Currency Crisis: Evolution of Beliefs, Experiments with Human Subjects and Real World Data

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Abstract

We study a model of currency crisis where agents' beliefs are the only source of volatility containing the potential for currency devaluation. Using the basic framework of Arifovic and Masson (2003), we examine dynamics generated in two types of agent-based models; one involving social learning, and another aimed at representing individual learning. As part of our methodology, we conduct a large number of simulations over diverse parameter values in order to verify robustness of the results. As our evaluation methods, we use both experiments with human subjects, and real world data. Our results show that the individual learning model matches the experimental outcomes better than the social learning model. However, we also find that a variant of social learning fits the real world data well. Analysis of our extensive simulations has helped us isolate the elements of our ACE models that influence the dynamics and that can be accounted for the difference between the results in the two types of learning models, as well as between the experimental outcomes and real world data.

JEL classification: D83, C63, C92, H41

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1 Introduction

Developing methodology for evaluation of agent-based computational models is one of the crucial building blocks for further advancement of this modeling approach and for assessment of its contributions to the research in economics. Two approaches to evaluation have been widely used. One is the comparison of the features of the data generated by ACE models to the features of the real world data. A number of these studies have been in the area of financial applications of ACE models where the time series features of the artificially generated data are compared to the features of the real world time series of stock returns, exchange rates, etc. (e.g., LeBaron et al., 1999, Lux and Schorstein, 2005)) The other one is the approach where the qualitative and quantitative features of the data generated in ACE simulations are compared to the outcomes of the experiments with human subjects (e.g Arifovic, 1996, Duffy 2001).

The two approaches compliment each other. Features of the real world time series are the ones that we would like to capture with our ACE models. However, a number of ACE models make different assumptions about the way in which agents' beliefs and decisions change over time and about the "degree" of boundedly rationality inherent in them. In turn, the ACE outcomes depend on the above assumptions. Thus, testing the soundness of a particular approach to modeling agents' expectations is a crucial component of the evaluation methodology. However, we do not have access to expectations and beliefs of actual economic agents and investors, and how they change in response to the environment. [1] Some of the ACE models, like Arifovic's (1996) model of the exchange rate have been compared both to the experimental data (Arifovic, 1996) as well as to the real world data (Lux and Schorstein, 2005).

In this paper, we take an ACE model of currency crisis (Arifovic and Masson, 2003) and subject it to both of the evaluation methods. Arifovic and Masson (2003) study an agent-based, dynamic model of currency crisis in which heterogenous expectations of boundedly rational agents change through an evolutionary algorithm that involves imitation and experimentation. Their model generates recurrent crises that result from investors' changes in expectations; periods of excessive optimism are followed by periods of excessive pessimism. It is important to emphasize that currency crises characterized by recurrent periods of devaluations are purely expectationally driven.

Arifovic and Masson's model is based on the idea of *social learning* where a population of beliefs of a large number of agents evolves together over time. This concept captures well the fact that a large number of investors participate in trading in real markets. Investors in real markets can also observe the behavior of some of the other investors (captured well by imitation). We refer to this setup as a model of Social evolutionary learning (SEL). SEL has been widely used in the ACE literature to model the evolution of agents' beliefs and decision rules. (See Arifovic (2000) and Dawid (1999) for surveys of the literature).

In addition, we implement the model of Individual evolutionary learning, IEL, (see Arifovic and Ledyard, 2003) where each investor has a collection of alternative expectational rules and chooses one of them probabilistically. [2] This model has exhibited superior performance in the recent mechanism design applications in terms of capturing features observed in the experiments

[1] We are able to make only indirect inference by comparing the features of the ACE outcomes and the real world outcomes.

[2] IEL shares some features with other commonly used individual learning algorithms, such as Reinforcement learning (see Roth and Erev, 1998), and Experience-weighted attraction learning (see Camerer and Ho, 2000). However, it does have distinctive features of its own. One of the important ones is that it is a lot more amenable for applications in the environments with large strategy spaces. For a thorough study of the differences in the updating process as well as in the observed dynamics, see Arifovic and Ledyard, 2004. They study the behavior of RL, EWA, and IEL in the context of the Groves-Ledyard mechanism.

with human subjects (see, Arifovic and Ledyard 2003, 2006). [3]

Our economic environment is a simple model of portfolio balance in which agents/investors make a decision of whether to invest their wealth into an emerging market, with a risky rate of return, or a domestic market, with a safe rate of return. This environment, when endowed with rational agents and stochastic behavior of the trade balance can result in multiple rational expectations equilibria, and if an exogenously specified sunspot process is added to it, it can also result in sunspot equilibria with recurrent currency crisis. We simplify the model by setting the trade balance to a constant value. This way, we have a model in which rational expectations equilibria with sunspot equilibria cannot exist. We endow the model with boundedly rational agents and focus on the evolution of their beliefs as the only source of potential currency crisis.

We model the evolution of agents' assessments of probability of devaluation. These are, in our setup, agent's expectational rules. The central bank of the emerging market economy sets the rate of return on the investment that reflects agents' average sentiment about the probability of devaluation. Then, agents, using their expectational rules and the announced rate of return in the emerging market, make their portfolio decision. These decisions will determine the total amount of deposits invested in the emerging market in a given period. The total amount of deposits invested in this period together with the amount that the emerging market economy has to pay out in terms of principal and interest earned for the past investments, determines the level of reserves. If it is above an exogenously given threshold, there is no devaluation. Otherwise, there is currency devaluation in the percentage sufficient to bring the reserves back to the threshold level.

In our SEL model, each agent is represented by a single expectational rule, and a population of agents/expectational rules evolves jointly over time. In our IEL model, each agent has an entire collection of these expectational rules. In each time period, one of the rules is chosen stochastically and determines agent's portfolio decision. Similarly, in our experiments with human subjects, each subject is asked to solicit her assessment of the probability of devaluation. The rest of the environment works in the same manner as in our artificial economies.

In addition to being interested in the robustness of the dynamics with respect to two different learning paradigms,[4] we employ a model of IEL as it is better suited for direct mapping into the design of the experiments with human subjects.

We simulate both SEL and IEL models for a large number of different parameter values, and examine the observed dynamics. Both models result in recurrent currency crisis. Other features such as duration of periods of devaluation and no-devaluation and the characteristics of the times series of the models' variables that are generated vary across different types of simulations.

In order to test predictions of our model, we needed data on the behavior and evolution of investors' expectational rules. However, it is hard to obtain this type of data.[5] The other is to collect the data on investors' expectational rules in the controlled experiments with human subjects.[6] That is the approach we take. This way, we can directly observe the evolution

[3]Due to space limitations, we do not provide an extensive comparative study of different individual learning models in this paper. However, as an illustration, we conducted a number of simulations of EWA in our currency crisis environment. We did not observe any interesting dynamics similar to the experiments. The results of our simulations are available upon request.

[4]Some recent work has demonstrated that the same algorithms when applied to the two different paradigms can result in different outcomes. However, the outcomes crucially depend on the values of the economics parameters as well (see, for example, Arifovic and Maschek, 2006)

[5]One approach is to use survey data analysis, e.g., Frankel and Foot, 1987, Allen and Taylor, 1990, Shiller, 1999)

[6]Only a fairly small number of experiments on expectations has been done. These include, for example,

of investors' expectational rules over time and compare the properties of the distributions generated in a model to those resulting from experiments with human subjects. We find that the observed experimental behavior matches that of the boundedly rational, artificial agents a number of dimensions. Most importantly, experiments result in recurrent instances of currency crises. In terms of the duration of periods of devaluation and no-devaluation, the IEL model performs better in capturing the behavior observed in the experiments. Both are characterized by more frequent switching between devaluation and no-devaluation periods than the SEL simulations, as well as the smaller magnitude of fluctuations of the average assessment of the probability of devaluation.

We provide detailed analysis of the impact of the interaction between the number of agents/number of expectational rules and the imitation process. This interaction affects directly the periods of no-devaluation, and also indirectly, the magnitude of the average assessment fluctuations.

Finally, we examine the time series properties of the returns generated by simulation of our models and those collected in the experiments. We compare these properties with the time series properties of the actual emerging market data reported in Masson (2003). First differences in spreads on returns of the emerging markets are characterized by negative skewness indicating that they are not normally distributed and high value of kurtosis, indicating fat tails in their distribution.

The set of SEL and IEL simulations that were conducted with fixed level of investors' wealth, as well as the outcomes from 2 experimental sessions [7] share common time series properties, positive skewness and high value of kurtosis. Thus, they are characterized by fat tails, but the sign of the skewness is opposite from the one calculated for real world data.

However, a set of our SEL simulations with endogenous wealth that is evolving over time resulted in both negative skewness and high value of kurtosis. The data from our third experimental session (with larger number of subjects) also had negative skewness. This suggests that our further research should be directed towards more simulations with endogenous wealth, experiments with human subjects where wealth is evolving, as well as a larger number of subjects. In addition, further analysis of the dynamics of simulations with fixed wealth and with endogenous wealth should provide insights into what the differences in outcomes are and why one of the sets results in positive, while the other results in negative skewness.

We proceed as follows. In section 2, we outline a model of currency crisis that has been used in the literature to study the existence of sunspot equilibria. We then present a simplified, representative agent, version of this model, in which there are no exogenous stochastic shocks to the economic fundamentals. This is the model that we use in our simulations and in our experiments. We then extend the model to discuss the incorporation of heterogenous agents with different expectational rules about the probability of occurrence of devaluation. In section 3, we describe our SEL and IEL models.

In section 4, we discuss the design and the results of the simulations. For both SEL and IEL, the simulations exhibit recurrent devaluations. In section 5, we describe the design of our experiments with human subjects, and the results of the experiments that we conducted.

In section 6, we discuss the features of the spread statistics of the simulated data and experimental data, and compare those to the same statistics of the real world data. We provide concluding remarks in section 7.

Smith et al. (1988) experiments on bubbles in a double-auction setup, Marimon et. al. (1993) and Marimon and Sunder (1993) in overlapping generations economies. More recently, Hommes at al. (2004) and Sonnemans et al. (2005).

[7] All of our experimental sessions were conducted with a fixed level of wealth

2 A Model of Currency Crises

Our economic environment is based on a portfolio balance model of currency crisis (Krugman, 1979). It is a simple model of a portfolio allocation between mature (domestic) and emerging markets in which risk neutral investors decide to put their wealth either in an emerging market country or the domestic market. An emerging market central bank defends a currency peg using its foreign exchange reserves until those reserves reach some minimum value.

The domestic asset is riskless, and pays a known rate r^*, while the emerging market asset's return, r_t, is subject to devaluation (or default) risk as well as potentially decreasing returns to the amount invested. The agent puts a fraction λ_t of her fixed wealth $\overline{W}$ in emerging market assets, such that expected returns on the two assets are equalized.

Making explicit the dependence of r_t on λ_t, letting π_t be the probability of a devaluation and δ_t the size of devaluation, the condition for portfolio equilibrium becomes [8]

$$r^* + \pi_t \delta^e = r_t = r(\lambda_t) \tag{1}$$

Inverting (1), we can write this dependence as

$$\lambda_t = \lambda(\pi_t) \tag{2}$$

As in the canonical currency crisis model (Krugman, 1979), devaluations are triggered by the decline of reserves to some threshold level, which we assume to be zero. The change in reserves is equal to the capital inflow plus the trade balance, minus the interest payments on outstanding debt:

$$R_t = R_{t-1} + T_t + D_t - D_{t-1} - r_{t-1} D_{t-1} \tag{3}$$

where $D_t = \lambda_t \overline{W}$. The trade balance T_t is stochastic and is assumed to follow a Markov process; that is, it depends only on its lagged value.

A rational expectation for the devaluation probability will satisfy

$$\pi_t = \Pr_t(R_{t+1} < 0 | no \;\; devaluation) \tag{4}$$

This probability can be rewritten

$$\pi_t = \Pr_t(R_t + T_{t+1} + \lambda(\pi_{t+1})\overline{W} - (1 + r^* + \pi_t \delta_t^e)\lambda(\pi_t)\overline{W} < 0) \tag{5}$$

Jeanne and Masson (2000) show that the simplified version of the model can have multiple solutions. Due to the features of the underlying fundamentals that exhibit multiple (static) equilibria, it is possible to add an exogenous sunspot process that governs switches between the neighborhoods of these equilibria. As a result, such sunspot equilibria generate dynamics of the recurrent currency crises. However, they require coordination of investors' beliefs on a particular sunspot process, falling short of explanation of why and how this coordination might take place.

Rational expectations equilibria that result in currency crisis in this model depend crucially on the stochastic nature of the trade balance. It is the interaction of these stochastic shocks with the investors' behavior that brings about potential for currency crisis.

As our objective is to isolate the impact of agents' beliefs on the dynamics of the model, we abstract from the stochastic nature of the trade balance and set it to a constant value. As

[8] For convenience, cross product terms are ignored here.

we intend to compare the results of our agent based model to the results from the experiments with human subjects, we simplify our model further by assuming a constant value for δ_t^e. [9]

As the focus of this paper is on studying evolution of agents' expectational rules and how this evolution can trigger recurrent currency crisis, we adopt and work with a version of the model where there are no stochastic shocks to the trade balance. In addition, a simplified framework in which only expectational rules about probability of devaluation change is more suitable for our design of the experiments with human subjects. [10]

Thus, we abstract from an evolving trade balance to one in which T_t equals zero for all periods. In addition, we assume that all individuals share the same expectation regarding the size of devaluation. Specifically, $\delta_{i,t}^e = \delta^e = 1$ for all i and over all periods t.

Once the trade balance is set to a constant value, the version of the model with rational (and identical) agents, the equilibrium is no longer characterized by an infinite number of solutions. (The inclusion of a non-stochastic trade balance will instead decrease the number of rational expectation solutions to just two.) We first analyze the framework which corresponds to a repeated one-period stage game.

Reserve levels are determined identically to the specification in equation (3), setting T_t equal to zero for all t.[11] The rational expectation solution for an individual's probability assessment is therefore still characterized by equation (5). We make the following assumption for the function $\lambda_t = \lambda(\pi_t)$

$$\lambda'(\pi_t) < 0 \tag{6}$$

ensuring that as individuals become more pessimistic, their investment in the emerging market decreases (ceteris paribus). We also assume $\lambda(0) = 1$ and $\lambda(\pi_{max}) = 0$. Under these simplifying assumptions, the rational expectations solution for π_t (equation (5)) therefore becomes

$$\pi_t = Pr_t(R_t + \lambda(\pi_t)\overline{W} - (1 + r^* + \pi_{t-1})\lambda(\pi_{t-1})\overline{W} < 0 | no \quad devaluation) \tag{7}$$

In any situation in which $R_t > (1 + r^* + \pi_{t-1})\lambda(\pi_{t-1})\overline{W}$ holds, the solution to this assessment is unique. Specifically, $\pi_t = 0$. Here, even as no funds are invested in the emerging market, it is impossible for a devaluation to occur. The reserve level of the emerging market's central bank is sufficient to cover *all* of its economy's current debt.

A unique solution also results in any situation in which it is impossible to meet a shortfall in reserves with incoming emerging market investment. That is, when $(1 + r^* + \pi_{t-1})\lambda(\pi_{t-1})\overline{W} - R_t > \overline{W} > 0$ holds, a devaluation is certain, and $\pi_t = \pi_{max}$ is the unique solution.

Multiple solutions exist for situations that fall between these two extremes. That is, when incoming emerging market investment can meet reserve shortfalls, or when $\overline{W} > (1 + r^* + \pi_{t-1})\lambda(\pi_{t-1})\overline{W} - R_t > 0$ holds, there are two possible solutions for π_t: $\pi_t = 0$ and $\pi_t = \pi_{max}$. It

[9] Arifovic and Masson (2003) have shown that the model of social learning in which heterogenous expectational rules about π_t, and δ_t that evolve over time results in recurrent currency crisis. In order to test the robustness of their model, they also examined the behavior of a simplified model in which only expectational rules about π_t evolved, and the belief about δ_t was kept at the constant level. This model resulted in the same type of dynamics. A further simplification in which there is no stochastic element of the trade balance (resulting in $T_t = T_{t+1}$ for all t) did not affect the qualitative features of the dynamics.

[10] In the development of experimental design, we wanted to focus on the examination of the effect of changes in agents' belies as the results of ACE simulations suggested that the crisis could be triggered solely by changes in agents' expectations. Therefore, we did not want to complicated out design and make interpretation of the results more difficult by keeping the stochastic balance of trade process.

[11] Setting T_t equal to $\overline{T}$ rather than zero does not change the solutions' characterization in any significant manner

is impossible, without further specification, to select one of these solutions over the other. When π_t takes the value of π_{max}, a self-fulfilling devaluation takes place in which $\lambda(\pi_t = \pi_{max}) = 0$ and through a devaluation of currency, $R_{t+1} = 0$.[12]

As is the nature of self-fulfilling phenomena, when investors do not expect a devaluation, that is, when π_t takes the value of 0, a devaluation does not take place. Importantly, this cannot occur indefinitely, as interest payments on emerging market debt will slowly diminish the level of reserves that are available. Eventually, the economy will find itself with too few reserves to cover its interest outflow and a devaluation occurs.

All of the above analysis is based on a framework where a one-period model (stage game) is repeated over time. In this respect, agents really have expectations of probability of devaluation in the following period. However, if we assumed investors were forward looking, then their rationality will imply the logic of backward induction, i.e. in case that devaluation can occur in some period t, no investment in the emerging market will ever occur.

2.1 Agent-based model with heterogenous beliefs

We now turn to the model with heterogeneous agents. There are n investors, each with constant wealth $\overline{W}$, who form expectations of the devaluation probability, π_t^i.[13] Since investors are risk neutral, they will be indifferent between investing in the two assets when their *ex ante* returns are equal, and choose between putting all their beginning-of-period wealth into the safe foreign asset, at rate r^*, or into emerging market claims, at rate r_t, depending on which expected return is greater.[14]

We assume that each investor is a price taker, and does not influence the marginal product of capital in the emerging market economy. Short selling of either asset is ruled out; neither portfolio proportion can be negative.[15] If λ_t^i is the share of i's wealth in emerging market debt, then $\lambda_t^i = 0$ or 1 as $(1 + r^*) >$ or $< (1 + r_t)/(1 + \pi_t^i)$.[16] Thus, at any period t, the amount of emerging market deposits held by all foreign investors is

$$D_t = \sum_{i=1}^{n} \lambda_t^i \overline{W}. \tag{8}$$

Emerging market banks set the interest rate on bank deposits to reflect market expectations of the return on emerging market debt. We assume that banks do not form expectations of devaluation themselves; they just use the average of all investors' expectations as a measure of the expected value of devaluation. Thus, the interest rate on emerging market deposits r_t

[12]This result is in essence a stag-hunt game with a payoff dominated equilibrium. In a model incorporating Bayesian learning, Chamely (2003) considers speculative attacks in a similar spirit. Agents update their expectations regarding the number of other agents that believe the current fundamentals are sufficient for a successful attack. Essentially, there are two states of the economy, one in which there is sufficient speculators for devaluation, and one in which there is not. The mass of these speculators is an uncertain parameter of this economy. While both models are essentially a game of timing, in Chamely's work multiple periods are necessary for the existence of speculative attacks and these attacks are not recurrent. However, the emphasis of Chamley's work is examining policies' ability to defend the currency peg, not in explaining recurrence.

[13]We continue to assume that each investor has an identical expectation regarding the devaluation size and that this expectation does not change over time, $\delta_t^{e,i} = \delta^e = 1$.

[14]We have also examined a model with risk averse agents with various values for the parameter determining risk aversion. The results are consistent with those reported within this paper and are excluded for brevity.

[15]Similar qualitative results can be obtained if borrowing is allowed, but there are limits on leverage (such as a minimum capital requirement).

[16]If the US rate were equal to the gross expected emerging market return discounted by the expected devaluation, λ_t^i would be indeterminate.

is set equal to the U.S. rate plus a weighted average of the expected rate of devaluation. This equation, which is analogous to an interest parity (no arbitrage) condition, can be written

$$r_t = (1 + r^*) \prod_{i=1}^{n} (1 + \pi_t^i)^{1/n} - 1. \tag{9}$$

With different expectations, expected returns will be equalized only for the marginal investor whose expectation equals the average expectation. Each individual investor will make her investment choice on the basis of a comparison with the average expectation embodied in the interest rate. If she is more optimistic on emerging markets, in the sense of estimating a lower probability of devaluation than the average, then she will put all her wealth into emerging market debt; otherwise, she will put it all into U.S. assets. In this model, investor heterogeneity is key to determining the amount of emerging market assets held.

As in the above described representative agent model, a balance of payments identity relates the change in reserves to the trade balance (assumed for simplification to equal zero in all periods) plus the purchase of new debt by investors minus the principal and interest on maturing debt.

Reserves earn no interest, but they could just as easily have been assumed to earn r^*. Provided that R_t is above some threshold level (which we assume without loss of generality to be zero), there is no devaluation at t, i.e. $\delta_t = 0$ (absence of superscript indicates that this is the realized value of depreciation, not its expectation). However, if reserves would otherwise be negative, there is a devaluation or default which reduces the amount that will be repaid on borrowing undertaken at t. That is, the ex post return for the lender will be $(1 + r_t)/(1 + \delta_t)$, where the amount of the devaluation is equal to the shortfall in the balance of payments that would have pushed R_t negative, divided by D_t:

$$\delta_t = \frac{-R_t}{D_t} = \frac{[(1 + r_{t-1})D_{t-1} - R_{t-1} - D_t]}{D_t} \tag{10}$$

Though the devaluation/default reduces the amount owed at $t + 1$, not t, we assume that, in this case, balance of payments arrears are accumulated within the period such that reserves at t do not go negative but instead equal zero.

3 Evolution of Heterogenous Expectational rules

In this section, we first describe in detail the details of the SEL, model followed by a description of the IEL model .

3.1 Social Evolutionary Learning

The SEL algorithm describes imitation-based adaptation of the agents' expectational rules (here a rule is just a point estimate for π_t^i). Investors consider their own success and that of other investors and try to imitate those rules yielding above-average returns. In addition, they occasionally experiment with new expectational rules.

The measure of performance of the expectational rule i, at time t, μ_t^i, is evaluated based on the ex post return on the emerging market asset.

$$\mu_t^i = (1 + r_t)/(1 + \delta_t) - 1 \tag{11}$$

if investor i invested her wealth in the emerging market and to

$$\mu_t^i = r^* \tag{12}$$

if she invested in the US market. In the case that due to devaluation the performance value of an expectational rule takes a negative value ($\delta_t > r_t$), it is truncated to zero. Thus all the expectational rules that resulted in $\lambda_t^i = 1$ receive the same performance measure even though they may have different values of π_t^i. Similarly, all those that resulted in $\lambda_t^i = 0$ receive the same performance measure even though they may have different π_t^i's.

Investors update their expectations of π_t^i at the end of each period by imitating rules that have proven to be relatively successful and by occasional experimentation with new expectational rules. The updating process is described below:

Imitation At the beginning of each period t, investor i, $i \in [1, \ldots, n]$ compares her expectational rule to a rule of a probabilistically selected investor j. The probability, Pr_t^j, that an expectational rule j is selected for comparison is equal to the expectational rule's relative performance:

$$Pr_t^j = \frac{\mu_t^j}{\sum_{i=1}^n \mu_t^i}. \tag{13}$$

Rules that performed better get larger slots than rules that did worse in the previous period, and thus well-performing rules have higher probability of being selected. Rules are selected with replacement. Once j is selected, investor i compares the performance of her own expectational rule to the performance of investor j's expectational rule. If the performance of her own rule is equal or higher, she keeps her own rule. Otherwise, investor i imitates (adopts) the expectational rule of investor j.

Note that in case of devaluation, if $\delta_t > r_t$, expectational rules of the investors who invested in the emerging market yield a negative return, which is truncated to zero. Thus expectations of all investors who invested in the emerging market will receive performance values equal to 0 and will not be imitated. Only the expectations of those investors who invested in the US market receive positive, equal probabilities of being selected in this case.

Experimentation Once the imitation is completed, each investor, $i \in [1, ..., n]$, can experiment with her expectational rule. Experimentation takes place with probability p_{ex}. If the investor experiments with the expected probability of devaluation, a new expected probability of devaluation is determined by drawing a random number from the uniform distribution over the interval $[0, \pi_{max}]$.

3.2 Individual Evolutionary Learning

Next we combine the currency crisis framework of Arifovic and Masson with the IEL model used by Arifovic and Ledyard (2003, 2005, 2006) and Arifovic and Maschek (2006).

3.2.1 Agent behavior

At the beginning of period t, each investor, i, has a collection A_t^i of possible alternative expectational rules. Each expectational rule of investor i is given by a real number that represents $\pi_{j,t}^i$ at time t. A_t^i consists of J alternatives, $a_{j,t}^i$, for $j \in \{1, \ldots, J\}$.[17] At each t, an investor

[17] J is a free parameter of the behavioral model that can be varied in the simulations. It can be loosely thought of as a measure of the processing and/or memory capacity of the agent.

selects an alternative randomly from A_t^i using a probability density Π_t^i on A_t^i.[18] This alternative then becomes the expectational rule that agent implements at time period t. We construct the initial set A_1^i by randomly selecting, with replacement, J expectational rules from the set of all possible rules within a predefined range. We construct the initial probability Π_1^i by letting $\Pi_1^i(a_{j,1}^i) = 1/J$.

After each investor chooses her expectational rule, we compute the emerging market interest rate, r_t. The next step is to determine the value of each investor's $\lambda_i(t)$. This is accomplished in the same manner as has already been described in the previous section. We use the rest of the model's equations to compute the level of reserves in the emerging market and extent of possible devaluation.

Based on the information obtained at t, each investor updates her collection of alternative expectational rules. This process consists of three parts, computation of foregone returns, imitation and experimentation.

3.2.2 Foregone return

In updating A_t^i and Π_t^i, the first step is to calculate what we call *foregone* returns for each alternative expectational rule in the collection. This is the (expected) return, given the information at t, that the alternative $a_{j,t}^i$ would have received if it had been actually used, taking the behavior of other investors as given. We use the notation $r^i(a_j^i|s_t^i)$ to compute the foregone (hypothetical) return of the alternative j that belongs to investor i's set of alternatives.

For each alternative j, we determine the value of hypothetical $\lambda_{j,t}^i$, given the value of $\pi_{j,t}^i$. Finally, using this value of $\lambda_{j,t}^i$, we compute the rules' foregone return. In this model, this represents their performance measure.

3.2.3 Updating $\mathbf{A}_t^i$

We modify A_t^i with processes of experimentation and replication analogous to the ones described above for social learning. Foregone returns play the role of performance measures. The process of replication (algorithmically analogous to imitation in case of social learning) results in the increase in frequency of the better performing rules. It can be interpreted as a reinforcement of those expectational rules that resulted in higher foregone returns. [19]

While algorithmically the process of experimentation is performed the same way in the two models, it has different interpretation and impact on the dynamics. In SEL model, it can be interpreted as a trembling hand random experimentation. However, in IEL model, newly generated expectational rules will not be automatically tried out when they are generated. They have first to increase their frequency, based on high foregone payoffs, in order to increase their probability of actually being selected.

[18] In essence the pair (A_t^i, Π_t^i) is a mixed strategy for i at t.

[19] Replication reinforces frequencies of well performing rules. It is analogous to the similar process of reinforcement that is implemented in RL and EWA learning algorithms. The main difference is that IEL starts out with a randomly selected set of rules while RL and EWA implementation requires representation of the whole strategy space which is accomplished through its discretization.

4 Simulation Results

4.1 Design of Simulations

In order to examine the robustness of our results, we simulated both our models for a large number of different parameter values. [20]

For both SEL and IEL algorithms, we conduct simulations with 12, 50, 75, and 100. We use the following rates of experimentation: 0.04, 0.0825, 0.165, and 0.33. In case of IEL model, the size of collection of an individual investor's expectational rules, J, was set equal to: 5, 15, and 45.

The values of initial external debt, and reserves, US interest rate, as well as the value of total wealth were taken from Arifovic and Masson (2003). Thus, the initial values for external debt, and reserves were taken to be those prevailing in Argentina at the end of 1996. In these "fixed - δ^e" simulations, the trade balance does *not* evolve. Interest rates and flows are converted to monthly data. All stocks and flows are expressed as ratios to GDP, so the relevant interest rates are actually the difference between the nominal interest rate and the growth of nominal GDP. For r^*, the U.S. interest rate used was $(0.05 - 0.03)$, or 0.001666.[21]

Regarding investors' wealth, we conduct two sets of simulations. In the first set, the wealth that investor has is kept constant over time, i.e. in each time period, she invests the same amount $\bar{W}$. In the second set, the wealth is allowed to evolve over time. In these endogenous wealth simulations, the wealth is set equal across the investors in the first period. Following that, this wealth increases or decreases according to the ex post returns of the previous period earned by that particular investor on their individual specific level of wealth. The measure of performance attributed to each investment rule remains identical between fixed and endogenous wealth simulations; the ex post return earned on investment.

4.2 Duration Statistics over Parameter Permutations

In our simulations of SEL, the observed dynamics are identical to those reported by Arifovic and Masson. The model exhibits recurrent instances of devaluations. We now consider the average duration of devaluation and no-devaluation periods over the various permutations of parameter specifications, using our two models of learning. In each simulation, the SEL initial values described above are used.[22]

Tables 1, 2, 3, and 4 present the average duration of periods of devaluation and no-devaluation for each of the simulations. (Tables 1 and 2 refer to constant wealth simulations, while tables 3 and 4 refer to endogenous wealth simulations.) We differentiate between two definitions of devaluation. Our first definition corresponds to the standard definition of devaluation (the same was used in Arifovic and Masson). That is, a simulation is within a period of devaluation if δ_t is greater than zero (or, anytime reserves fall below their the threshold value). We refer to these as simply *devaluations*. They occur whenever the emerging market's currency

[20]The data presented in this paper represents a subset of 120 different parameterizations of the simulations. We selected to present the results of those simulations that are the most representative, interesting and capture the features exhibited in the rest of the simulations. The results of the entire set of 120 simulations can be found at http://www.sfu.ca/ arifovic/crisis. As distinct parameterizations of simulations are associated with unique simulation numbers, the non-sequential numbering of simulations in the tables has been maintained to facilitate comparison with the entire sample used in our work.

[21]Variables of interest include $D_1 = 412.8$, $R_1 = 73.2$, $T_1 = -0.3$ and $n\overline{W} = 825.6$. The value for total wealth, $n\overline{W}$, was arbitrarily chosen to be twice D_1; π_{max} was chosen as 0.1, and $\delta^e_{max} = \delta^{e,i}_t = 1$.

[22]As in Arifovic and Masson, each simulation is run for $10,000$ periods.

undergoes a depreciation against the domestic. The *ex post* emerging rate of return is lower than *ex ante* rate of return.

However, the fact that the emerging market's currency depreciated does not guarantee that the resulting rate of return earned from investing in the emerging market is lower than that of investing in the domestic market. A depreciation arising from reserves shortages may not be enough to make investing in the domestic market more attractive. Therefore, we also include a definition of devaluation periods that only include those in which the *ex post* rate of return in the emerging market is strictly lower than that of the domestic. We refer to these periods as *effective devaluations*.

This distinction is important for the evaluation of the payoff function used both in simulations and in experiments with human subjects. Although a devaluation may have occurred in the previous simulation period, if it was not large enough to drive the *ex post* emerging market return below that of the domestic market, rules that translated into investment in the emerging market will still propagate. Therefore, simulation dynamics are more likely to be based on the effective devaluations rather than the standard definition of devaluation. We discuss the results across different types of simulations.

4.2.1 SEL Algorithm Simulations - Fixed Wealth

First, consider the SEL simulations (simulations 1 through 20). Consistent with the results of Arifovic and Masson (2003), holding the numbers of investors constant, decreasing the rate of experimentation (p_{ex}) decreases the average duration of periods of devaluation. Upon the onset of a devaluation, those investment rules associated with domestic investment earn higher rates of return than those associated with investment in the emerging economy. For a devaluation to continue, investment must favor the domestic market, therein pulling wealth out of the emerging economy. This occurs when those rules associated with domestic investment are imitated by investors; a process that is inherent in the social learning algorithm. However, with higher rates of experimentation, this imitation is not as effective and the favoring of the domestic economy is less prominent. Increased experimentation decreases the ability of imitation and therefore the swing towards domestic investment required for sustained devaluations is less probable.

Additionally, holding the rate of experimentation constant, lowering the level of investors (n) in the social evolutionary learning simulations tends to decrease the average duration of periods of devaluation. However, this result does not hold for the two lowest specifications of p_{ex} where the duration measures for these parameterizations are already near their lower bound. As such, no decrease in the duration of devaluations is possible. This holds as well when considering periods without devaluations. Generally, decreasing the number of investors in the social evolutionary learning simulations (ceteris paribus) has the effect of lowering durations of both devaluation and no-devaluation periods.[23]

4.2.2 IEL Algorithm Simulations - Fixed Wealth

Our simulations of IEL result in shorter duration of no-devaluation periods when the size of agents' collections of alternative rules is relatively small. In these simulations, we observe a more frequent switching between states of devaluation and those with no devaluation. Specifically,

[23] With respect to the two lowest specifications of p_{ex}, comparing the average duration of periods without devaluations between the highest and lowest specification for n, the value for the lowest is smaller. However, for specifications of n that fall between these two values, the duration of these periods without devaluation tends to be higher.

simulations in which agents have a collection of five rules and experimentation rates equal to 0.04 (simulations 76 through 80, inclusive) have average durations of successive periods without devaluation two to three times smaller than their SEL counterparts (simulations 16 through 20). This result holds across both specifications of the experimentation rate. This decrease in duration measures from the SEL model does not hold when the number of rules in the investors' collections increases to it's largest specification ($J = 45$, simulations 61 through 65). Here, duration measures for no-devaluation periods are very comparable to the SEL model counterparts.

We conclude that decreasing the diversity of rules available for each agent is very important for decreasing the duration of of no-devaluation periods.[24] Smaller collections of rules are associated with shorter periods without devaluations.

Decreasing the size of each agent's collection has the effect of increasing the duration of devaluation periods. For both specifications of p_{ex}, the duration of devaluations is longest with the lowest specification of the number of rules in this collection (and with the number of investors, n equal to 100). Holding the number of rules per agent and the rate of experimentation constant, decreasing the number of agents has the effect of lowering both the duration of devaluation and no-devaluation periods (consistent with the SEL results). Decreasing the experimentation rate does not seem to have any general effects in IEL simulations with high numbers of rules in agents' subsets. However, when these subsets are quite low (5 rules), lowering the experimentation rate decreases the duration of devaluation and no-devaluation periods.

4.2.3 Endogenous Wealth

Tables 3 and 4 contain the duration statistics for the simulations allowing for investors' wealth to evolve endogenously according to ex post return. With respect to changes in population size (n), rate of experimentation (p_{ex}), and the size of the IEL rule set (J), the dynamics are still characterized by recurrent currency crisis. However, endogenously evolving wealth increases the average duration of consecutive periods in which there is no devaluation. Keeping all other parameters the same across fixed and endogenous wealth, the duration of the periods between devaluations is always larger in the endogenous wealth simulations over those with fixed wealth counterparts. In many instances, this increase is quite significant. (For example, in the SEL simulation with 100 investors and an experimentation rate of 0.165 (simulation number 6), the average duration of periods between devaluations increased from 25.15 to 95.90.)

4.3 Duration Statistics and the Process of Replication

Algorithmically, replication is performed in the same way in SEL and IEL models. However, it has different interpretation. In SEL model, replication results in the increase in the number of well performing, actually used expectational rules/ investors. In IEL, replication works at the level of the individual investor's collection of expectational rules. The size of the population in case of SEL or collection, in case of IEL, interacts with replication in an interesting way. For the purposes of the analysis of the impact of replication on duration statistics, we will refer to population of rules (SEL) or collection of rules (IEL) as a set of expectational rules.

The performance of each expectational rule, μ_t^i is evaluated according to the *ex post* return earned in the preceding period. Recall from equation (15), the likelihood of a particular rule, i, being selected for comparison is determined according to that rule's *relative* performance, Pr_t^i.

[24] Note that decreasing the number of agents in the SEL model would have the same effect on diversity. As described above, the resulting impact on duration statistics is the same.

Consider any particular set of k rules. Within this set of rules, we will assume m rules resulted in full investment in the emerging market. As a result, $k - m$ rules resulted in full investment in the riskless asset.

We begin by considering periods in which no devaluation has taken place. Although there may exist considerable heterogeneity in the m rules associated with emerging market investment, each of these rules will have an identical measure of performance. We will refer to this level of performance as $\overline{\mu_t}$. Similarly, those rules associated with investment in the riskless asset will be associated with a performance measure ($\underline{\mu_t}$) equal to r^*. The difference in these two measures of performance, $\overline{\mu_t} - \underline{\mu_t}$, will be referred to as $\Delta\mu_t$.

The selective advantage of a particular rule over another is the increased likelihood of its selection in the imitation process. As mentioned, this is determined according to the difference between the two rules' relative fitness values. Consider a particular rule i associated with investment in the emerging marker versus one that would dictate riskless investment. The difference in relative performance, ΔPr_t^i, is given by the following equation.

$$\Delta Pr_t^i = \frac{\Delta\mu_t}{kr^* + m(\Delta\mu_t)}$$

Note that regardless of the absolute advantage in performance associated with this rule, $\Delta\mu_t$, as the number of rules (k) increases, the selective advantage associated with this rule falls. As such, the likelihood of its selection within the imitation process also falls.

This decrease in selective advantage affects the evolution of the set of expectational rules during periods of no-devaluation. *Ceteris paribus*, as the number of rules within the set over which imitation takes place increases, rules associated with investment in the riskless asset have a higher chance of being selected for comparison. On the contrary, those rules associated with investment in the emerging market are less likely to be selected in the imitation process despite the higher levels of performance associated with them.

Overall, under-performing rules have a greater chance of surviving the imitation process (remaining in the population of rules) when the number of rules within the set increases. This occurs because the likelihood of selecting another under-performing rule for comparison increases. Thus, increasing the size of the set over which imitation takes place has the effect of lessening the selective advantage of better performing rules. For SEL simulations, increasing the population size, n, also increases the size of this set. In individual evolutionary learning simulations, increases in this set are associated with increases in the J parameter.

This effect does not exist within periods of *effective* devaluations. During these periods, investment in the emerging market results in losses. The fitness of rules associated with this outcome is negative. As such, their relative fitness equals zero and there is no possibility of selecting such a rule during the selection process.[25]

Rules that result in investment in the riskless asset are the only rules associated with strictly positive measures of fitness (r^*). During the imitation process, each will be selected with equal probability. If it is being compared to a rule associated with investment in the emerging market, it will replace that rule. The likelihood of this occurring is not affected by the level of rules within the population; it occurs with certainty. Therefore, the dynamics of the evolution of the set of rules available to each agent during periods of devaluation will not be affected by increasing the level of population.

Of course, these effects are not occurring in isolation. Especially within the SEL simulations with large values for the rate of experimentation (p_{ex}), we do observe changes in the duration

[25] Negative fitness measures are truncated to zero.

of devaluation periods associated with changes in the size of sets of rules.

4.4 Increasing the number of agents (n)

Lux and Schornstein (2005) apply a model of genetic learning in the Kareken-Wallace model of exchange rate formation in a two-country overlapping generations world. Their main interest is in whether the dynamics resulting from this learning process helps explain the main stylized facts of free-floating exchange rates. The genetic algorithm model they implement (which is based on Arifovic, 1996) is similar to our SEL model. The major difference is that in their model, an election operator tests newly generated rules before letting them enter the population; only rules with fitness greater than or equal to their parents are accepted. They find that realistic time series are crucially dependent on the number of agents within the simulation. Populations with larger number of agents, when coupled with relatively low rates of mutation, exhibit simulation dynamics that break down into regular periodic oscillations of the agents' choice variables.

Our work shares some key characteristics with that of Lux and Schornstein (2005). Importantly, the non-uniqueness of the equilibrium is derived from the total absence of typical macroeconomic fundamentals associated with other monetary models. However, the systematic tendency of the genetic process is highly dependent on an absence of significant random distortions over the population of rules (hence the additional requirement of low mutation rates). Removing the election operator may allow significant introduction of distortions even when coupled with lower mutation rates.

In order to investigate the impact of the larger number of agents on the dynamics of our agent-based framework, we increase the simulations' parameterization of the number of agents, in both the SEL and IEL simulations. We set (n) equal to 1000, 2000, 4000, and 10,000. Holding the rate of experimentation constant (p_{ex}) at a level of 0.04, we conduct one SEL (table 5, simulations 121 - 124) and one IEL simulation (table 5, simulations 125-128) for each value of n. Our IEL simulations occur with the parameter J held at 15.[26]

Duration statistics for these simulations are presented in Table 5. Plots of average assessment, $\overline{\pi_t}$, for a representative SEL and IEL simulations with the highest specification for the number of agents ($n = 10,000$) are presented in Figures 5 and 6, respectively. The results in Table 5 show evidence of the positive dependence of the average duration of no-devaluation periods on the size of n. When simulations allow for endogenous wealth, the increase in this measure is very large. The average duration between periods of devaluation reaches a maximum of 496 in simulation number 122 when one allows for endogenous wealth compared to its fixed wealth counterpart where the same measure is only 60.13. Overall, the question of whether the dynamics of our models become similar to that of Lux and Schorstein when the number of agents is very large requires further investigation.

4.5 The Dynamics of Average Assessment

Data for a subset of periods of a representative SEL and IEL simulation are given in Figures 1 and 2, respectively. These figures plot the average assessment of devaluation ($\overline{\pi_t}$) and devaluation size (δ_t) over time. A defining characteristic separating the dynamics of SEL and IEL simulations is the range over which average individual assessment fluctuates through transitions from devaluation to no-devaluation periods. Generally, in response to the onset of devaluation

[26]Except for our variation of n, these simulations are otherwise comparable in all parameter choices to those numbered 16-20 and 71-75 (SEL and IEL, respectively).

in SEL simulations, the average assessment climbs quickly and concertedly towards its upper bound before reversal occurs. This is followed by a steady decrease in this measure until it once again pushes against its lowest bound. This is not the scenario that occurs within IEL simulations. Here, average assessments need not meet their upper or lower bound before onset or reversal of devaluations occurs. In addition to the difference in duration statistics, this is a key distinguishing characteristic separating social from IEL when applied in this environment.

Interestingly, as the size of the population within these simulations increases, it appears that these ranges converge. Plots of representative simulations with the highest specification of n are contained within Figures 5 and 6. In each, devaluations are associated with an increase in average assessment that is close to the highest allowable value prior to a devaluation reversal. Similarly, average assessment closer to its lower bound prior to the onset of a devaluation period. In this sense, the qualitative dynamics of the SEL and IEL frameworks begin to converge.[27]

As the analysis of section 4.3 suggest, the reason for the longer no-devaluation periods in SEL simulations is related to the way in which the imitation operates. Its selective pressure decreases as the number of agents increases. In addition, increasing the number of agents in both the SEL and IEL simulations results in a decreased percentage of wealth that each agent invests. In effect, this lessens each agent's individual ability to affect the occurrence of devaluations and thus, results in longer no-devaluation periods.

5 Experiments with Human Subjects

Our experimental design follows closely that of our IEL simulation design in which δ_t^i is equal to one for all investors and over all experimental periods.[28]

Subjects were economics third and fourth year SFU undergraduates. They volunteered, i.e. none were participating for fulfillment of any course requirement, were paid a "show-up" fee and awarded an additional payment dependent on performance.[29] We used Z-tree software for experimental economics developed by Urs Fischbacher to create our experimental environment.

At the beginning of each experiment, subjects are given the following information: (1) the balance of payments identity that governs the currency reserves of the emerging economy's central bank in the following period; (2) the equation determining the rate of return in the emerging economy's asset market; (3) the fixed rate of return in the U.S. economy, r^*, and an initial value of the emerging market rate of return, r_0; (4) the initial level of investment in the emerging market, D_0; (5) The fixed level of wealth available for investment $\overline{W}$ in each experimental period; (6) the equation governing their portfolio allocation; (7) and the method according to which experimental payoff is determined. [30]

[27] In both the work of Lux and Schornstein (2005) as well as ours, the results with respect to the application of evolutionary algorithms with large numbers of agents suggest that this is an interesting issue worth further investigation.

[28] A different experimental design for a model of currency crisis can be found in Heinemann, Nagel and Ockenfels (2004). Their work tests the predictions of the global game theory with respect to private information using a reduced form of Morris and Shin (1998) model. However, due to the fundamentally static nature of these experiments (consecutive experimental periods are in no way related in terms of endogenous variables), their results cannot be used to address the issues of the recurrence or duration of devaluation and no-devaluation periods.

[29] The "show-up" fee was equal to 15 dollars. The performance dependent payment was calculated in a manner such that the average total payment across subjects amounted to approximately 25 dollars. Subjects were informed about the nature of the total payment prior to participation in the experiment.

[30] This information is contained in the set of experimental instructions.

At the beginning of each period, subjects are prompted to enter the probability of devaluation, an integer number between 1 and 100. In order to make this assessment more intuitive, they are asked to enter a probability over the span of $[0, 10]$ rather than $[0, .10] = [0, \pi_{max}]$. Their assessment is then converted to a $\pi_t^{e,i}$ by dividing by 100.[31] Once all subjects enter their individual π_t^i's, the geometric average of pi_t^i's and the rate of return in the emerging market r_t are calculated. When r_t is computed, an investment decision for each subject (investor) is determined. If $(1 + r_t)/(1 + \pi_t^i) > (1 + r^*)$ then subject's λ_t^i is set equal to 1, and it is set to 0 otherwise.[32]

In each time period, each subject receives a fixed amount of experimental francs (68.8) that is, once λ_t^is are determined invested either into the domestic or the emerging market. Using (9) the total amount of deposits in the emerging market for that time period D_t is computed, and δ_t is determined. As described above, if the reserves R_t do not fall short of 0, δ_t is set equal to 0. Otherwise, equation (11) is used to compute the amount of devaluation. Finally, each subject's payoff for that period is computed.

Payoffs

A per period payoff for each subject is based on earnings in excess of the per period investment. That is, a subject earns $r^* \frac{\overline{W}}{n}$ when invested in the domestic market, $r_t \frac{\overline{W}}{n}$ when invested in the emerging market, and $[(1 + r_t)/(1 + \delta_t) - 1] \frac{\overline{W}}{n}$ when invested in the emerging market in periods in which a devaluation takes place. Wealth, $\overline{W}$, is not accumulating; each subject has the opportunity to invest a constant amount in each period that is not dependent on previous investment performance. Importantly, as was the case in the simulations' fitness functions, experimental profit is bounded below by zero. Cumulative experimental profit translates into cash payment via a conversion factor. Total payment to the subject is the sum of a "show-up fee" and the converted experimental profit.

Information

In each time period subjects obtained information on r_t, D_t, δ_t and their own payoff. They could also view the history of these variables as well. Subjects did not have information about the threshold level of reserves, nor about other subjects' assessments of the probability of devaluation.

Subjects are shown their resulting portfolio and rate of return, and their experimental payoff for that period. They are also informed of that periods' *ex ante* and *ex post* rates of return in the emerging market (before and after any devaluation, r_t, δ_t and $(1 + r_t)/(1 + \delta_t)$), and of the total level of investment in the emerging market from the previous period, D_{t-1}.

It is important to emphasize which variables are in the participants' information set and which are excluded. Each participant knows the complete history of total foreign investment, the *ex ante* and *ex post* emerging market return, and the extent of devaluation. However, they do not have information on the following: (i) the current level of currency reserves of the emerging market's central bank, and (ii) the devaluation threshold. We assume that in reality, although reserve levels may be known by investors, the threshold under which devaluation occurs is unknown. We remove knowledge regarding the current level of reserves in order to

[31] The parameterization of π_{max} is taken from the original work of Arifovic and Masson (2003) in order to maintain comparability of results. They specified this range in order to generate interest rate spreads in simulations compatible with those of actual data on monthly spreads.

[32] Under the unlikely scenario that a subject's assessment equals the geometric mean of all assessments, the rules governing subject's investment are the following. If $\pi_t^i < \pi_{max}/2$, all of the subject's wealth is invested in the emerging market. If $\pi_t^i > \pi_{max}/2$, subject's wealth is invested in the domestic market. Finally, if $\pi_t^i = \pi_{max}/2$, the investment is split equally between the emerging and domestic markets. However, these rules did not have to be implemented in any of the sessions.

avoid subjects' learning the devaluation threshold through repeated observation of devaluations.

5.1 Experimental Results

In this section we compare the results of our simulations to those obtained in the experiments with human subjects. We conducted a total of three experimental sessions.[33] We had 15 subjects in our first experimental session, and 11 subjects in the last two experimental sessions. The summary statistics on average duration of devaluation and no-devaluation periods are presented in Table 2.

The average duration of periods of devaluation and periods with no-devaluation are generally quite small when compared to those of the SEL social evolutionary learning simulations (simulations 1, 6, 11, and 16). Although we conducted only one session with 15 subjects, it is noteworthy that the treatment with a larger number of subjects also has larger durations of devaluation and no-devaluation periods. These are not unexpected outcomes. Our discussion above refers to falling durations for specifications with a smaller number of agents; though these smaller durations are still larger than those of the treatments, especially with respect to no-devaluation periods. When we allow for smaller number of agents, the SEL simulations reasonably approximate the experimental results.

We have also noted that the simulations of our IEL model are normally associated with far more switching between devaluation and no-devaluation states. A final point with respect to durations is that simulations of IEL match the experimental data very well. Consider, for example, simulation number 80: an IEL model with 12 agents, 5 rules each and an experimentation rate of 0.04. Its duration measures of 2.07 and 4.83 match the 11 subject sessions quite well with respect to duration of devaluation and no-devaluation periods, respectively. Similarly, simulations with slightly larger collections of rules (15 rules per agent, simulations 71 through 75) perform well in matching the 15 subject session.

We now turn to the analysis of the behavior of the average assessment of devaluation in experiments with human subjects and in simulations of our SEL and IEL. Figures 1 - 3 plot the average assessment of devaluation ($\overline{\pi_t}$) and devaluation size (δ_t) over time. Data for a subset of periods of a SEL and IEL simulations are given in Figures 1 and 2, respectively. The results of one of the experimental session are contained in Figure 3.[34]

A defining characteristic of the plots of the experimental average assessment is the relatively small range in which these measures fall when compared to those of the standard SEL simulations. For example, in the final ninety experimental periods of the session presented in Figure 3, average assessment is never larger than 0.05, and in only a very few periods does it fall below 0.02.[35] A similar lower bound exists for the plots associated with the standard SEL simulations. However, in the majority of periods of devaluation, the average assessment climbs as high as 0.08. One may argue that the SEL simulation and the experimental results share a common lower bound for the average assessment. It is important to note that within experiments, there are many situations wherein the onset of a devaluation is not associated

[33] We set π_{max} 0.10 to match the number used in our simulations.

[34] In order to facilitate comparison between simulation and experimental results, the following parameter choices are used for the SEL (Figure 1) and IEL (Figure 2) simulation plots. The SEL simulation has 12 agents, one rule per agent, and a probability of experimentation set to 0.0825. The IEL simulation is one in which 12 agents have 5 rules in their collections and experiment with a probability equal to 0.0825. Figure 3, that of the experimental data, is a session with 11 subjects.

[35] With respect to average assessment, the results of the other sessions are both qualitatively and quantitatively similar. Importantly, there is nothing particular to the specific experimental session we are discussing that cannot also be said of the other two sessions.

with an average assessment close to the lower bound. This is rarely the case for the SEL simulation results. Additionally, the upper bound placed on assessment does not appear relevant for reversing these periods of devaluation in treatments, as average assessment rarely crosses the 0.05 level.

The plots of the average assessment for our IEL look much more like experimental data. Consider Figure 2, plotting the IEL simulation's results. Here, the plot of average assessment looks very much like those plotted for the experimental session. Periods of devaluation are not necessarily associated with the lower bound on assessment, and the reversal of these devaluation periods occurs far before average assessment can climb to its upper boundary. In this respect, IEL,individual learning simulations appear to match the experimental dynamics much better than the SEL specification.

The IEL simulations compare more favorably to the experimental results with respect to duration statistics. Specifically, they exhibit more frequent devaluation periods, and substantially shorter durations of no-devaluation periods. The range under which the average assessment occurs for the IEL simulations is quite smaller than that of the single-rule simulations. In addition to duration of devaluation and no-devaluation periods, this is a key characteristic the IEL simulations share with the experimental results.

6 Spread Statistics

Masson (2003) reports on empirical regularities of the returns on emerging market debt.[36] As our simulations and experiments are calibrated to monthly data, we use Masson's results on monthly data as well. The data (table 5) indicate that monthly changes in spreads are definitely not normally distributed, exhibiting much fatter tails (kurtosis of 82.34) with negative skewness (-5.70). We report distribution statistics for the first difference in the emerging market's interest rate spread, $[r_t - r^*]$ in Tables 6, 7, and 8. We will compare the qualitative features of these distributions to those of Masson (2003).

Fixed Wealth We calculate the measure of skewness and kurtosis from the distribution of the first difference in the emerging market interest rate spread for each simulation (tables 6 and 7). In all of the simulations, the skewness statistic from this distribution falls in the range $[0.2264, 3.8765]$. Thus, the skewness generated with our simulated data has the opposite sign from the real world data. These simulations, however, generated measures of kurtosis similar to the empirical ones. Masson (2003) finds a high value of kurtosis, in excess of 80. In the majority of our simulations (47 out of 58), the kurtosis measure exceeds that of a normal distribution, reaching a maximum of approximately 199 in the SEL simulation with 100 agents, experimentation with probability 0.04 (simulation number 16).

In all of our experimental sessions we observe kurtosis measures (table 7) greater than that associated with the normal distribution. Skewness measures are negative in one of the three treatments, where there were eleven subjects. Kurtosis measures vary between treatments, but

[36] He uses a set of spreads on emerging market debt compiled by JP Morgan using daily data from 31 December 1993 to 19 July 2002. This data base comprises virtually the universe of all developing countries issuing Brady bonds and Eurobonds. The list of countries is the following (those included in JP Morgan's so-called EMBI+ index, see JP Morgan, 1995): Argentina, Brazil, Bulgaria, Colombia, Ecuador, Korea, Mexico, Morocco, Panama, Peru, Philippines, Poland, Qatar, Russia, South Africa, Turkey, Ukraine, and Venezuela. However, not all countries had bonds outstanding during the whole period 1993-2002; what observations existed were pooled to study the distribution of spreads.

are generally too small to compare well with the simulative SEL results. Rather, this characteristic better associates itself with IEL simulations, where Kurtosis measures are significantly smaller.

Endogenous Wealth Some of our simulations with endogenous wealth were more successful in generating the right sign of the skewness (tables 8 and 9). We find that negative skewness exists in seven of these simulations with identical parameterizations to those considered above. Importantly, all of the simulations that result in negative skewness utilize the SEL algorithm, generally with the lower two specifications of the rate of experimentation. Simulation 16, which utilizes 100 agents and an experimentation rate of 0.04, results in the smallest skewness measure, -2.48. This simulation also captures qualitatively the high degree of kurtosis with a measure of 30.77.

7 Concluding Remarks

We propose a methodology for evaluating ACE models that consists of implementation of ACE models, examination of different variants, a number of different parameter values, and comparison with the outcomes generated in the experiments with human subjects as well as real world data.

To implement our methodology, we use a model of currency crisis where the only source of volatility that contains potential for speculative attacks and devaluation of currency is agents' expectational rules. These expectational rules are heterogenous and evolve over time. We use two different agent-based frameworks, Social and Individual evolutionary learning. As part of our methodology, we conduct a large number of simulations for different parameter values to check for the robustness of the results of our agent-based models. All of the simulations for both learning models resulted in recurrent currency crisis. Our analysis of the impact of the number of agents/expectational rules on the duration of devaluations provides a valuable insight into what the driving force of the dynamics of recurrent currency crisis are.

Further, we designed and conducted experiments with human subjects in which we simulated the same type of the environment. The experiments resulted in the recurrent currency crisis as well. IEL simulations match the experimental data better than the SEL simulations in terms of the duration and frequency of devaluations.

Finally, we compared the statistics calculated from our computer generated data and experiments with human subjects to the statistics calculated using the real world data (Masson, 2003). Real world data on first differences in spreads are characterized by large kurtosis and negative skewness. Our fixed wealth simulations and two of the experimental sessions resulted in positive skewness and the level of kurtosis that 'indicates fat tails'. Thus, these simulations and experimental sessions failed to capture negative skewness of the real world distributions. However, some of our endogenous wealth simulations (SEL with relatively low rates of experimentation) and one experimental session, in addition to high values of kurtosis, generated negatively skewed distributions of the first differences in spreads.

Our results illustrate to what extent we can take experimental results to be approximation of the real world phenomena. With our approach, and using the results from our ACE simulations, we are able to determine exactly where the differences might be and what should be changed in the experimental design in order to obtain results that are more in line with real world observations.

We are not aware of any other studies in ACE literature, as well as in behavioral economics, that have brought these two evaluation methods together. Our approach provides guidance as to how this can be accomplished. From the overall study, our hypothesis is that the increase of the number of subjects in the experiments, as well as the introduction of the endogenously evolving wealth, coupled with a design where subjects are given an opportunity to observe other subjects' beliefs (using a biased roulette wheel type of mechanism) will bring the features of the data generated in the experiments close to the features of the real world data. Of course, we would not have been able to draw these conclusions and formulate the hypothesis had we not extensively analyzed two types of learning algorithms for a wide range of the parameter values. This approach provides guidance as to how we can implement and analyze ACE models, with a help of experiments with human subjects, to capture and understand the dynamics of the real world phenomena.

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Simulation No.	Agents	Rules	p_m	Count(deval)	Ave.deval	Ave.non-deval
1	100	1	0.33	410	4.02	20.37
2	75	1	0.33	493	3.28	17.00
3	50	1	0.33	550	2.91	15.30
4	25	1	0.33	779	1.94	10.89
5	12	1	0.33	935	1.45	9.24
6	100	1	0.165	361	2.55	25.15
7	75	1	0.165	375	2.29	24.38
8	50	1	0.165	441	2.02	20.70
9	25	1	0.165	453	1.62	20.46
10	12	1	0.165	540	1.24	17.28
11	100	1	0.0825	464	1.24	20.31
12	75	1	0.0825	385	1.43	24.54
13	50	1	0.0825	427	1.11	22.31
14	25	1	0.0825	328	1.13	29.36
15	12	1	0.0825	503	1.13	18.75
16	100	1	0.04	526	1.02	17.98
17	75	1	0.04	516	1.01	18.37
18	50	1	0.04	338	1.03	28.55
19	25	1	0.04	391	1.08	24.49
20	12	1	0.04	621	1.15	14.95
41	100	45	0.0825	334	1.12	28.82
42	75	45	0.0825	460	1.24	20.50
43	50	45	0.0825	507	1.31	18.41
44	25	45	0.0825	583	1.19	15.96
45	12	45	0.0825	545	1.09	17.26
46	100	30	0.0825	417	1.48	22.50
47	75	30	0.0825	443	1.25	21.32
48	50	30	0.0825	481	1.38	19.41
49	25	30	0.0825	656	1.19	14.05
50	12	30	0.0825	605	1.14	15.41
51	100	15	0.0825	387	2.80	23.03
52	75	15	0.0825	404	2.07	22.68
53	50	15	0.0825	524	1.56	17.52
54	25	15	0.0825	664	1.32	13.74
55	12	15	0.0825	787	1.18	11.54

Table 1: Count and Duration Measures - Effective Devaluations

Simulation No.	Agents	Rules	p_m	Count(deval)	Ave.deval	Ave.non-deval
56	100	5	0.0825	433	5.02	18.08
57	75	5	0.0825	532	3.93	14.86
58	50	5	0.0825	697	3.02	11.33
59	25	5	0.0825	1001	1.79	8.19
60	12	5	0.0825	1198	1.37	6.98
61	100	45	0.04	404	1.97	22.78
62	75	45	0.04	422	1.36	22.33
63	50	45	0.04	539	1.33	17.22
64	25	45	0.04	580	1.19	16.04
65	12	45	0.04	615	1.13	15.13
71	100	15	0.04	405	3.92	20.77
72	75	15	0.04	458	3.45	18.39
73	50	15	0.04	635	2.30	13.44
74	25	15	0.04	827	1.51	10.59
75	12	15	0.04	859	1.31	10.34
76	100	5	0.04	961	3.70	6.71
77	75	5	0.04	1360	2.57	4.78
78	50	5	0.04	1525	2.21	4.35
79	25	5	0.04	1547	1.80	4.66
80	12	5	0.04	1545	1.49	4.98
121	1000	1	0.04	17	1.82	56.94
122	2000	1	0.04	16	2.31	60.13
123	4000	1	0.04	19	2.15	50.42
124	10000	1	0.04	12	3.83	79.42
125	1000	15	0.04	20	9.45	40.50
126	2000	15	0.04	17	11.00	47.76
127	4000	15	0.04	16	13.80	52.80
128	10000	15	0.04	15	16.07	50.53
Treatment	15	–	–		2.73	10.75
Treatment	11	–	–		1.27	4.64
Treatment	11	–	–		1.32	4.12

Table 2: Count and Duration Measures - Effective Devaluations (Cont'd)

Simulation No.	Agents	Rules	p_m	Count(deval)	Ave.deval	Ave.non-deval
1	100	1	0.33	27	3.41	33.59
2	75	1	0.33	37	2.89	24.11
3	50	1	0.33	101	1.58	8.31
4	25	1	0.33	107	1.50	7.83
5	12	1	0.33	150	1.47	5.19
6	100	1	0.165	10	4.00	95.90
7	75	1	0.165	15	2.93	63.67
8	50	1	0.165	18	2.33	53.17
9	25	1	0.165	23	1.17	42.26
10	12	1	0.165	30	1.17	32.13
11	100	1	0.0825	24	1.17	40.46
12	75	1	0.0825	22	1.14	44.27
13	50	1	0.0825	13	1.00	75.85
14	25	1	0.0825	23	1.00	42.43
15	12	1	0.0825	19	1.05	51.53
16	100	1	0.04	48	1.02	19.79
17	75	1	0.04	19	1.00	51.58
18	50	1	0.04	47	1.02	20.23
19	25	1	0.04	26	1.00	37.42
20	12	1	0.04	27	1.00	36.00
41	100	45	0.0825	21	1.33	46.24
42	75	45	0.0825	20	1.40	48.55
43	50	45	0.0825	42	1.12	22.67
44	25	45	0.0825	27	1.00	36.00
45	12	45	0.0825	26	1.19	37.23
51	100	15	0.0825	14	2.50	68.86
52	75	15	0.0825	12	3.42	79.83
53	50	15	0.0825	34	1.12	28.26
54	25	15	0.0825	30	1.40	31.90
55	12	15	0.0825	32	1.06	30.16

Table 3: Endogenous Wealth Count and Duration Measures - Effective Devaluations

Simulation No.	Agents	Rules	p_m	Count(deval)	Ave.deval	Ave.non-deval
56	100	5	0.0825	11	5.73	85.09
57	75	5	0.0825	22	2.82	42.59
58	50	5	0.0825	31	2.74	29.48
59	25	5	0.0825	59	1.42	15.51
60	12	5	0.0825	59	1.49	15.71
61	100	45	0.04	14	2.43	68.93
62	75	45	0.04	28	1.46	34.21
63	50	45	0.04	27	1.63	35.37
64	25	45	0.04	12	1.58	81.67
65	12	45	0.04	40	1.05	23.93
71	100	15	0.04	11	3.55	87.27
72	75	15	0.04	16	1.94	60.50
73	50	15	0.04	18	2.11	53.39
74	25	15	0.04	36	1.50	26.25
75	12	15	0.04	25	1.24	40.33
76	100	5	0.04	25	3.20	36.76
77	75	5	0.04	41	2.27	22.10
78	50	5	0.04	33	1.03	29.24
79	25	5	0.04	49	1.22	19.16
80	12	5	0.04	74	1.09	12.58
121	1000	1	0.04	3	3.00	330.00
122	2000	1	0.04	2	3.50	496.00
123	4000	1	0.04	2	4.50	495.00
124	100000	1	0.04	3	2.00	331.00
125	1000	15	0.04	4	8.25	241.50
126	2000	15	0.04	3	10.00	323.00
127	4000	15	0.04	4	7.25	242.50
128	10000	15	0.04	2	12.00	487.50

Table 4: Endogenous Wealth - Count and Duration Measures - Effective Devaluations (Cont'd)

First Difference in Interest Rate Spread *Summary Statistics - Masson (2003)*	
Standard Deviation	0.05585
Skewness	-5.70
Kurtosis	82.34
Jarque-Bera	347,207
Observations	1,297

Table 5: First Difference in Interest Rate Spread - Summary Statistics - Masson (2003)

Simulation No.	Agents	Rules	p_m	Std.Deviation	Skewness	Kurtosis	Jarque-Bera	AC(1) Coef.	Confidence Int.	
1	100	1	0.33	0.0091	0.5127	5.5040	13035.6990	0.2993	0.2806	0.3180
2	75	1	0.33	0.0092	0.4944	4.6378	9351.0910	0.2359	0.2168	0.2550
3	50	1	0.33	0.0094	0.4318	3.7688	6216.0740	0.1886	0.1694	0.2079
4	25	1	0.33	0.0099	0.4443	1.9918	1977.3317	-0.0068	-0.0264	0.0128
5	12	1	0.33	0.0111	0.2401	0.5333	213.8430	-0.2279	-0.2470	-0.2088
6	100	1	0.165	0.0077	1.2972	13.2932	76308.0661	0.3176	0.2990	0.3362
7	75	1	0.165	0.0077	1.3998	12.9326	72834.4806	0.2711	0.2523	0.2900
8	50	1	0.165	0.0075	1.1540	11.3626	55921.1954	0.2056	0.1864	0.2248
9	25	1	0.165	0.0077	0.8566	9.3357	37472.9769	0.0093	-0.0103	0.0289
10	12	1	0.165	0.0087	0.3384	2.9588	3830.0576	-0.2972	-0.3159	-0.2784
11	100	1	0.0825	0.0041	3.6698	65.8064	1824027.9196	0.1963	0.1771	0.2155
12	75	1	0.0825	0.0050	2.6872	44.5675	838351.5928	0.2286	0.2095	0.2477
13	50	1	0.0825	0.0038	2.4136	40.7258	699705.7608	-0.0851	-0.1047	-0.0656
14	25	1	0.0825	0.0047	0.2264	13.7378	78590.1710	-0.2888	-0.3076	-0.2701
15	12	1	0.0825	0.0061	0.4415	5.0447	10907.4362	-0.3544	-0.3727	-0.3361
16	100	1	0.04	0.0017	3.0018	198.5952	16423523.5662	-0.1816	-0.2007	-0.1625
17	75	1	0.04	0.0018	3.2966	121.1293	6122273.3598	-0.3743	-0.3924	-0.3563
18	50	1	0.04	0.0024	2.7412	126.8562	6707545.9660	-0.2853	-0.3040	-0.2665
19	25	1	0.04	0.0032	1.2228	33.3954	466448.2024	-0.2979	-0.3166	-0.2792
20	12	1	0.04	0.0047	1.2422	30.4967	389480.0011	-0.1950	-0.2142	-0.1758
41	100	45	0.0825	0.0041	1.0875	19.4909	160002.6478	-0.3617	-0.3799	-0.3434
42	75	45	0.0825	0.0064	1.0613	10.2143	45271.7094	-0.1798	-0.1991	-0.1605
43	50	45	0.0825	0.0071	0.9126	7.7630	26451.7540	-0.1737	-0.1930	-0.1544
44	25	45	0.0825	0.0082	0.8966	4.5400	9909.4879	-0.3238	-0.3424	-0.3053
45	12	45	0.0825	0.0086	0.6200	3.1745	4829.8700	-0.4811	-0.4983	-0.4639
51	100	15	0.0825	0.0081	0.9140	7.2758	23407.9618	0.2382	0.2192	0.2572
52	75	15	0.0825	0.0071	0.9443	8.9320	34668.3468	0.0721	0.0525	0.0916
53	50	15	0.0825	0.0072	1.0776	7.2041	23519.4089	-0.0995	-0.1190	-0.0800
54	25	15	0.0825	0.0080	0.9976	4.9102	11683.2420	-0.2729	-0.2917	-0.2540
55	12	15	0.0825	0.0094	0.7716	3.0537	4868.2602	-0.4258	-0.4436	-0.4081

Table 6: First Difference in Interest Rate Spread - Distribution Statistics

Simulation No.	Agents	Rules	p_m	Std.Deviation	Skewness	Kurtosis	Jarque-Bera	AC(1) Coef.	Confidence Int.	
56	100	5	0.0825	0.0062	0.9715	3.8255	7656.7156	0.3269	0.3084	0.3454
57	75	5	0.0825	0.0065	0.9585	3.2130	5821.9478	0.2094	0.1902	0.2285
58	50	5	0.0825	0.0069	0.8943	2.5814	4101.6903	0.0535	0.0340	0.0731
59	25	5	0.0825	0.0076	0.8765	2.1780	3250.8436	-0.2578	-0.2767	-0.2389
60	12	5	0.0825	0.0084	0.7425	2.2301	2985.1361	-0.4307	-0.4484	-0.4130
61	100	45	0.04	0.0070	1.0798	10.3452	46458.2009	0.1466	0.1272	0.1659
62	75	45	0.04	0.0053	1.4803	16.2791	113887.6196	-0.0382	-0.0578	-0.0186
63	50	45	0.04	0.0063	1.2612	10.8344	51474.7298	-0.0891	-0.1086	-0.0696
64	25	45	0.04	0.0068	1.2401	7.5790	26451.8283	-0.2477	-0.2667	-0.2287
65	12	45	0.04	0.0078	0.9752	4.9357	11714.5018	-0.3580	-0.3763	-0.3397
71	100	15	0.04	0.0069	1.0829	5.1691	13064.3355	0.3247	0.3062	0.3432
72	75	15	0.04	0.0071	1.0350	4.4505	10020.2264	0.2647	0.2458	0.2836
73	50	15	0.04	0.0072	1.0494	4.1444	8975.8392	0.0632	0.0437	0.0828
74	25	15	0.04	0.0071	1.1048	4.2055	9386.9137	-0.2049	-0.2240	-0.1857
75	12	15	0.04	0.0076	0.9565	4.2609	9072.8453	-0.3640	-0.3823	-0.3458
76	100	5	0.04	0.0042	0.5120	0.7610	676.7226	0.0046	-0.0150	0.0242
77	75	5	0.04	0.0046	0.5166	0.4158	515.9622	-0.1854	-0.2046	-0.1661
78	50	5	0.04	0.0050	0.4390	0.3818	381.2936	-0.2942	-0.3129	-0.2755
79	25	5	0.04	0.0055	0.5907	1.2644	1245.0845	-0.3666	-0.3848	-0.3483
80	12	5	0.04	0.0063	0.5532	1.8727	1966.7852	-0.4721	-0.4894	-0.4548
121	1000	1	0.04	0.0045	3.8765	58.8815	144733.9043	0.4774	0.4236	0.5311
122	2000	1	0.04	0.0046	3.8897	56.6335	134094.6077	0.4865	0.4331	0.5400
123	4000	1	0.04	0.0051	2.7596	46.4530	89784.8908	0.5117	0.4591	0.5644
124	100000	1	0.04	0.0054	2.4732	45.3351	85326.4244	0.5257	0.4735	0.5779
125	1000	15	0.04	0.0058	1.2290	10.1291	4451.1671	0.5629	0.5122	0.6137
126	2000	15	0.04	0.0057	1.2218	10.9800	5184.6350	0.5895	0.5400	0.6390
127	4000	15	0.04	0.0055	1.0158	12.9438	7034.6978	0.5955	0.5464	0.6446
128	10000	15	0.04	0.0054	0.8331	14.2937	8486.6156	0.5903	0.5409	0.6397
Treatment	15	–	–	0.0133	0.4775	4.2401	110.0687	-0.0729	-0.2306	0.0848
Treatment	11	–	–	0.0135	-0.1458	1.7901	14.2501	-0.3321	-0.4978	-0.1665
Treatment	11	–	–	0.0123	0.5701	2.3902	34.9046	-0.3668	-0.5238	-0.2098

Table 7: First Difference in Interest Rate Spread - Distribution Statistics (Cont'd)

Simulation No.	Agents	Rules	p_m	Std.Deviation	Skewness	Kurtosis	Jarque-Bera	AC(1) Coef.	Confidence Int.	
1	100	1	0.33	0.0072	0.5369	9.3309	3612.0824	0.189	0.128	0.25
2	75	1	0.33	0.0078	0.6021	6.3183	1692.3086	0.0779	0.0159	0.1398
3	50	1	0.33	0.0103	0.7056	2.8462	412.0985	-0.163	-0.2244	-0.1017
4	25	1	0.33	0.0116	0.5082	1.3155	112.4754	-0.1458	-0.2072	-0.0843
5	12	1	0.33	0.014	0.4202	1.5285	123.6065	-0.2079	-0.2688	-0.1471
6	100	1	0.165	0.0057	1.7501	24.195	24510.6161	0.2604	0.2007	0.3201
7	75	1	0.165	0.0062	1.1289	18.7903	14684.3902	0.2227	0.1623	0.2831
8	50	1	0.165	0.0058	1.2525	18.0018	13543.1508	0.1245	0.063	0.186
9	25	1	0.165	0.0061	0.3796	7.4619	2301.8401	-0.2618	-0.3216	-0.202
10	12	1	0.165	0.0088	0.0539	1.0793	47.2321	-0.3892	-0.4464	-0.332
11	100	1	0.0825	0.0031	2.6736	78.4835	253929.9208	0.0376	-0.0232	0.0983
12	75	1	0.0825	0.0032	3.3232	68.4563	194108.4075	-0.0683	-0.1293	-0.0072
13	50	1	0.0825	0.003	-0.2867	1.3515	87.2411	-0.4194	-0.4747	-0.3642
14	25	1	0.0825	0.0045	-0.1541	0.4332	11.2116	-0.4984	-0.5517	-0.445
15	12	1	0.0825	0.0066	-0.01	2.6874	293.7977	-0.4029	-0.4597	-0.3461
16	100	1	0.04	0.0016	-2.4821	30.7747	39862.2085	-0.3106	-0.3637	-0.2575
17	75	1	0.04	0.0019	-0.9726	9.9125	4179.7349	-0.4079	-0.4612	-0.3547
18	50	1	0.04	0.0023	-0.6664	5.2112	1182.9975	-0.3954	-0.45	-0.3408
19	25	1	0.04	0.003	-0.0911	1.2349	62.7519	-0.4441	-0.4987	-0.3894
20	12	1	0.04	0.0041	0.0099	1.0593	45.0319	-0.4966	-0.5501	-0.443
41	100	45	0.0825	0.0053	1.218	14.9121	9358.003	-0.1516	-0.2124	-0.0908
42	75	45	0.0825	0.0048	0.428	20.0991	16591.6484	-0.296	-0.3546	-0.2373
43	50	45	0.0825	0.0064	1.0567	8.1375	2894.6997	-0.3802	-0.4373	-0.3231
44	25	45	0.0825	0.0053	0.4285	2.6928	325.3576	-0.6292	-0.6771	-0.5812
45	12	45	0.0825	0.0082	0.4688	2.1102	217.0453	-0.4592	-0.5143	-0.4041
51	100	15	0.0825	0.005	1.3787	19.6006	16064.011	0.0657	0.0044	0.127
52	75	15	0.0825	0.0052	0.9356	21.0883	18377.4582	0.0636	0.0022	0.125
53	50	15	0.0825	0.0049	1.286	12.1836	6354.3603	-0.4453	-0.5	-0.3906

Table 8: Endogenous Wealth - First Difference in Interest Rate Spread - Distribution Statistics

Simulation No.	Agents	Rules	p_m	Std.Deviation	Skewness	Kurtosis	Jarque-Bera	AC(1) Coef.	Confidence Int.	
54	25	15	0.0825	0.0063	1.1054	9.2292	3689.2076	-0.392	-0.4488	-0.3352
55	12	15	0.0825	0.0081	0.6359	2.7457	373.6337	-0.5362	-0.5886	-0.4839
56	100	5	0.0825	0.0039	1.6649	16.0754	11049.66	0.1321	0.0713	0.1928
57	75	5	0.0825	0.0043	1.783	10.4922	5034.3027	-0.0471	-0.1086	0.0143
58	50	5	0.0825	0.0054	1.3216	7.2869	2461.3139	-0.1547	-0.2158	-0.0936
59	25	5	0.0825	0.0061	1.1953	4.8114	1181.8937	-0.4231	-0.4791	-0.367
60	12	5	0.0825	0.0071	0.582	3.1272	454.3483	-0.5015	-0.5551	-0.4479
61	100	45	0.04	0.0049	1.3299	23.4189	22780.5214	0.1874	0.1271	0.2477
62	75	45	0.04	0.0049	1.4553	19.7123	16280.0468	-0.1581	-0.2188	-0.0973
63	50	45	0.04	0.0056	1.2669	14.9883	9471.5063	-0.0045	-0.0663	0.0573
64	25	45	0.04	0.005	1.1974	11.7835	5924.8139	-0.3861	-0.4432	-0.3291
65	12	45	0.04	0.0072	1.0831	6.3146	1824.5665	-0.4842	-0.5384	-0.43
71	100	15	0.04	0.0039	1.7397	20.7061	18078.1631	0.1467	0.0866	0.2069
72	75	15	0.04	0.0038	1.9726	20.1523	17293.1189	-0.0963	-0.1569	-0.0358
73	50	15	0.04	0.0042	2.1297	18.0247	14067.9279	-0.0614	-0.1226	-0.0002
74	25	15	0.04	0.0046	1.3751	9.9623	4376.7189	-0.2036	-0.2642	-0.1431
75	12	15	0.04	0.0056	0.5932	4.4455	864.9883	-0.4959	-0.5499	-0.442
76	100	5	0.04	0.0029	1.4087	10.364	4726.9123	-0.1071	-0.1675	-0.0467
77	75	5	0.04	0.0032	1.0238	7.0448	2203.5348	-0.2508	-0.3096	-0.192
78	50	5	0.04	0.0031	1.0707	8.931	3454.7382	-0.4903	-0.5428	-0.4377
79	25	5	0.04	0.0039	0.8925	4.2543	870.5089	-0.5295	-0.5815	-0.4775
80	12	5	0.04	0.0052	0.8181	4.2418	845.0802	-0.5301	-0.5828	-0.4774
121	1000	1	0.04	0.0022	4.3921	209.8957	1811279.737	0.4707	0.4195	0.5218
122	2000	1	0.04	0.0023	4.3035	246.7464	2501807.514	0.525	0.4756	0.5743
123	4000	1	0.04	0.0023	2.3076	210.9187	1826637.629	0.5989	0.5534	0.6444
124	100000	1	0.04	0.0023	4.7527	238.1694	2331778.008	0.509	0.4595	0.5585
125	1000	15	0.04	0.0029	2.011	46.0869	87803.5864	0.5599	0.511	0.6089
126	2000	15	0.04	0.0026	2.5704	76.4976	241209.3454	0.5368	0.4877	0.586
127	4000	15	0.04	0.0026	2.4529	70.8132	206746.9879	0.5138	0.4641	0.5636
128	10000	15	0.04	0.0022	1.5788	95.1258	371732.5651	0.6202	0.5769	0.6635

Table 9: Endogenous Wealth First Difference in Interest Rate Spread - Distribution Statistics (Cont'd)

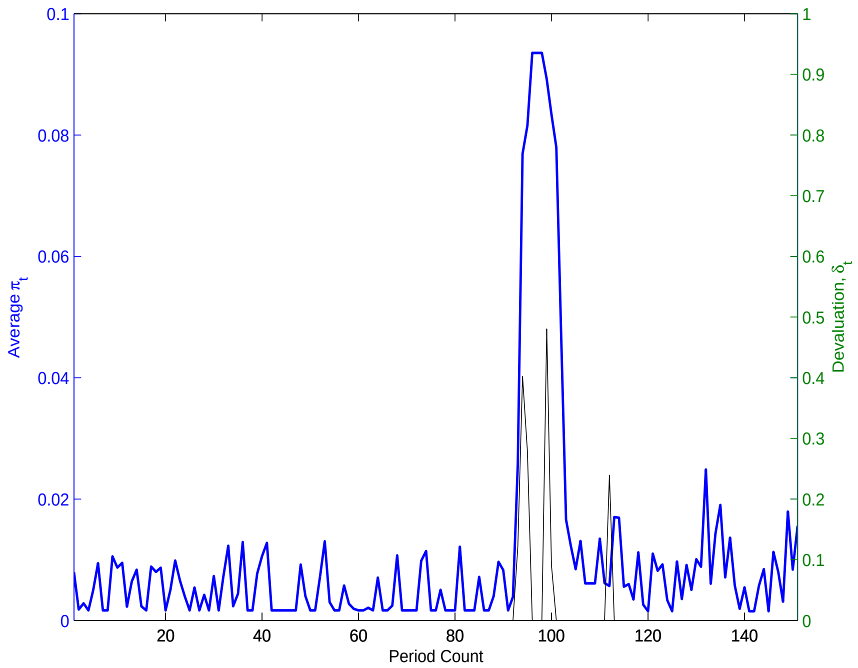

Figure 1: Baseline Simulation - 12 agents, 1 rule per agent, probability of experimentation 0.0825

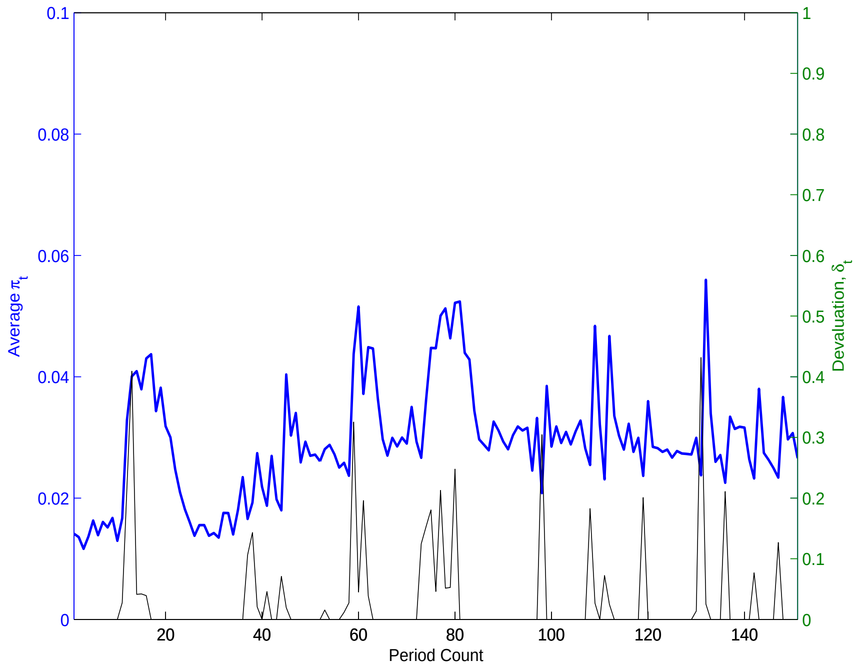

Figure 2: Extended Simulation - 12 agents, 5 rules per agent, probability of experimentation 0.0825

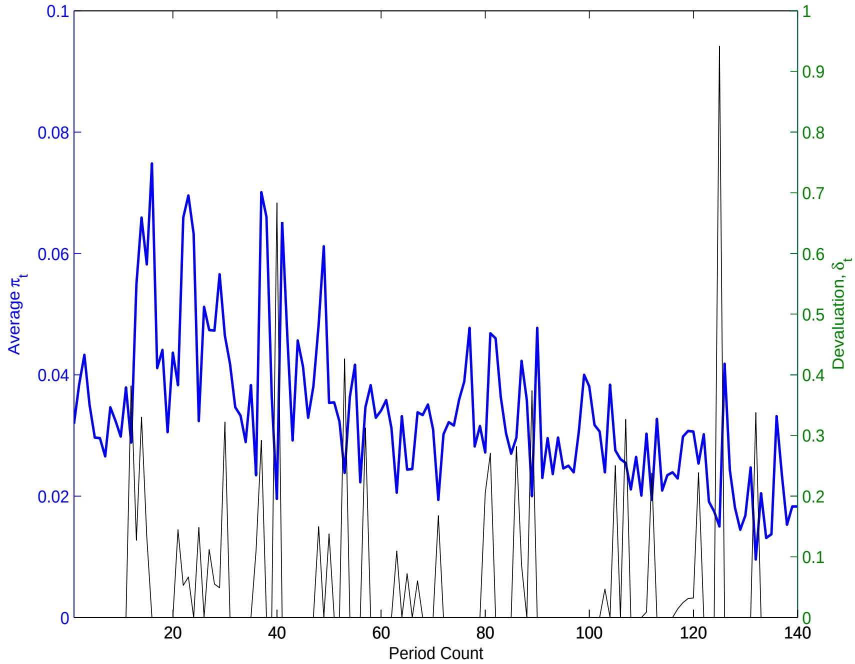

Figure 3: Treatment - 11 subjects

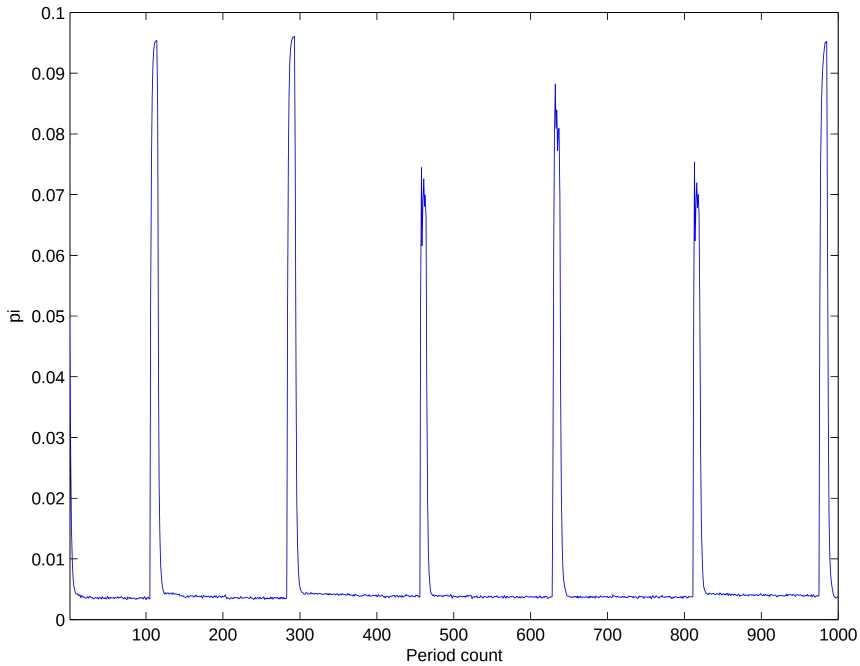

Figure 4: Baseline Simulation - 10,000 agents, 1 rule per agent, probability of mutation 0.04

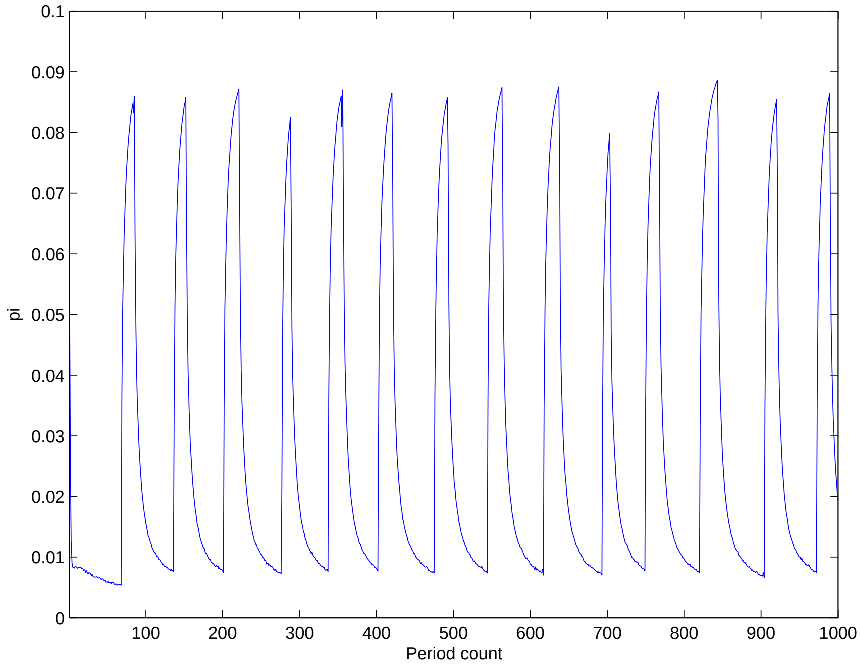

Figure 5: Extended Simulation - 10,000 agents, 15 rules per agent, probability of experimentation 0.04