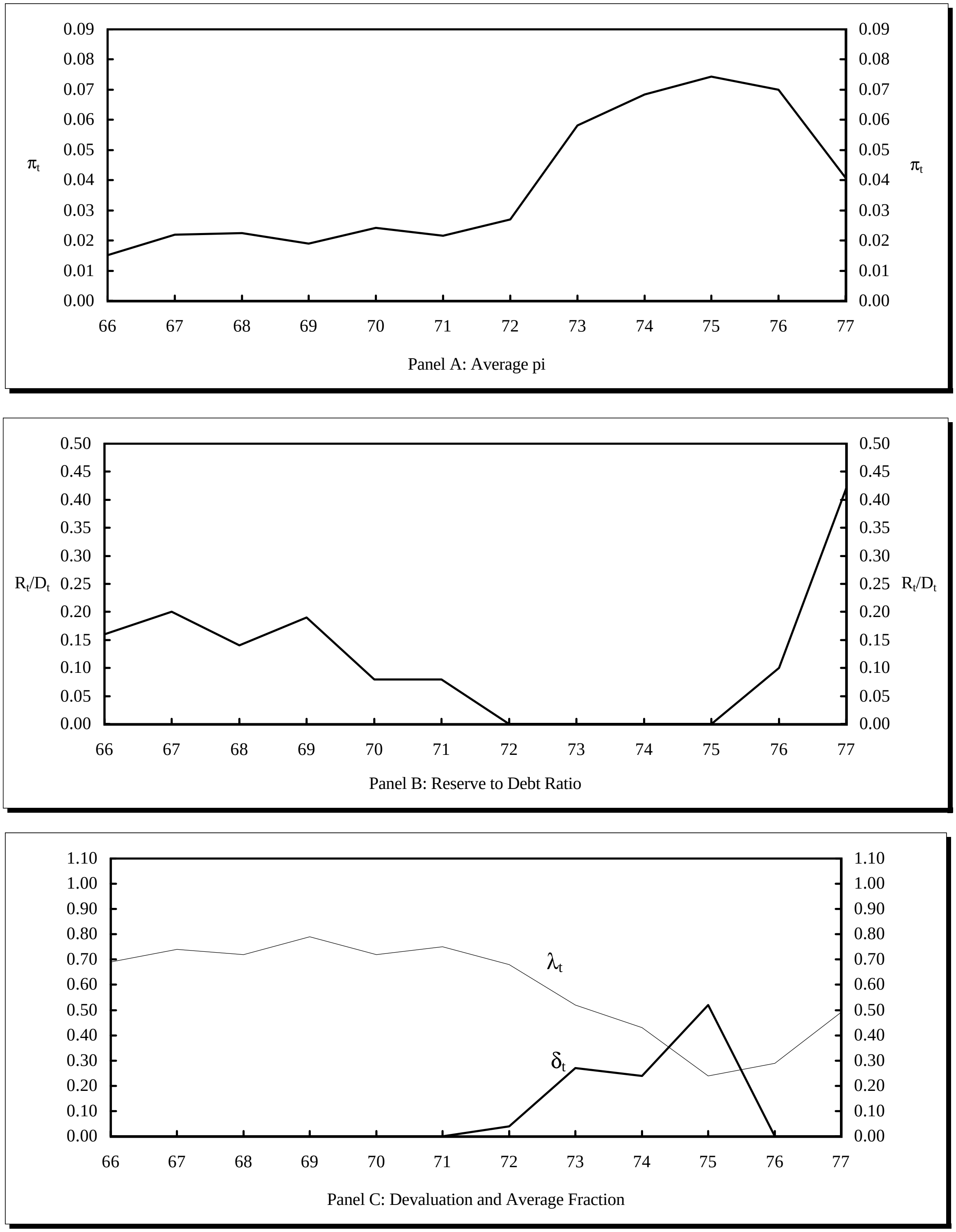


Figure 4. Simulated Distribution of Devaluation Probabilities

(With Fixed δ^e, $p_{ex} = 0.33$)

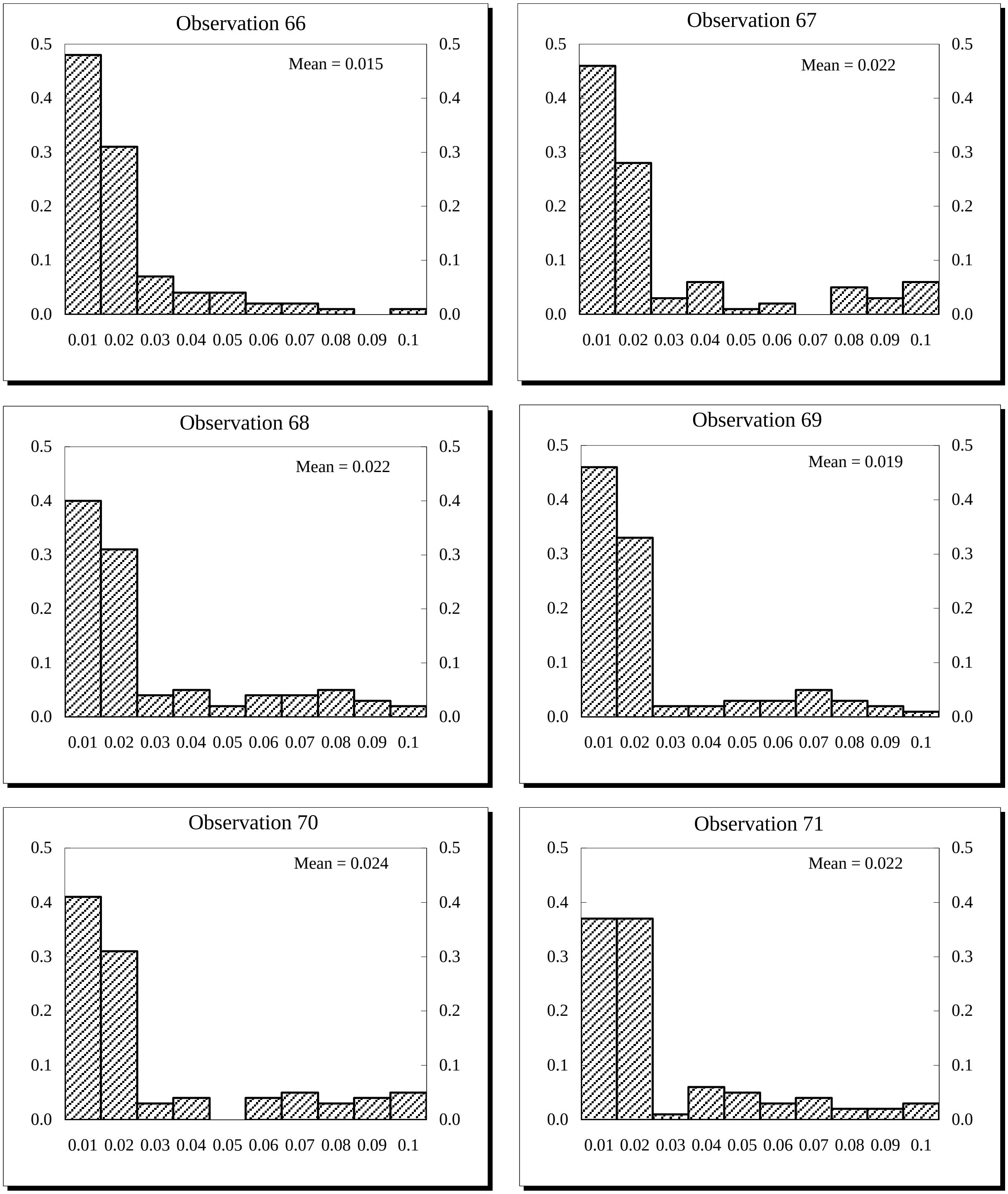


Figure 4 (concluded). Simulated Distribution of Devaluation Probabilities
(With Fixed δ^e, $p_{ex} = 0.33$)

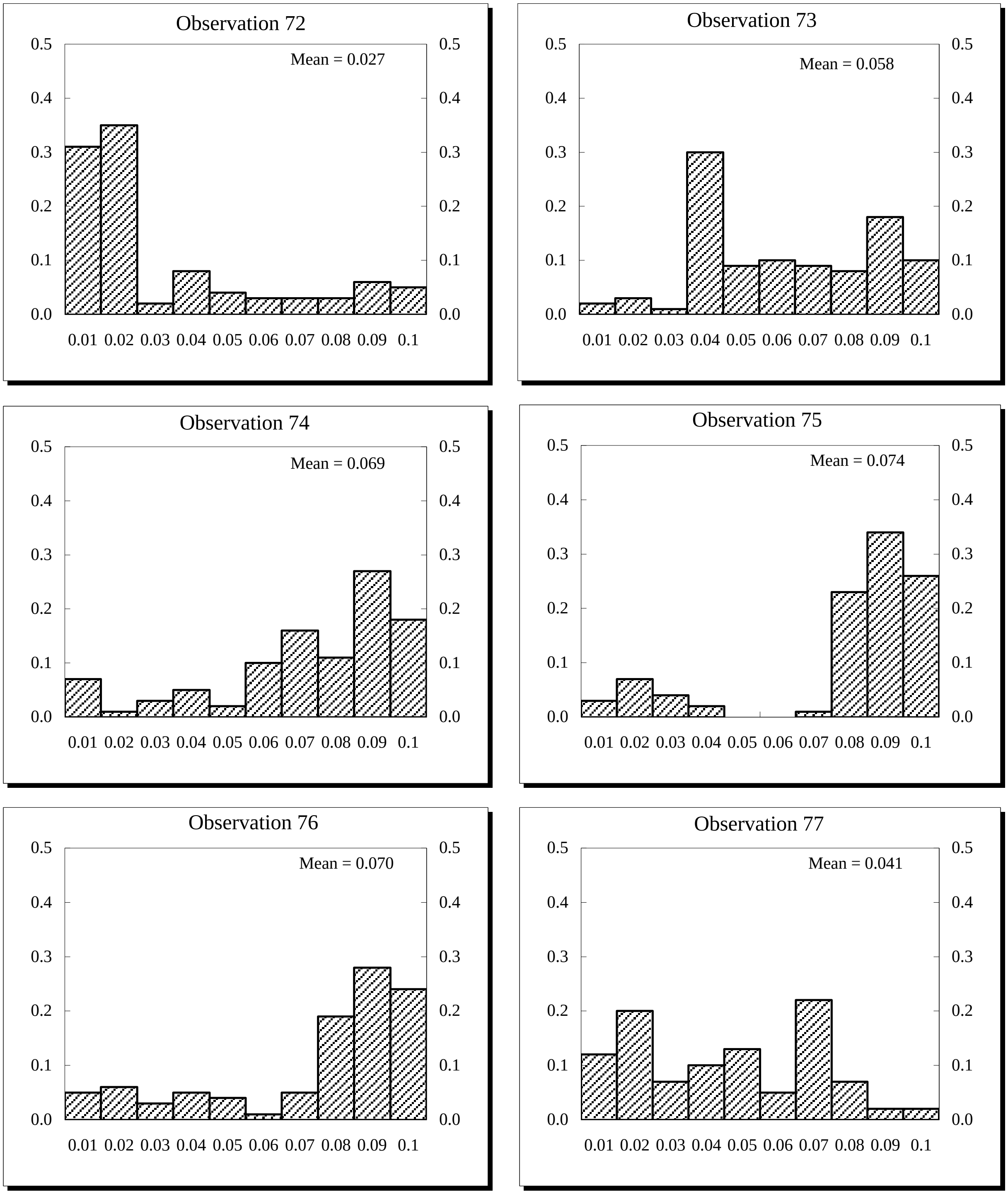


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Figure 5: Solutions for Reserves, lambda, and Devaluation Boundary