

Evolutionary Dynamics of Currency Substitution

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Abstract

This paper examines the issues related to the competition between two currencies in an agent-based computational economic model. The economic environment is a two-country overlapping generations economy with no restrictions on foreign currency holdings. Governments of both countries finance their deficits via seignorage. Agents make decisions about their savings and portfolio decisions. They use the genetic algorithm to update their decision rules. The results presented in the paper show that the currency of the country that finances larger of the two deficits becomes valueless. The adjustment process is characterized by a flight away from the currency used to finance the larger of the two deficits. The economy converges to a stationary equilibrium that corresponds to a single-currency economy.

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1. Introduction

This paper examines the evolutionary dynamics of the exchange rate determination in a two-country economy with a flexible exchange rate system and perfect currency substitution. If countries conduct different monetary and fiscal policies in this type of the environment, the question arises as to what happens with the value of ‘low inflation’ and ‘high inflation’ currencies. Does the currency competition impose fiscal and monetary discipline on authorities of those countries who conduct inflationary policies by establishing as an anchor a currency linked to the restrictive monetary policy ?

In the absence of governments’ intervention in the foreign exchange market, the willingness of the private sector to substitute one currency for another becomes crucial for any analysis of exchange rate determination. When currencies are not substitutable, the general conclusion of the Monetary Approach to Exchange Rate Determination holds. On the other hand, if the private sector views the currencies as perfect substitutes, the outcome of the general equilibrium models based on optimizing microeconomic behavior shows that the exchange rate must be constant over time. This result is based on the no arbitrage condition which requires equal rates of return on both currencies. The equality of the rates of return implies equal inflation rates in terms of all currencies. Thus, increasing the substitutability of currencies may in fact have perverse effects. It does not provide any guarantee that the currency of the country with restrictive monetary policy will dominate and force the other country to adopt the same policies. In fact, residents of the country that conducts restrictive monetary policy may have to face higher inflation rates (and lower welfare) in the world of free currency competition.

In addition, the equivalence of the currencies as savings instruments results in the indeterminacy of the equilibrium exchange rate. Thus this theoretical framework is silent about the actual exchange rate determination.

This paper studies competition between two currencies in an agent-based com-

putational economic (ACE) model where agents adjust their decision rules using the genetic algorithm (GA). The economic environment is a version of the Kareken-Wallace (1981) overlapping generations (OG) environment with two currencies, flexible exchange rate system and no restrictions on holdings of foreign currency. Agents make decisions about how much to save and what fraction of their savings to place in each currency. The two countries finance deficits of different sizes by issuing new quantities of money.

GAs have been increasingly employed to model the behavior of economic agents in macroeconomic models, both as equilibrium selection devices and as models of transitional, out-of-equilibrium dynamics. See, for example, Arifovic (1995, 1996, 1998), Arifovic, Bullard and Duffy (1997), Bullard and Duffy (1998a, 1998b), Dawid (1996). They have also been successful in capturing the main features of the behavior of human subjects in controlled laboratory settings (Arifovic 1995, 1996). GAs impose low requirement on the computational ability of economic agents. Moreover, they provide a convenient framework for modeling decentralized learning with heterogeneity of agents' beliefs. The success of different decision rules depends on the payoffs that agents receive. In addition, there is experimentation with new rules through random changes and recombination of the parts of the existing rules.

The results presented in the paper show that, in the environment in which there is a free competition between currencies, the currency used to finance the larger of the two deficits cannot survive in the ACE environment. Evolution of agents' decision rules results in a flight away from this currency until it eventually becomes valueless. At the end of the adjustment process, agents hold all of their savings in the currency used to finance the lower of the two deficits. Thus, the economy converges to the equilibrium in which only the low-deficit currency is valued. This equilibrium is equivalent to a stationary equilibrium of the single currency model. The speed of adjustment depends on the size of the difference between the two deficits. The larger the difference, the smaller the number of periods that it takes

to complete the process during which agents bring down the holdings of the high-deficit currency to zero.

The selection of a single currency equilibrium is the global result because it is not sensitive to the initial conditions. Most of the simulations were initialized randomly, meaning that, on average, agents held equal amounts of savings in each currency. In order to examine the sensitivity of the results to different initial conditions, a set of simulations with relatively large initial fractions placed in the high-deficit currency was conducted. Large initial fractions of savings placed in the high-deficit currency did not slow down the adjustment process. Thus, initial conditions do not seem to have any effect on the speed of convergence. The only factor that affects the speed is the difference in the levels of deficits. Moreover, a stationary rational expectations equilibrium in which both currencies are valued is unstable under the GA dynamics in the sense that, in case that a simulation is initiated in such an equilibrium, the genetic algorithm takes the economy away from it and towards a stationary, single-currency equilibrium.

The dynamics that take place in this ACE environment are driven by agents' attempts to exploit perceived arbitrage opportunities and put more of their savings into the currency that had a higher rate of return in the previous period. Agents' portfolio decisions determine the rates of return on the two currencies. In turn, the rates of return affect the performance of the rules and their survival in this co-evolutionary environment.

The paper proceeds as follows. The description of the economic model is given in section 2. The updating of agents' rules is described in section 3. The analysis of the dynamics is given in section 4. Finally, concluding remarks are presented in section 5.

2. The Economy

The economy is a two-country OG model with fiat monies (Kareken and Wallace, 1981). At each date t with $t \geq 1$, there are born N young people, in each country,

said to be of generation t . They are young at period t and old at period $t + 1$. Each agent of generation t is endowed with w^1 units of a single consumption good at time t , and w^2 of the good at time $t + 1$, $w^1 > w^2$,¹ and consumes $c_t(t)$ of the consumption good when young and $c_t(t + 1)$ of the good when old. Agents in both countries have common preferences given by: $u_t[c_t(t), c_t(t + 1)] = \ln c_t(t) + \ln c_t(t + 1)$. It is a free-trade, flexible-exchange-rate regime economic environment in which agents in the two countries are permitted freely to borrow and lend to each other and to hold each other's currencies. An agent of generation t solves the following maximization problem at time t :

$$\begin{aligned} & \max \ln c_t(t) + \ln c_t(t + 1) \\ & s.t. \ c_t(t) \leq w^1 - \frac{m_1(t)}{p_1(t)} - \frac{m_2(t)}{p_2(t)} \\ & \quad c_t(t + 1) \leq w^2 + \frac{m_1(t)}{p_1(t + 1)} + \frac{m_2(t)}{p_2(t + 1)} \end{aligned}$$

where $m_1(t)$ are the agent's nominal holdings of currency 1, $m_2(t)$ are the agent's nominal holdings of currency 2 acquired at time t , $p_1(t)$ is the nominal price of the good in terms of currency 1 at time t and $p_2(t)$ is the nominal price of the good in terms of currency 2 at time t . Agent's savings, $s(t)$, in the first period of life, are equal to the sum of real holdings of currency 1, $m_1(t)/p_1(t)$ and real holdings of currency 2, $m_2(t)/p_2(t)$.

The exchange rate $e(t)$ between the two currencies is defined as $e(t) = p_1(t)/p_2(t)$. Since there is no uncertainty in the model, an equilibrium condition requires equal rates of return on all assets. Thus, the rates of return on currency 1 and currency 2 have to be equal to:

$$R(t) = \frac{p_1(t)}{p_1(t + 1)} = \frac{p_2(t)}{p_2(t + 1)}, \quad t \geq 1, \quad (1)$$

¹The necessary condition for the existence of a monetary equilibrium is that $w^1 > w^2$. This endowment pattern ensures that, for a given range of rates of return on money holdings, agents have incentive to save in the first period of their life through acquiring money holdings.

where $R(t)$ is the gross real rate of return between t and $t + 1$. Rearranging (1), we obtain

$$\frac{p_1(t+1)}{p_2(t+1)} = \frac{p_1(t)}{p_2(t)} \quad t \geq 1. \quad (2)$$

From equation (2) it follows that the exchange rate is constant over time.

$$e(t+1) = e(t) = e, \quad t \geq 1. \quad (3)$$

Individual's savings $s(t)$ that are derived from the agent's maximization problem are given by

$$s(t) = \frac{m_1(t)}{p_1(t)} + \frac{m_2(t)}{p_2(t)} = \frac{1}{2} \left[w^1 - w^2 \frac{1}{R(t)} \right] \quad (4)$$

and then the equilibrium condition in the loan market requires that aggregate savings equal real world money supply, i.e. that

$$S(t) = N \left[w^1 - w^2 \frac{p_1(t+1)}{p_1(t)} \right] = \frac{H_1(t)}{p_1(t)} + \frac{H_2(t)e}{p_1(t)} \quad (5)$$

where $H_1(t)$ is the nominal supply of currency 1 at time t and $H_2(t)$ is the nominal supply of currency 2 at time t .

At time $t \geq 1$, the government of country i ($i = 1, 2$) finances the purchases of $G_i \geq 0$ units of the consumption good by issuing currency i . The government i 's policy at time t is given by

$$G_i = \frac{H_i(t) - H_i(t-1)}{p_i(t)} \quad (6)$$

Combining (5) with (6) gives the condition for the monetary equilibrium in which both governments finance their purchases via seignorage:

$$G_1 + G_2 = S(t) - S(t-1)R(t-1). \quad (7)$$

These equilibrium dynamics do not yield a specific value of the exchange rate. In fact, in this model, the exchange rate is indeterminate. Any constant exchange rate $e \in (0, \infty)$, is an equilibrium that supports the same stream of government

deficits G_i , $i = 1, 2$ and the same equilibrium gross rate of return $R(t)$ for $t \geq 1$ (see Sargent, 1987). Consider the condition for equilibrium in the currency market at $t = 1$:

$$G_1 + G_2 = S(1) - \frac{H_1(0) + eH_2(0)}{p_1(1)}. \quad (8)$$

Next, define the exchange rate $\hat{e} \in (0, \infty)$ such that $\hat{e} \neq e$ and the nominal price level $\hat{p}_1(1) \neq p_1(1)$ such that

$$\hat{p}_1(1) = \frac{[H_1(0) + \hat{e}H_2(0)]p_1(1)}{H_1(0) + eH_2(0)}. \quad (9)$$

Then, solve for $\hat{p}_1(1)$ and substitute into equation (8):

$$\frac{H_1(0) + \hat{e}H_2(0)}{\hat{p}_1(1)} = S(1) - [G_1 + G_2]. \quad (10)$$

Use $\hat{p}_2(1) = \hat{p}_1(1)/\hat{e}$ to solve for $\hat{p}_2(1)$. Using equation (1), the values of $\hat{p}_i(t)$, $i = 1, 2$, $t \geq 1$ are calculated. The price levels $\hat{p}_1(1)$ and $\hat{p}_2(1)$ adjust sufficiently to leave the aggregate savings function (and thus gross real rate of return) unaltered with the same levels of expenditures G_1 and G_2 .

The equality of the rates of return implies the equality of the rates of inflation. The residents of both countries face a single world-wide inflation rate, ($\pi_w(t) = 1/R(t)$), which is influenced by the values of G_i streams chosen by both countries. The world deficit G_w is defined by

$$G_w = G_1 + G_2 = \frac{H_w(t) - H_w(t-1)}{p_1(t)}. \quad (11)$$

where $H_w(t) = H_1(t) + H_2(t)e$. Using equations (5) and (7), the paths of equilibrium world inflation rates are

$$\pi_w(t+1) = \frac{w^1}{w^2} + 1 - \frac{G_w}{2Nw^2} - \frac{w^1}{w^2} \frac{1}{\pi_w(t)}. \quad (12)$$

From the solution to equation (12) it follows that there are two stationary inflation rates, a low and a high one, for each feasible level of the world deficit, G_w , except

for the maximum seignorage level, for which there is a unique stationary rate of inflation. The low stationary world inflation rate, $\pi_{w,1}^*$, and the high stationary world inflation rate, $\pi_{w,2}^*$ are given by

$$\pi_{w,1,2}^* = \frac{\left[\frac{w^1}{w^2} - \frac{G_w}{2Nw^2} + 1 \right] \pm \sqrt{\left[\frac{w^1}{w^2} - \frac{G_w}{2Nw^2} + 1 \right]^2 - 4\frac{w^1}{w^2}}}{2}. \quad (13)$$

The high stationary world inflation rate is the stable stationary equilibrium for all $\pi_w(1) \in (\pi_{w,1}^*, w^1/w^2)$. If $\pi_w(1) = \pi_{w,1}^*$ then $\pi_w(t) = \pi_{w,1}^*$ for $t \geq 1$.

From (13), the maximum value of the world deficit, $G_{w(max)}$, that can be financed via seignorage is given by: $G_{w(max)} = \left(\frac{w^1}{w^2} + 1 - 2\sqrt{\frac{w^1}{w^2}} \right) 2Nw^2$ with a unique stationary inflation rate. In a stationary equilibrium, both currencies grow at the same rate, i.e. the rates of growth are equal to the stationary inflation rate.

Note that there are also equilibria in which one of the currencies is not valued. The government whose currency is not valued raises no revenue while the government whose currency is valued raises revenue by taxing the residents of both countries. The model becomes equivalent to the single-currency OG economy. In that case, there are again two stationary equilibria, with low and high inflation rate. Finally, if the country whose currency is valued keeps the supply of money constant, the model has the unique stationary equilibrium with a constant price level.

It is worthwhile to consider what happens if one of the countries does not raise any revenue via seignorage, i.e. $G_i = 0$. Suppose that $G_1 = 0$ and that $G_2 > 0$. Then, if the two-country economy reaches a stationary equilibrium, the rate of growth of the world money supply will be given by the stationary rate of growth of currency 2, and this rate is equal to a stationary inflation rate. In the limit, as $t \rightarrow \infty$, currency 2 will drive currency 1 out of agents' portfolios.²

Given this result, it might be in the interest of a country with restrictive monetary policy to impose a degree of capital control. Lapan and Enders (1983) consider a

²For a discussion of the similar result in the context of a money-in-the utility function model and perfect currency substitution, see Weil (1990).

situation where governments may randomly impose capital controls. This form of government intervention will render different currencies imperfect substitutes. Under this scenario, even if domestic agents can hold the foreign currency, the exchange rate is determinate in a manner similar to that predicted by the Monetary Approach. The ACE dynamics presented in this paper result in the outcomes that are different from the perfect foresight equilibrium dynamics. Different outcomes imply different policy recommendations. A country that conducts a restrictive monetary policy does not have to impose probabilistic capital controls in order to avoid transmission of higher rates of inflation. In addition, countries with looser fiscal and monetary policies have to align them with the more restrictive policies or risk that their currencies become valueless. However, as will be described in the subsequent sections, the adjustment process which occurs prior to convergence to the one-currency stationary equilibrium is characterized by high volatility of the exchange rates which itself might be undesirable from the standpoint of the country with restrictive monetary policy.

3. The Genetic Algorithm Economy

At each integer point in time $t \geq 1$, there are two populations of binary strings, one being the new population of generation t , the young ones, the other being the population of generation $t - 1$, the old ones. Each population consists of a fixed size of agents $N = 30$. In each period t , only the *young* strings, members of generation t , take actions that affect the market outcomes. At the same time, the market outcomes of these actions affect the payoffs received by the *old*, strings of generation $t - 1$. The payoffs of generation $t - 1$ affect the rules of generation $t + 1$.

Each agent i , $i \in [1, N]$, is characterized by a decision rule which is encoded as a binary string of length ℓ , $\ell = 30$. A binary string i , $i \in [1, N]$, consists of two parts (two binary numbers) that are decoded into integer numbers and normalized in order to obtain two real number values. The first represents the quantity $c_{i,t}(t) \in [0, w^1]$ that agent i of generation t decides to consume at time t and the second represents

the fraction $\lambda_i(t) \in [0, 1]$ that is the fraction that agent i 's savings placed in currency 1.

The difference between w^1 and $c_{i,t}$ gives the savings decision, $s_i(t)$, of agent i at time t . Agent i places the fraction $\lambda_i(t)$ of the savings $s_i(t)$ into currency 1 and the fraction $1 - \lambda_i(t)$ into currency 2. Once the savings in terms of currency 1 and 2 are determined, the prices of the consumption good in terms of currency 1, $p_1(t)$, and currency 2, $p_2(t)$ are computed:

$$p_1(t) = H_1(t-1) / \left(\sum_i^N \lambda_i(t) s_i(t) - G_1 \right) \quad (14)$$

$H_1(t-1) = \sum_i^N \lambda_i(t-1) s_i(t-1) p_1(t-1)$ and

$$p_2(t) = H_2(t-1) / \left(\sum_i^N (1 - \lambda_i(t)) s_i(t) - G_2 \right) \quad (15)$$

where $H_2(t-1) = \sum_i^N (1 - \lambda_i(t-1)) s_i(t-1) p_2(t-1)$.

Note that $p_1(t)$ is not defined if $\sum_i^N \lambda_i(t) s_i(t) \leq G_1$ and that the level of $p_2(t)$ is not defined if $\sum_i^N (1 - \lambda_i(t)) s_i(t) \leq G_2$. If the holdings of currency i , $i = 1, 2$, do not exceed the level of deficit G_i , currency i becomes valueless and the economy from that time period on evolves as a single-currency economy where only currency j , $j \neq i$, is valued. When this happens, there will be agents with positive holdings of the valueless currency and some of them might continue holding positive amounts for several more periods. However, they are not able to acquire consumption good in exchange for their holdings. Eventually, these holdings are driven down to zero for each individual. It is also possible for both nominal price levels not to be well defined. This outcome is interpreted as a breakdown of the monetary economy.

Given the market clearing prices, $p_1(t)$ and $p_2(t)$, the fraction $\lambda_i(t)$ will determine the nominal holdings of currency 1 of agent i , $i \in [1, N]$, of generation t

$$m_{i,1}(t) = \lambda_i(t) s_i(t) p_1(t) \quad (16)$$

and the nominal holdings of currency 2

$$m_{i,2}(t) = (1 - \lambda_i(t)) s_i(t) p_2(t). \quad (17)$$

Then the second period consumption values of members of generation $t - 1$ are determined:

$$c_{i,t-1}(t) = w^2 + s_i(t-1)\bar{R}_i(t-1), \quad i \in [1, N] \quad (18)$$

where $\bar{R}_i(t-1)$ is the *individual rate of return* of agent i of generation $t - 1$. It is equal to:

$$\lambda_i(t-1)R_1(t-1) + (1 - \lambda_i(t-1))R_2(t-1)$$

where $R(t-1) = p_1(t-1)/p_1(t)$ is the rate of return on currency 1 and $R_2(t-1) = p_2(t-1)/p_2(t)$ is the rate of return on currency 2 between periods $t - 1$ and t .

Finally, fitness values of the members of generation $t - 1$ are computed. The *fitness*, $\mu_{i,t-1}$, of a string i is given by the ex post value of the utility function of agent i of generation $t - 1$:

$$\mu_{i,t-1} = \ln c_{i,t-1}(t-1) + \ln c_{i,t-1}(t).$$

Once the fitness of each member of generation $t - 1$ is computed,³ their beliefs are updated in order to generate rules that will be used by generation $t + 1$. The updating is performed using four operators: reproduction, crossover, mutation, and election.

Reproduction makes copies of individual strings. Strings with higher fitness values get more copies over time. Reproduction is accomplished by conducting one *selection tournament* for each newborn. Two strings are randomly selected, with replacement, from the population of generation $t - 1$. The string with the higher fitness is copied and put into the group of newborn strings. In case of a tie, a string that is copied is chosen randomly. The number of strings in a population is kept constant. Thus, reproduction yields N copies of strings that enter into a *mating pool* to undergo application of other operators.

³In case that $c_{i,t}(t) = 0$, the fitness $\mu_{i,t}$ is set equal to $\ln c_{i,t}(t+1)$. If $c_{i,t}(t) = w^1$, then $c_{i,t}(t+1) = w^2$ and $\mu_{i,t} = \ln w^1 + \ln w^2$. These first period consumption values occurred occasionally at the very beginning of simulations, but quickly disappeared from the GA populations.

Crossover operates on members of the mating pool. First, a pair of strings is selected randomly. Second, an integer number k is selected from $[1, \ell - 1]$, again at random. Two new strings are formed by swapping the set of values to the right of the position k . The total number of pairs that are selected is $N/2$ (where N is an even integer). Crossover takes place on each pair with the probability p_{cross} .

Mutation randomly alters the value of a position within a string to the other value taken by the binary alphabet. The probability of mutation, p_{mut} , is independent and identical across the positions.

Election (Arifovic, 1994, 1996) tests newly generated offspring *before* they are allowed to enter into a new population as the members of generation $t + 1$. New strings are first decoded and normalized in order to obtain values for the first period consumption and for the fraction of savings placed in currency 1. Utility associated with a decision rule i , generated at the beginning of period $t + 1$, is computed using the rates of return on two currencies from period t . Then a *potential* fitness value, $\mu_i^p(t + 1)$, for each new rule is calculated. This fitness is calculated using a new rule's consumption and portfolio decisions and using rates of return from the previous period, $R_1(t - 1)$ and $R_2(t - 1)$.

Each pair of strings that is selected for crossover is recorded as a parent pair. The resulting strings are recorded as offspring. If no crossover takes place, then two offspring are just identical to the two parents. Prior to the application of the election operator, offspring undergo mutation.

The election operator is then applied in the following way. Two offspring and two parents are ranked based on their fitness values (potential fitness values of the offspring and actual fitness values of the parents are taken into account), from the highest to the lowest. Then the two strings which have the first and the second rank are chosen and placed into the population of a new generation. In case of a tie between a parent and an offspring, an offspring is placed into the population.⁴

⁴For a detailed discussion of the effects of the election operator see Arifovic (1994).

This operator is used in order to offset the effects of mutation that otherwise maintain considerable population diversity and prevent the GA's convergence in OG environments (see Arifovic, 1995).

4. Dynamics of Adaptation

The GA simulations were conducted for 18 sets of the OG model parameter values. Two endowment patterns, $w \in [10, 4]$ and $w \in [10, 1]$ were combined with nine different pairs of values of G_1 and G_2 to generate a total of 18 different OG economies given in table 1. Simulations presented in this paper all had $G_1 = 0$ and $G_2 > 0$.⁵

[Table 1 around here]

Each set of the OG parameter values was combined with two sets of the GA parameter values. The first set had $p_{cross} = 0.6$ and $p_{mut} = 0.0033$ and the second set had $p_{cross} = 0.6$ and $p_{mut} = 0.033$.⁶ Five simulations using different random number generator seed values were used for each combination of the parameter values. Thus the total number of simulations was equal to 180. Initial GA populations were randomly generated. At $t = 1$, $H_1(1) = 300$ units of currency 1 and $H_2(1) = 300$ of currency 2 were distributed to the initially old, i.e. the members of generation 0 who used all of the fiat monies to acquire the consumption good. The criterion for terminating a simulation was the convergence to a stationary equilibrium. The maximum number of simulation periods was set to 20,000.

The adaptation of the GA agents resulted in the convergence to the equilibrium in which only currency 1 was valued. After initial fluctuations, the fraction of holdings in currency 1 starts to increase (the fraction of holdings in currency 2

⁵This choice was made in order to make the presentation of the results and the analysis of the dynamics clearer and more concise. The qualitative features of the dynamics are the same to those of simulations with positive values of both G_1 and G_2 .

⁶GA parameter values such as the size of a population ($N = 30$), the length of a binary string ($\ell = 30$), the values of crossover and mutation correspond to the values commonly used in the genetic algorithm simulations (see for example Goldberg, 1989).

starts to decrease) until it reaches the value of one.⁷ This behavior results in the convergence of the exchange rate ($e(t) = p_1(t)/p_2(t)$) to zero.

Table 2 shows, for each combination of the OG and the GA parameter values, the average number of time periods (averaged over 5 simulations initialized with different random seed numbers) that was required for the convergence to the single-currency equilibrium. The speed of convergence depends on the difference between the two deficit values. The larger the difference, the faster the convergence. In addition, the table shows that the speed of adjustment is, with few exceptions, generally greater for the second set of the GA values, the one with the higher mutation rate ($p_{mut} = 0.33$). Thus, higher exogenous rate of experimentation results in faster adjustment.

[Table 2 around here]

Figures 1 – 4 show the behavior of the exchange rate, the average portfolio fraction, the average first period consumption, and the inflation rate in terms of currency 1 for the set 15 of the OG parameter values and the set 2 of the GA values. As the value of the average portfolio fraction approaches 1, i.e. as agents hold more and more of their savings in currency 1, the average portfolio fraction approaches 1, and the exchange rate approaches the value of 0. The economy becomes equivalent to the single-currency OG economy. Since $G_1 = 0$, this is the economy with constant money supply that has the unique, Pareto optimal, stationary equilibrium with constant price level and the complete consumption smoothing. The GA adaptation takes the economy to that equilibrium. Figure 3 shows that value of the gross inflation rate becomes equal to 1. This corresponds to the constant price level. The value of the average first period consumption is equal to 5.5 (figure 4). Since the price level is constant, the second period consumption value is also equal to 5.5.

[Figures 1-4 around here]

Figures 5 – 8 show the behavior of the exchange rate, the average portfolio

⁷Once the average portfolio fraction converges to one, the realized mutation rate is equal to zero.

fraction, the average first period consumption, and the inflation rate in terms of currency 1 for the set 11 of the OG parameter values and the set 2 of the genetic algorithm parameter values. This simulation was conducted with larger values of G_2 ($G_1 = 0$) than the simulation whose variables are presented in figures 1 – 4. The general patterns of the behavior of the variables are the same, i.e., the average portfolio fraction converges to 1, the exchange rate converges to 0, and the gross inflation rate in terms of currency 1 converges to 1. The difference is the speed of convergence, that is much faster than in figures 1 – 4.

[Figures 5-8 around here]

If the economy is initialized at the point where agents hold larger fractions of currency 2, the outcome is still the same. Table 3 shows the difference in the average number of periods it takes for the convergence to the single-currency equilibrium for three different values of initial $\bar{\lambda}(1)$ and two sets of values of G_1 and G_2 . The same endowment pattern $w \in [10, 4]$ was used for all of the six economies. Again, five simulations with different random seed numbers were conducted for each economy. Table 3 shows the averages over five simulations. In each simulation, all agents of both populations were initialized at the same value of $\bar{\lambda}(1)$ and randomly assigned values of the first period consumption. The small initial values of $\bar{\lambda}(1)$ were chosen in order to give as much advantage as possible to currency 2.

Table 3 shows that the number of periods that it takes for the value of $\bar{\lambda}(t)$ to go to 1 does not seem to depend on the initial value of $\bar{\lambda}(1)$. It again depends on the difference between the two values, and the larger the difference, the smaller average number of periods required for $\bar{\lambda}(t)$ to converge to 1.

[Table 3 around here]

If a simulation is initialized at a stationary monetary equilibrium in which both currencies are valued, the GA takes the economy away from this equilibrium and the final outcome of the adjustment will again be the convergence towards the single-currency equilibrium. Figure 9 shows the behavior of the exchange rate and figure

10 the behavior of the average portfolio fraction in a simulation that was initialized at the stationary equilibrium in which all agents placed equal amounts of savings in each currency. The figures show that a stationary equilibrium of this model in which both currencies are valued is unstable under the GA dynamics.

[Figures 9-10 around here]

Consider a GA economy that has been in a stationary equilibrium for two periods, $t - 2$ and $t - 1$. The stationary equilibrium implies the following:

1. All N agents, in both GA populations, have identical consumption and portfolio decisions, $c_i(t - 1) = c_i(t) = c^*$ for all i and $\lambda_i(t - 1) = \lambda_i(t) = \lambda^*$ for all i .

2. There is a world wide stationary inflation rate equal to π^* . Thus, $R_1(t - 1) = R_2(t - 1) = R^* = 1/\pi^*$.

3. Given R^* , $s^* = 1/2[w^1 - w^2/R^*]$ and λ^* such that, given $p_1(t)$, $p_2(t)$, $H_1(t)$ and $H_2(t)$, and e , the equilibrium condition in equation (5) holds.

4. Stability check: Suppose that at time t , an invading rule $\lambda_i(t) \neq \lambda^*$ is assigned to agent i , $i \in N$ and all other $N - 1$ agents keep the same portfolio fraction λ^* . All N agents have the same savings s^* . No changes take place in a population of generation $t - 1$, they all hold equilibrium rules and play the role of the old at t .

The result of the invasion by a single rule different than the equilibrium one will be that at the end of t , $R_1(t) \neq R_2(t)$. The direction of inequality will depend on the value of $\lambda_i(t)$. Suppose that $\lambda_i(t) > \lambda^*$. This will result in $R_1(t) > R_2(t)$. Then at the end of time t , rule of generation $t - 1$ will be evaluated. Since they are all identical, they will all receive the same fitness, lower than the one they had in the stationary equilibrium. This population of rules will then be used to generate a population of rules of generation $t + 1$. The application of reproduction and crossover will not result in any changes in the population since they will operate on the population of identical rules. But, mutation may result in the introduction of new rules that prescribe different levels of $\lambda_i(t + 1)$ and $c_{i,t+1}(t + 1)$.

Suppose that a new rule is generated such that $\lambda_i(t+1) > \lambda^*$ and $c_{i,t+1}(t+1) = c^*$. The election operator will accept it into the population of rules of generation $t+1$. This will have further impact on the inequality between the two rates of return and will result in further changes of the rules of the successor generation. Even if there is no change in the rules of generation $(t+1)$, the inequality between the two rates will still remain, and eventually mutation will bring about new rules that will try to take advantage of the inequality. Thus, the inequality of the rates of return will set off a process of adaptation and changes in portfolio fractions through which agents try to exploit the perceived arbitrage opportunity.

It is worth pointing out that the particular way in which the tie between a parent and an offspring is resolved in the election operator implementation does not affect either the dynamics or the outcomes of GA adaptation. The election operator presented above favors an offspring over a parent in case of equal fitness values. What happens if a *modified* version of the election operator where the tie is resolved in favor of a parent is used? In this case, a GA simulation that is initialized at the stationary monetary equilibrium in which both currencies are valued, will not move away from the equilibrium values. The stability of the equilibrium can be checked by inserting a rule with $|\lambda_i(t) - \lambda^*| = \epsilon > 0$ into the population and examining whether the economy returns to the equilibrium under the genetic algorithm dynamics. Simulations with the modified election operator that were initialized in this way resulted in the algorithms' divergence away from the monetary equilibrium in which both currencies are valued and convergence to the single-currency equilibrium. Similarly, simulations with the modified election operator and with randomly initialized populations also converged to the single-currency equilibrium.

4.1 *The Role of the Average Portfolio Fraction*

The behavior of the average portfolio fraction, $\bar{\lambda}(t)$, is crucial for the observed GA dynamics. Changes in $\bar{\lambda}(t)$ result in changes in the rates of return and these in turn induce further changes in $\bar{\lambda}(t)$. How do the actual dynamics take place?

To simplify the analysis, consider the case where $G_1 = 0$ and $G_2 > 0$. Thus the supply of currency 1 is kept at the constant level $H_1(t) = H_1$ for all t and the rate of growth of supply of currency 2 is determined endogenously. The rate of return on the two currencies are given by:

$$R_1(t) = \frac{p_1(t-1)}{p_1(t)} = \frac{\bar{\lambda}(t)\bar{s}(t)}{\bar{\lambda}(t-1)\bar{s}(t-1)} \quad (19)$$

and

$$R_2(t) = \frac{p_2(t-1)}{p_2(t)} = \frac{\sum_i(1 - \lambda_i(t))s_i(t) - G_2}{\sum_i(1 - \lambda_i(t-1))s_i(t-1)} = \frac{(1 - \bar{\lambda}(t))\bar{s}(t) - G_2/N}{(1 - \bar{\lambda}(t-1))\bar{s}(t-1)}. \quad (20)$$

where $\bar{\lambda}(t)$ is the average portfolio fraction and t , and $\bar{s}(t)$ is the average amount of savings at t . In equilibrium $R_1(t) = R_2(t)$. However, out-of-equilibrium, $R_1(t) \neq R_2(t)$.

The out-of-equilibrium heterogeneity of the portfolio fraction values results in the inequality of the rates of return on two currencies. The GA agents seek to exploit this arbitrage opportunity by placing larger fractions of their savings into the currency that had a higher rate of return in the previous period. In general, increasing sequences of $\bar{\lambda}(t)$ are required to preserve $R_1(t) > R_2(t)$, and decreasing sequences to preserve $R_1(t) < R_2(t)$. But, much smaller changes are required for the preservation of the former inequality than the changes (in terms of absolute value) than are required for the preservation of the latter. Due to the fact that currency 2 is used to raise seignorage, larger changes are required to maintain $R_1(t) > R_2(t)$ than to maintain $R_1(t) < R_2(t)$. The condition for $R_1(t) > R_2(t)$ is that

$$\frac{\bar{\lambda}(t)}{\bar{\lambda}(t-1)} > 1 - \frac{G}{N\bar{s}(t)}. \quad (23)$$

The above inequality shows that $\bar{\lambda}(t)$ does not have to be greater than the previous period $\bar{\lambda}(t-1)$ in order that $R_1(t) > R_2(t)$. It can be smaller as long as the above inequality is satisfied. The higher the size of G_2 , the smaller $\bar{\lambda}(t)$ can be,

relative to $\bar{\lambda}(t - 1)$. On the other hand, for $R_2(t)$ to be greater than $R_1(t)$, it is not sufficient that $\bar{\lambda}(t) < \bar{\lambda}(t - 1)$. It has to be sufficiently smaller so that the above equation holds with reversed inequality. Notice that, as the values of $\bar{\lambda}(t)$ are increasing (decreasing), the probability that there will be further increases (decreases) is getting smaller, because there is smaller and smaller number of bits that can be changed from 0 to 1 (from 1 to 0). Consequently, the probability of the change in the direction of inequality increases. However, it is always higher for currency 2 because larger decreases in $\bar{\lambda}(t)$ are required.

Moreover, an arbitrarily small positive value of $\bar{\lambda}(t)$ is sufficient for valued currency 1. However, there is a lower bound on the value of $(1 - \bar{\lambda}^b(t))$ if currency 2 is to be valued. This bound is determined by the smallest amount of savings in terms of currency 2 such that $(1 - \bar{\lambda}^b(t))\bar{s}(t) > G_2/N$. Obviously, the higher G_2 , the higher value of $1 - \bar{\lambda}^b(t)$, for given $\bar{s}(t)$, required for valued currency 2. This implies that once $\bar{\lambda}(t) < 1$ exceeds the value given by $\bar{\lambda}^b(t)$, currency 2 ceases to be valued. Given that sequences of $R_1(t) > R_2(t)$ are longer than sequences of $R_1(t) < R_2(t)$, and that the value of $\bar{\lambda}^b(t)$ gets smaller for larger G_2 , the time it takes to reach a single-currency equilibrium gets shorter for larger values of G_2 .

4.2 *Results of the GA learning in other OG environments*

While GAs are good global search algorithms, their ability to find global optima is less interesting from the standpoint of using them to model adaptation of economic agents unless that feature generates behavior similar to the behavior of the actual time series or the experiments with human subjects. When compared to other commonly used adaptive algorithms, GAs are much more successful in capturing the behavior exhibited in the experiments with human subjects and capturing some of the features of the actual time series of the exchange rates and asset prices. This superior performance is related to the following behavioral patterns. First, there are economies where GAs, other adaptive algorithms and experiments with human subjects all select the same equilibria, but the patterns of the GA adjustment are

much closer to the patterns observed in the experiments than the patterns generated by other algorithms. Second, there are economies in which GAs and experiments with human subjects converge, while other algorithms exhibit divergence. Third, it is possible that other algorithms converge to a rational expectations equilibrium, while, at the same time, GAs and experiments with human subjects fail to converge and instead exhibit persistent fluctuations. These three behavioral patterns are illustrated with the overview of the GA application to the OG single-currency environment (Arifovic, 1995) and to the two-country OG environment with constant supply of monies (Arifovic, 1996). The first application will be used to illustrate the first and the second behavioral pattern, while the second application will be used to illustrate the third behavioral pattern.

Arifovic (1995) studies the GA adaptation in the OG environment with single currency and two different types of monetary policies, constant money supply and inflationary deficit financing. The economy where the constant money supply policy is conducted has two stationary equilibria, autarky, in which fiat money has no value, and a stationary, Pareto optimal, monetary equilibrium which is unstable under the perfect foresight dynamics. GA simulations converged to the stationary monetary equilibrium. This equilibrium is also the point of convergence of the algorithm that uses the sample average of past price levels for the price forecasting (Lucas, 1985). Likewise, the experiments with human subjects (Lim, Prescott, and Sunder, 1994) exhibited price paths close to the stationary monetary equilibrium. However, the sample average of past prices exhibits smooth convergence to the equilibrium, while both the GA patterns and the patterns generated in the experiments with human subjects are much more erratic. (These results illustrate the first behavioral pattern.)

The second environment where the policy of inflationary deficit financing is conducted has two stationary equilibria, the low-inflation and the high-inflation one. (They correspond to those given in equation (13).) The GA converges to the low-

inflation equilibrium. The study of the least squares algorithm (Marcet and Sargent, 1989) and the evidence from the experiments with human subjects (Marimon and Sunder, 1993; Arifovic, 1995) showed convergence to the low-inflation stationary equilibrium. The differences between the GA and the least squares algorithm is in the features of the simulated inflation paths. The GA ones are characterized by irregular fluctuations (prior to convergence) that provide a better match for the inflation paths observed in the experiments with human subjects. At the same time, the least squares ones are much smoother. (This result illustrates the first behavioral pattern.) In addition, the GA economies converged for those relatively high values of G , ($G < G_{max}$) and initial conditions that result in the divergence of the least squares algorithm. Again, the same behavior (convergence to the low-inflation equilibrium) was observed in the human subject experiments. (This result illustrates the second behavioral pattern.)

Arifovic (1996) studies a two-country economy identical to the one described in this paper except that the money supplies of both currencies are kept constant over time. This economy is also characterized by a constant and indeterminate exchange rate. For each value of the exchange rate, there is a unique stationary, Pareto optimal equilibrium characterized by complete consumption smoothing. While the stochastic approximation algorithm (Sargent, 1993) converges to a stationary rational expectations equilibrium with a constant value of the exchange rate and equal rates of return on two currencies, the GA economy exhibits persistent fluctuations of the exchange rate that do not die out over time. This result is qualitatively different from the results obtained for single currency OG economies where the GA selects the same equilibrium as the other algorithms. In a model with 2 currencies, the GA does not select Pareto optimal equilibrium. Instead, it generates persistent volatility of the exchange rates whose time series features are very similar to those of the actual time series. Moreover, the experiments with human subjects conducted in the same economic environment exhibit the same type of volatility.

(This illustrates the third behavioral pattern.)

These results are surprising regarding both the GA economies as well as the economies with human subjects. First, a number of economic applications of the GA demonstrated the algorithm's good convergence properties. Second, there is an overwhelming evidence that, after a period of initial adjustment, experimental market economies with human subjects exhibit convergence to equilibrium. In addition, this volatility is particularly interesting in light of the fact that there is no exchange rate model based on economic fundamentals that can produce these types of fluctuations which are important features of the actual exchange rate time series.⁸

5. Conclusion

In the two-country OG economy with no capital control and perfect currency substitution in which at least one of the countries uses inflationary deficit financing, the GA selects an equilibrium in which the currency used to finance the larger of the two deficits becomes valueless. The larger the difference between the two deficit values, the faster convergence to this equilibrium. The exchange rate converges to zero, as the nominal price in terms of the high-deficit currency diverges towards infinity. The exhibited dynamics are robust in regard to the changes in the parameter values of both the economic model and the GA. Initial conditions which favor the savings in terms of the high-deficit currency (agents are forced initially to put larger fraction of their savings into the high-deficit currency) do not affect either the selection of the equilibrium or the speed of adjustment. The analysis of the evolutionary dynamics shows how the adjustment of portfolio fractions results in a single-currency equilibrium outcome.

⁸It is worthwhile to point out that in spite of the exchange rate volatility, the values of GA agents' first period consumption, after initial adjustment, stay close to the stationary value, another feature shared with the data from the experiments with human subjects.

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Table 1
Overlapping generations model parameter values

a) Endowment pattern: $w^1 = 10, w^2 = 4$					
pair	1	2	3	4	5
G_1	0.00	0.00	0.00	0.00	0.00
G_2	0.60	1.50	2.10	3.00	6.00
pair	6	7	8	9	
G_1	0.00	0.00	0.00	0.00	
G_2	9.00	12.00	18.00	30.00	
b) Endowment pattern: $w^1 = 10, w^2 = 1$					
pair	10	11	12	13	14
G_1	0.00	0.00	0.00	0.00	0.00
G_2	0.60	1.50	2.10	3.00	6.00
pair	15	16	17	18	
G_1	0.00	0.00	0.00	0.00	
G_2	9.00	12.00	18.00	30.00	

Table 2
Convergence to a single-currency stationary equilibrium
Average Number of Periods

OG	1	2	3	4	5	6
GA						
1	7,682	1197	368	134	41	33
2	1,533	394	188	73	18	65
OG	7	8	9	10	11	12
GA						
1	12	3	5	468	190	127
2	7	21	9	530	415	254
OG	13	14	15	16	17	18
GA						
1	210	15	14	14	12	4
2	75	39	25	8	7	4

Table 3
Selection of currency 1
Average Number of Periods

	λ	0.001	0.01	0.1
G_2				
1.50		835	950	1,212
15.00		21	5.2	18.2

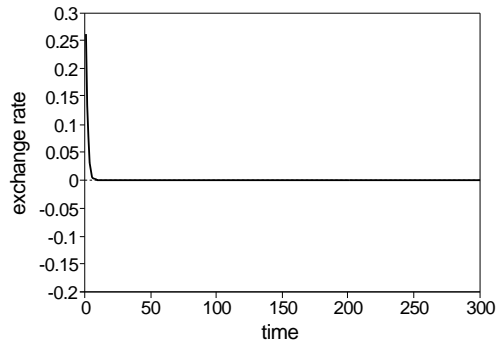


Fig. 1 Exchange rate, $G_1 = 0$, $G_2 = 9$, $p_{cross} = 0.6$, $p_{mut} = 0.033$

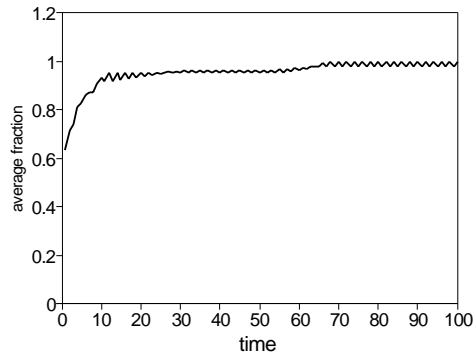


Fig. 2 Average portfolio fraction, $G_1 = 0$, $G_2 = 9$, $p_{cross} = 0.6$, $p_{mut} = 0.033$

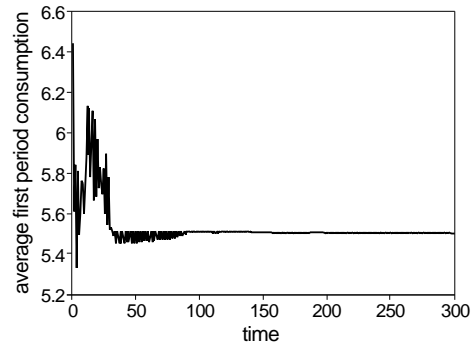


Fig. 3 Average first period consumption
 $G_1 = 0, G_2 = 9, p_{cross} = 0.6, p_{mut} = 0.033$

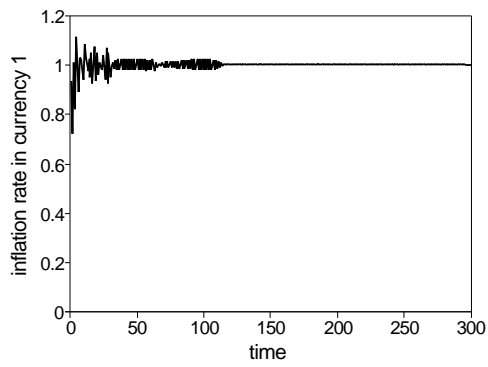


Fig. 4 Inflation rate - currency 1
 $G_1 = 0, G_2 = 9, p_{cross} = 0.6, p_{mut} = 0.033$

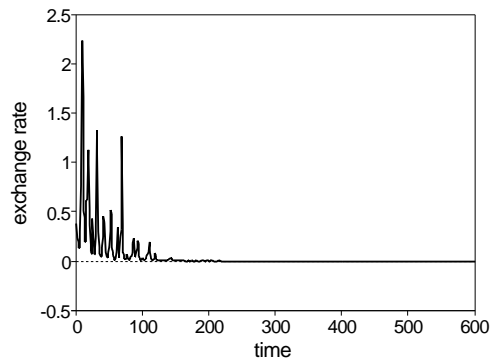


Fig. 5 Exchange rate, $G_1 = 0$, $G_2 = 1.5$, $p_{cross} = 0.6$, $p_{mut} = 0.033$

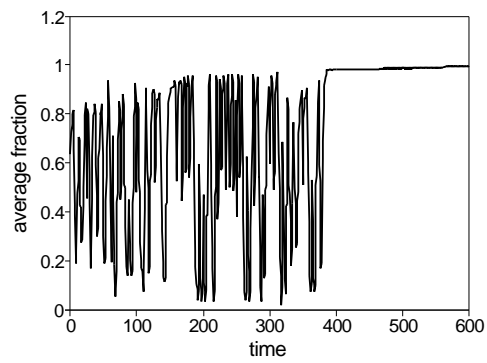


Fig. 6 Average portfolio fraction $G_1 = 0$, $G_2 = 1.5$, $p_{cross} = 0.6$, $p_{mut} = 0.033$

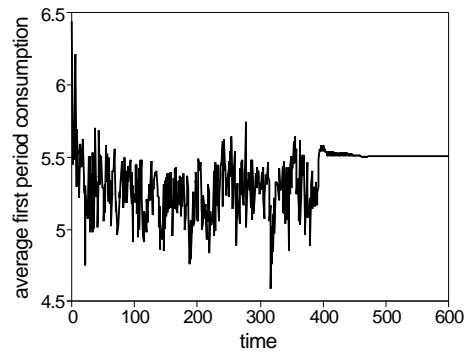


Fig. 7 Average first period consumption,
 $G_1 = 0, G_2 = 1.5, p_{cross} = 0.6, p_{mut} = 0.033$

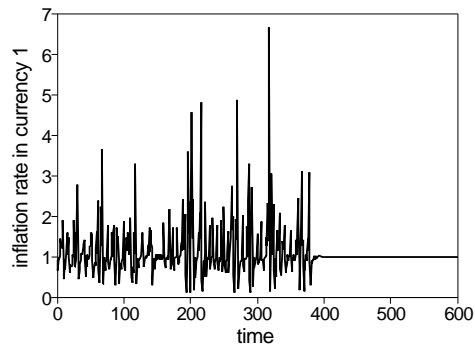


Fig. 8 Inflation rate - currency 1,
 $G_1 = 0, G_2 = 1.5, p_{cross} = 0.6, p_{mut} = 0.033$

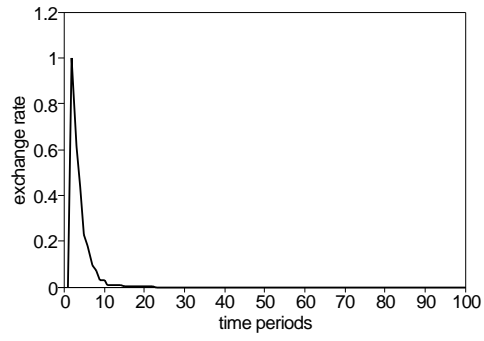


Fig. 9 Exchange rate, initialized at a stationary equilibrium

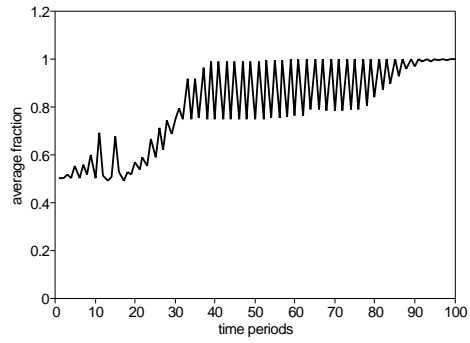


Fig. 10 Average portfolio fraction,
initialized at a stationary equilibrium