Genetic Drift in Tacit Coordination Games

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Abstract

This paper investigates evolutionary adaptation in a coordination game with strategic uncertainty. The game is characterized by the multiplicity of Nash equilibria that can be ranked according to the payoff that players obtain. Two different equilibrium refinement concepts predict the selection of different equilibria. Evidence from the experiments with human subjects suggests that the equilibrium selection depends on the number of players that take part in the game and on the number of repetitions of the game. In the model described in the paper, Nash equilibria are neutrally stable. This implies that any of the equilibria can be invaded by strategies that do not disappear from a population and can eventually, through the impact of genetic drift, take the population to a different Nash equilibrium. The results of simulations in which players use the genetic algorithm to update their strategies show that, regardless of the number of players that participate in the game, any equilibrium can be reached. The number of players has an impact on the time spent in each of the equilibria. In particular, the time spent in those equilibria that result in the higher payoffs is negatively related to the number of players.

1. Introduction

Coordination games with strategic uncertainty are usually analyzed in the context of a team-production model in which players make decisions about the level of effort they want to contribute towards production of a joint final good. When making their decisions, players do not have information on the actions taken by their fellow participants. The amount of final good produced is determined by the minimum amount of effort. Players obtain the highest payoff when each player contributes the maximum feasible amount of effort, but there is an element of strategic uncertainty since players are not sure about what the others will do. Thus, these games are characterized by multiplicity of equilibria that can be Pareto ranked. Criteria for choosing equilibrium strategies include payoff dominance and risk dominance. Evidence from the experiments with human subjects suggests that neither the payoff dominance nor risk dominance equilibrium selection criteria can be used as an equilibrium selection device. Experimental evidence also shows that group size, payoff parameter values, (Van Huyck, Battalio, and Beil,1990) and the number of repetitions of the game (Berninghaus and Ehrhart, 1998) have impact on the equilibrium selection process.

This paper uses an evolutionary model to examine the issues related to the equilibrium selection, long-run behavior and the impact of different variables on the dynamics of adaptation in a coordination game with strategic uncertainty. The equilibria of the model are neutrally stable which implies that invading strategies will not necessarily be driven out of a population. Because these equilibria are neutrally stable, transition from one Nash equilibrium to the other is possible through the effects of mutation and genetic drift. [1]

[1] Eaton and Morrison(1998) study an evolutionary model of an investment-opportunist-retaliation (leader/follower) game. The subgame perfect Nash equilibrium of the game in which retaliation does not occur is inefficient. They find that evolutionary stability does not rule out equilibria which involve retaliation since the equilibria of the game are neutrally stable, but are not evolutionary stable. Moreover, they analyze the dynamic behavior of the system in computer simulations and

The results of simulations in which players use the genetic algorithm to update their strategies show that mutation and genetic drift take populations of strategies through different Nash equilibria regardless of the group size. The results of simulations also show that populations with small and large group sizes spend fractions of time close to each of the Nash equilibria of the game. However, the fraction of time spent close to the high-effort equilibria decreases monotonically with increases in the number of players. Small group sizes spend more time in high effort equilibria, while large group sizes spend more time in the low effort equilibria. This behavior is due to the effect of the genetic drift which takes the populations through different equilibria. The transition from the low effort to the high effort equilibrium is more likely for small groups than for large groups. Likewise, the transition from the high effort to the low effort equilibrium is less likely for small groups than for large groups.

The rate of mutation has an impact on the evolutionary dynamics, i.e. too much mutation disrupts coordination on the high-effort equilibria.

Section 2 of the paper describes the basic game and the results obtained in the experiments with human subjects. The description of the evolutionary model and of the Nash equilibria in a model with a monomorphic population is given in section 3, while the genetic algorithm model is described in section 4. This section also contains the results of simulations and the discussion of the impact of the number of players, relative payoffs and rate of mutation on genetic algorithm's dynamics. Section 5 contains concluding remarks.

2. One-shot Game

This is a production game where a player i, $i \in [1, N]$ makes a decision about the level of effort e_i that he contributes to the joint production of output. Actions are

find that the system oscillates between equilibria which include retaliatory behavior and equilibria which do not.

restricted to the set of integers from 1 to n. Let $e_1, \ldots, e_N$ denote the actions taken by N players. The payoff function for player i is given by:

$$\pi(e_i, e_{min}) = a e_{min} - b e_i \qquad (1)$$

$$a > b > 0,$$

where e_{min} equals min $(e_1, \ldots e_N)$. Thus each player's payoff increases in the minimum level of effort contributed to the team production, and decreases in player's own level of effort.

The choice that results in the highest payoff for each player is the one where every player chooses the maximum feasible level of effort, n. But, if one of the players deviates and takes a lower level of effort, $e_i < n$, the best response for each player is to take the same level of effort, e_i. Thus, all feasible actions are potential Nash equilibrium outcomes which can be Pareto ranked. The payoff-dominant equilibrium is the one in which each player chooses the maximum level of effort, n. The equilibrium with the lowest payoff to each player is the one in which everyone chooses the least amount of effort, $e_i = 1$ for all i. If a selection principle is based on the "riskiness" of an equilibrium point, security selects the equilibrium with maximin actions, i.e. the equilibrium with the lowest payoff. The payoff-dominant selection principle, on the other hand, chooses an equilibrium with the maximum level of effort, n, for each player i, $i \in [1, N]$.

The results of the experiments with human subjects in which the above described game was simulated are described in Van Huyck, Battalio, and Beil (1990). These results showed that the number of players was an important determinant for equilibrium selection. The evidence from these experiments demonstrated that a small number of players (group size equal to 2) could coordinate on a payoff dominant equilibrium while a large number of players (group size equal to 14) coordinates on an equilibrium with the lowest payoff.

Following this experimental work, Berninghaus and Ehrhart (1998) showed that the number of repetitions of the game in the experiments with human subjects was another important determinant for equilibrium selection. If a period coordination game is repeated large number of times, the payoff dominant outcome can be observed even in games with 'many' players (group size equal to 6). By sufficiently reducing the number of repetitions, Berninghaus and Ehrhart were able to reproduce Van Huyck et al. experimental results for large groups.

Arifovic (1996) demonstrated that the populations of players that used the genetic algorithm to update their strategies coordinated on the high effort equilibrium in case of the group size equal to 2, and on the low effort equilibrium in case of the group size equal to 14. However, the examination of the long-run behavior showed that both group sizes could coordinate on any of the Nash equilibria. The group size did affect the frequency with which equilibria with different levels of effort were reached and the time spent in each one of them. Simulations with the group size equal to 2 spend most of the time in the maximum effort equilibrium, while simulations with the group size equal to 14 spent most of the time in the minimum effort equilibria.

3. Repeated Game

A population of N players participate in the team production game. Players have identical payoff functions and strategy sets. The time horizon is infinite.

Players are randomly placed into groups of size g. They play a repeated game within a given group for $T+1$ periods. The duration of the game within the same group will be referred to as an *epoch*. After a epoch, h, expires, players are randomly placed into new groups of size g and the new epoch begins. Within an epoch, at each $t \in [0, T]$, players make decisions about the level of effort they want to contribute. Then, the minimum level of effort in each group is determined and players' payoffs are calculated using equation (1). At the end of each epoch, an average payoff for each player is calculated by dividing the accumulated payoff by $T+1$.

Players' stategies are described in the following way. A strategy s_i, $i \in [1, n^3]$, is given by a vector of values:

$$[p_0, p_1, p_2, \ldots, p_n].$$

The value of p_0 indicates the level of effort that a player plays at the initial time period $t = 0$ of an epoch, and p_j indicates the level of effort that player plays at t given that the minimum group effort at time $t-1$ was j. Thus there is a total of n^3 possible strategies where n is the maximum level of effort.

Next we describe Nash equilibria of this game. There are two types of equilibria, type s stationary equilibria which corresponds to the number of equilibria of the one-shot game, and type c cyclical equilibria.

Denote the set of stationary, s, equilibria as E_s. The number of equilibria is equal to the number of Nash equilibria of the original one-shot game described in section 1, i.e. it is equal to the number of possible levels of effort. A strategy s_i^* is a Nash equilibrium strategy with the level of effort i if it induces a level of effort i at the initial position, i.e. $p_0 = i$ at $t = 0$ and a level of effort i at position p_i, i.e. $p_i = i$ for $t \geq 0$. A strategy is the best response to itself if a value of effort at the position p_0 and at the position i is equal to i.

Suppose that, in a group of size g, each player plays strategy s_i^*. Thus, at $t = 0$, each player contributes the level of effort i since they all have p_0 equal to i. Thus, the minimum level of effort at $t = 0$ is i. Then, at $t = 1$, given that $e_{min,0} = i$, all players take the action prescribed by the i^{th} position, p_i. For all players, $p_i = i$, and thus all of them contribute the level of effort i which is the minimum level of effort at $t = 1$. For all $t > 1$, players continue taking the action prescribed by p_i. The resulting payoff for a strategy s_i^* in every time period is equal to: $\pi^i = ai - bi$.

However s_i^* is not unique best response to itself. Any strategy s_k, $k \in [1, n^3]$, such that $p_0 = i$ and $p_i = i$ is a best response to s_i^* and is also a Nash equilibrium strategy. This strategy can have different levels of effort at some or all positions p_j, $j \neq i$. Thus, $p_0(s_k) = p_0(s_i^*) = i$, $p_i(s_k) = p_j(s^*) = i$, and $p_j(s_k) \neq p_j(s^*)$ for some $j \neq i$, $j \in [1, n]$. Define a set of strategies that is a best response to s_i, Σ_i where subscript i denotes the level of effort i. Thus, Σ_i is a set of strategies that are best responses to $s_i^* \in \Sigma_i$, i.e.

$$\Sigma_i(E_s) = \{s_k = [p_0, p_1, \ldots, p_n | p_0 = i, p_i = i\}$$

.

As noted above, all strategies in the set $\Sigma_i(E_s)$ are Nash equilibrium strategies for the level of effort i. Since s_i^* is not a unique best response to itself, Nash equilibria of type s are weak. The fact that these Nash equilibria are weak is important for the behavior of the evolutionary model of this game over time.

Note that there are also Nash equilibria where in the first $(m+1)$, $m \in [1, n]$, periods players do not play stationary Nash equilibrium. However, for $t \in [m+1, T]$, they play a stationary Nash equilibrium.

In addition there are Nash equilibria of type c. Denote the set of cyclical equilibria by E_c. These cyclical equilibria can of order 2 up to order n. Suppose that $n = 8$. Then the following are the cyclical equilibria of the game:

Equilibria of order 2:

$p_0 = i$, $p_i = j$, $p_j = i$

Equilibria of order 3:

$p_0 = i$, $p_i = j$, $p_j = k$, $p_k = i$

Equilibria of order 4:

$p_0 = i$, $p_i = j$, $p_j = k$, $p_k = l$, $p_l = m$

Equilibria of order 5:

$p_0 = i$, $p_i = j$, $p_j = k$, $p_k = l$, $p_l = m$, $p_m = i$

Equilibria of order 6:

$p_0 = i$, $p_i = j$, $p_j = k$, $p_k = l$, $p_l = m$, $p_m = n$, $p_n = i$

Equilibria of order 7:

$p_0 = i$, $p_i = j$, $p_j = k$, $p_k = l$, $p_l = m$, $p_m = n$, $p_n = o$, $p_o = i$

Equilibria of order 8:

$p_0 = i$, $p_i = j$, $p_j = k$, $p_k = l$, $p_l = m$, $p_m = n$, $p_n = o$, $p_o = q$,$p_q = i$

Any cyclical equilibrium strategy of order less than n is not the unique best response to itself. Thus, all of the cyclical equilibria of order 2 to $n-1$ are weak. The cyclical equilibrium of order n is a strong Nash equilibrium. An equilibrium strategy of a cycle of order n is the unique best response to itself. Whethere these cyclical equilibria are weak or strong will again be important for their evolutionary stability of the equilibrium strategies.

To gain understanding of the basic underlying dynamics, evolutionary stability of equilibria in case of the monomorphic populations of strategies is analyzed. Then, the evolutionary dynamics are examined in simulations in which players use the genetic algorithm to update their strategies. A number of simulations is conducted in order to study the impact of the group size, duration of the game, payoff parameter values, and different rates of mutation.

3.1. Monomorphic Population with Pairwise Matching

The analysis of the evolutionary stability of the Nash equilibrium strategies will be carried out for the group of size 2 and for the monomorphic equilibria, i.e. equilibria in which all players in the population use the same strategy. Consider a population with an infinite number of players who are matched randomly pairwise. (The frequency distribution of strategies in the population is independent of any one player's strategy with the assumption of infinitely many players.) In this model, there are again n sets of Nash equilibria, indexed by i. Each set, Σ_i consists of all the strategies that have $p_0 = i$ an $p_i = i$.

Suppose that s_i is a Nash equilibrium strategy with the level of effort i, $s_i \in \Sigma_i$. A strategy s_i is an *evolutionary stable strategy* (ESS) if, for all strategies $s_j \neq s_i$, and δ sufficiently small,

$$\delta\pi(s_i, s_j) + (1-\delta)\pi(s_i, s_i) > \delta\pi(s_j, s_j) + (1-\delta)\pi(s_j, s_i)$$

Since δ can be made arbitrarily small, the following is an equivalent definition. A strategy s_i is an ESS, if for all strategies $s_i \neq s_j$,

either (i) $\Pi(s_i, s_i) > \Pi(s_j, s_i)$

or (ii) $\Pi(s_i, s_i) = \Pi(s_j, s_i)$ and $\Pi(s_i, s_j) > \Pi(s_j, s_j)$.

If one of the above conditions is satisfied, a population of players in a monomorphic equilibrium cannot be successfully invaded by a small fraction of players using a different strategy. If the invasion is to fail, resident players must get a strictly larger expected payoff than invaders at the time when invasion occurs.

A Nash equilibrium strategy $s_i^* \in \Sigma_i$ is stable with respect to any strategy that is not a best response to s_i^*. However, s_i^* is not stable with respect to $s_j \in \Sigma_i$. From here, it follows that strategy s_i^* is not ESS since if all players are using s_i^*, an invasion of players using $s_j \in \Sigma_i$ will not be driven out (nor will they necessarily come to dominate the population). The only Nash equilibrium that is evolutionary stable is a cycle of order n.

Neutral stability is a stability concept that is weaker than the concept of evolutionary stability. A strategy s_i is a neutrally stable strategy (NSS) if, for all strategies $s_j \neq s_i$,

either $(i)\Pi(s_i, s_i) \geq \Pi(s_j, s_i)$

or (ii) $\Pi(s_i, s_i) = \Pi(s_j, s_i)$ and $\Pi(s_i, s_j) \geq \Pi(s_j, s_j)$.

All the Nash equilibria, except for a cycle of order n, of the minimum effort game are neutrally stable. Thus individual equilibrium strategies are neutrally stable, but are not evolutionary stable. In case of a NSS strategy, a small fraction of invaders will not be necessarily driven from the population, nor will they necessarily come to dominate the population.

The objective of the above analysis of the stability of equilibria in monomorphic populations with pairwise matching was to provide guidance in explaining the behavior observed in simulations of the evolutionary algorithm. The analysis suggests, that except for the cycle of order n, no other equilibrium in this game is evolutionary stable. They are all neutrally stable meaning that a fraction of invaders will not necessarily disappear from the population. This fact combined with the effects of genetic drift can result in switching of the population of strategies between different equilibria. In addition, in case of pairwise matching, i.e. in case of small group size in the terminology used in the experiments with human subjects, equilibria with higher levels of effort will be more frequent and last longer than the equilibria with lower levels of effort. The genetic algorithm and the results of simulations are described in the following section.

4. The genetic algorithm application

Players' decision rules are represented by a population of N binary strings. At each epoch h, players are randomly placed into groups of size g. Within each group, players repeatedly play the game for T iterations. A player i, $i \in [1 \dots N]$, uses a binary string i to make a decision about his action at each iteration, t, $t \in [0 \dots T]$. A binary string consists of $n + 1$ positions where $n = 8$ is the maximum feasible action. Each position p_j, $j \in [0, n]$, consists of l bits where l is the number of bits required to encode integer value n. The total length of a string is $(n+1)l$ bits. The first position,

p_0, encodes the action that player takes at iteration $t = 0$. A position p_j, $j \in [1, n]$ encodes the action that player i takes at iteration $t > 0$, given that a minimum effort of his group at iteration $t-1$, $e_{t-1,min}$, was equal to j. An example of a binary string that encodes integer actions in the interval $[1, 8]$ is given below:

position p_j	0	1	2	3	4	5	6	7	8
binary string:	000	101	001	110	011	111	000	011	100
action $e_{i,0}$	1								
action $e_{i,t}$:		6	2	7	3	8	1	4	5
$\bar{e}_{t-1,min}$		1	2	3	4	5	6	7	8

The strategy encoded by this binary string is interpreted in the following way. At iteration $t = 0$, player i takes action $e_{i,0} = 1$. For $t > 1$, the player decides on effort 6 at iteration t if $e_{t-1,min}$ was equal to 1; the player decides on effort 2 at t if $e_{t-1,min}$ was equal to 2; and so on.

The algorithm is implemented in the following way:

1. At each epoch h, $h \in [1, maxh]$, players are randomly placed into groups of size g. Each player's strategy is represented by a binary string. The size of the population of binary strings N divided by g, the group size, determines the number of groups in a given iteration.

2. Within each group, the players play the game that is repeated T times.

3. Each player i, $i \in [1, N]$, chooses his action. Initial action, at $t = 0$, is determined by decoding bits in the position p_0. For $t > 0$, $t \in [1, T]$, each player chooses his action at t by using the information about, $e_{t-1,min}$, the minimum group effort from the previous, $t-1$, iteration. Given $e_{t-1,min} = j$, $j \in [1, n]$, the player looks up the j^{th} position of his binary string and decodes the action that he takes at t.

4. Once each player chooses his action for iteration t, the minimum group effort of the current iteration is determined within each group.

5. The payoff for each player is computed.

6. The payoffs are accumulated during T iterations of the game. The average payoff for each player is computed as $\sum_{t=1}^{T} \pi_i/(T+1)$.

7. Players' strategies are updated using the genetic operators: reproduction, crossover and mutation.

8. Steps 2 through 8 are repeated for $maxh$ epochs.

9. Initially, at $h = 1$, a binary string for each player is chosen randomly.

The following genetic algorithm parameter values are used in simulations: population size, popsize =40 or 42, number of epochs, maxh = 100,000, number of iterations, T = 50, the probability of crossover, pcross=0.6, and the probability of mutation, pmut = 0.0033. Initial populations are randomly generated. Tournament selection in which two binary strings participate in a tournament is implemented as the reproduction operator.

In order to examine the effect of the group size on the behavior of the algorithm, group sizes of 2, 3,4,5,6,7, and 14 are used. The values of the payoff parameters are: $a = 1.2$ and $b = 0.6$.

5. Results

The results of simulations are summarized in tables 1 and 2. Table 1 presents the frequency distribution of the values of effort averaged over $T + 1$ iterations of each epoch, $\bar{e}_h$:

$$\bar{e}_h = \frac{\sum_{t=0}^{T} \sum_{i=1}^{N} e_{i,t}}{N(T+1)}$$

The value of the average effort calculated in this way represents a data point in the frequency distribution. All the values of $\bar{e}_h$ are truncated to the nearest smaller integer i and used in calculation of the frequency for $e = i$. Table 2 shows the frequency distribution of the average values of minimum effort, $\bar{e}min, h$, calculated in the following way:

$$\bar{e}_h = \frac{\sum_{t=0}^{T} \sum_{i=1}^{N} e_{i,t}}{N(T+1)}$$

The results clearly show that the frequency of high values of $\bar{e}_h$ and $\bar{e}_{min,h}$ decrease with increases in the group size. For example, populations with $g = 2$ spend most of the time playing $e = 8$, populations with $g = 3$ spend most of the time playing $e = 7$ etc.

Table 1

Frequency distribution of average effort over 100,000 epochs

e	1	2	3	4	5	6	7	8
frequency								
g=2	0.00	0.00	0.00	0.00	0.00	0.02	0.14	0.84
g=3	0.00	0.00	0.00	0.00	0.05	0.15	0.366	0.43
g=4	0.00	0.00	0.00	0.01	0.08	0.15	0.44	0.31
g=5	0.00	0.00	0.01	0.06	0.18	0.29	0.25	0.20
g=6	0.00	0.02	0.04	0.13	0.19	0.30	0.23	0.09
g=7	0.00	0.01	0.04	0.17	0.24	0.30	0.17	0.09
g=8	0.00	0.04	0.08	0.21	0.20	0.30	0.19	0.05
g=10	0.00	0.16	0.13	0.24	0.23	0.23	0.04	0.02
g=12	0.00	0.17	0.16	0.21	0.18	0.23	0.036	0.013
g=14	0.00	0.34	0.20	0.20	0.10	0.13	0.02	0.01

Table 2

Frequency distribution of average minimum effort over 100,000 epochs

e	1	2	3	4	5	6	7	8
frequency								
g=2	0.00	0.00	0.00	0.00	0.00	0.4	0.27	0.69
g=3	0.00	0.00	0.00	0.00	01	0.14	0.4	0.36
g=4	0.00	0.00	0.00	0.08	0.13	0.17	0.47	0.20
g=5	0.00	0.00	0.4	0.6	0.32	020	0.26	0.13
g=6	0.00	0.02	0.04	0.20	0.30			
g=7	0.01	0.01	0.12	0.16	0.35	0.16	0.17	0.03
g=8	0.00	0.06	0.16	0.17	0.35	0.15	0.10	0.01
g=10	0.05	0.11	0.24	020	0.31	0.06	0.03	0.00
g=12	0.07	018	0.23	0.15	0.29	0.07	0.02	0.00
g=14	0.18	0.26	0.23	0.14	0.15	0.04	0.01	0.00

At the same time, the common feature of the behavior observed in simulations with all group sizes is that a large fraction of genetic algorithm populations spends most of the time playing best response actions. This feature is present regardless of the group size. In order to examine how close an entire population of strings was to playing best response actions at each epoch, the *best response measure* was calculated in the following way.

$$br_h = 1 - \frac{\sum_{t=0}^{T} I_{i,t}}{N(T+1)}$$

where $I_{i,t}$ is an index variable equal to 0 when player's i effort at iteration t is equal to the minimum within the group in which he plays and 1 if it is greater than the minimum effort. Frequency distribution of this measure for all group sizes is given in table 3.

Table 3

Frequency distribution of best response measure

over 100,000 epochs

interval	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
frequency										
g=2	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.04	0.02	0.77
g=3	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.04	0.15	0.80
g=4	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.07	0.22	0.69
g=5	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.08	0.23	0.64
g=6	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.09	0.24	0.62
g=7	0.00	0.00	0.00	0.00	0.01	0.02	0.04	0.10	0.23	0.60
g=8	0.00	0.00	0.00	0.00	0.01	0.02	0.04	0.08	0.20	0.64
g=10	0.00	0.00	0.00	0.01	0.01	0.02	0.03	0.09	0.14	0.69
g=12	0.00	0.00	0.00	0.00	0.01	0.02	0.03	0.07	0.13	0.75
g=14	0.00	0.00	0.00	0.01	0.01	0.01	0.03	0.04	0.08	0.81

The frequency distribution shows that, regardless of the group size, the mass of the distribution is concentrated on the values of the measure close to one. This means that in both cases, for $g = 2$ and $g = 14$, the genetic algorithm populations spend most of the time close to the equilibria. More significant deviations, meaning lower values of the best response measure, occur only during brief periods of time during which the transition from one equilibrium to the other takes place.

In order to understand the dynamics of the evolutionary process, it is helpful to consider the following hypothetical example. Suppose that players are playing in groups of size $g = 2$. All players' strategies are such that they choose initial level of effort equal to 1, $p_0 = 1$ for all i and that they have $p_1 = 1$. Thus, all players choose best response actions and the economy is in the minimum effort Nash equilibrium. A positive rate of mutation will result in small deviations from the best response, but strategies prescribing actions $p_1 = k$, $k > 1$, at location 1 will be eliminated from the population since they will have lower average payoff than the strategies with $p_1 = 1$. At the same time, mutation will result in a genetic drift in other parts of players' strategies, not currently used in the game. Over time, the drift will generate a number of strings with strategies s' such that $p_0 = 1$ and $p_1 = 1$, but in addition $p_j = j$ at some $j > 1$. These changes will be irrelevant for the levels of effort that are actually chosen. Eventually positions that encode initial levels of effort within these strings will also be affected by mutation, i.e. mutation will result in the initial levels of effort equal to j. When a sufficient number of such strings is generated at the same time, they will earn higher average payoffs than the strings that encode $p_0 = 1$. Reproduction will promote these strings and they will quickly overtake the population. Continuing effects of the genetic drift eventually bring the population to the highest level of effort.

This process works in the reverse as well, with population going from the higher to the lower level of effort. As indicated in the section that analyzed evolutionary stability of the game with pairwise matching, this event has lower probability of occuring than the event of going from the lower effort equilibrium to the higher effort equilibrium. If the strings prescribing lower amounts of effort are not to be wiped out of the population, their number has to be large enough to result in lower payoffs for high effort strings.

Finally, for $g > 2$ similar dynamics take place. Due to the larger strategic uncertainty faced by players in larger groups, higher levels of effort are less likely to be observed.

Figures $1-4$ show the behavior of the average effort and average minimum effort over a thousand epochs for group sizes $g = 4$, $g = 6$, $g = 10$ and $g = 14$.

Another interesting feature of the dynamics is that, frequently, if the population is close to an equilibrium with $p_i = i$ (and $p_0 = i$ or $p_0 = j$, $p_j = i$, $p_i = i$), a number of positions $p_j \neq p_i$ encode the value of effort i. This works as a protection of the population against the invasion of mutatants.

6. Concluding Remarks

The analysis of the long-run behavior of the economies which adapt under the genetic algorithm shows that, regardless of the group size, any equilibrium point can be reached. Due to the larger amount of strategic uncertainty present in the case of large group sizes and larger amounts of mutation required for equilibrium points with higher levels of effort, economies with larger group sizes will spend most of the time in low effort equilibria. Economies in which players play in small group sizes that are characterized by smaller strategic uncertainty spend more time in high effort equilibria. Also, smaller amount of mutation is required to move these economies from low to high effort equilibria. On the other hand, larger amount of mutation is required to move them from high to low effort equilibria.

At the same time, regardless of the group size, all populations spend significant amount of time playing best response actions, i.e. close to Nash equilibria.

In order to compare the performance of the model to the experimental data, a set of experiments that matches the model's framework more closely should be designed and conducted. In particular, human subjects should submit their strategies in the form that corresponds to the representation of strategies in the evolutionary model. In addition, after every experimental period, human subjects should be given the opportunity to see a strategy of another, randomly chosen, player.

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Figure 1 - Average and average minimum effort; group size = 4

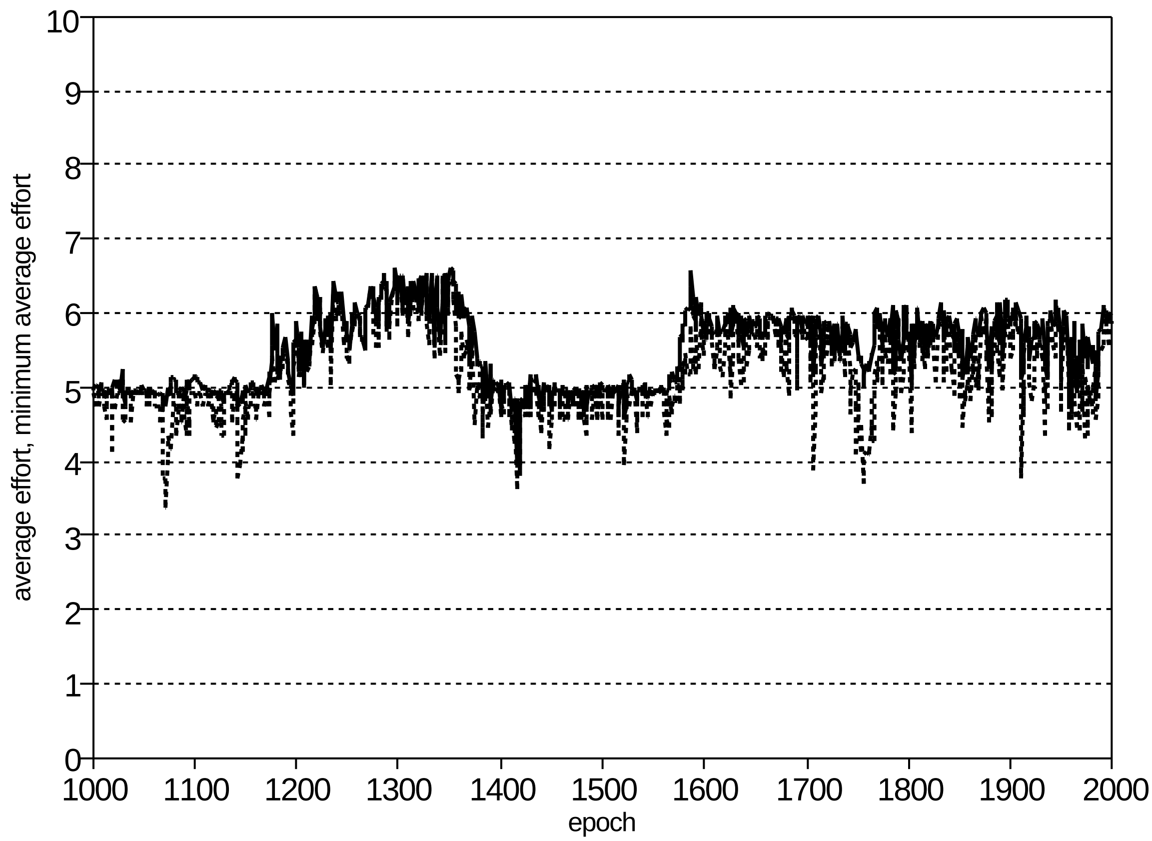

Figure 2 - Average and average minimum effort; group size = 6

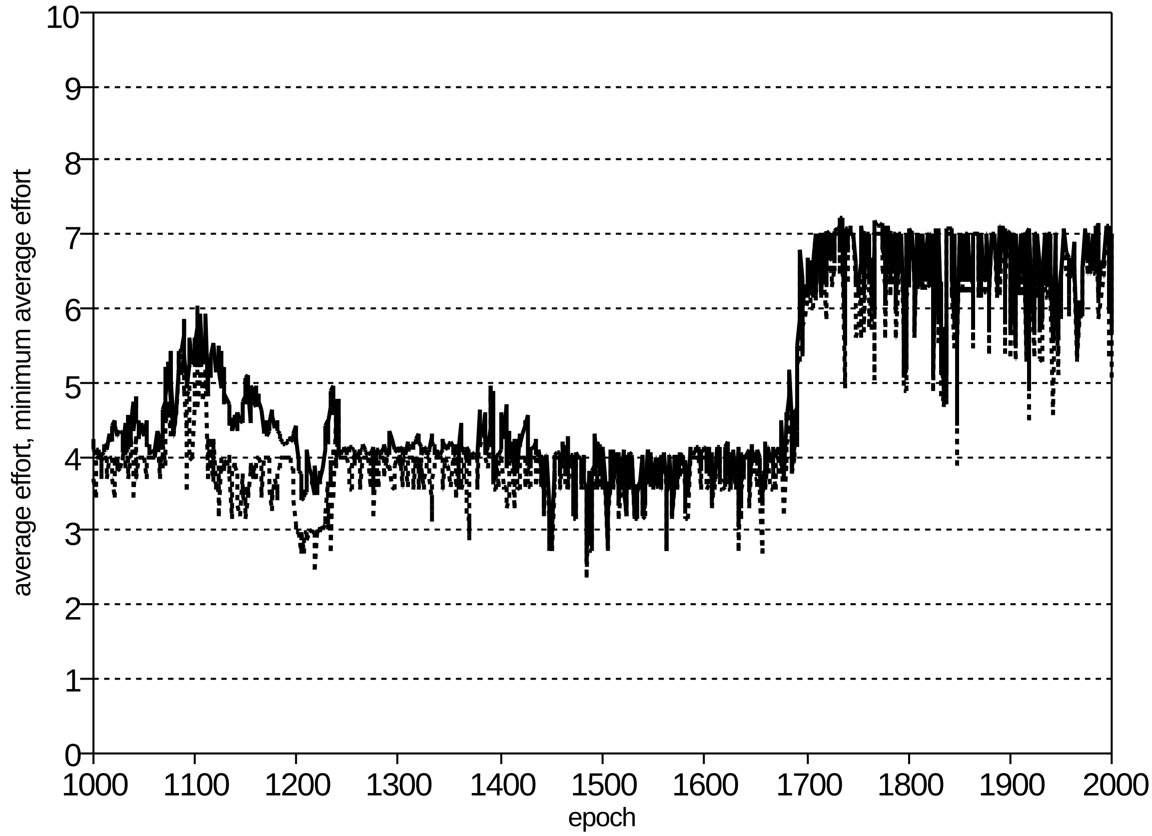

Figure 3 - Average and average minimum effort; group size = 10

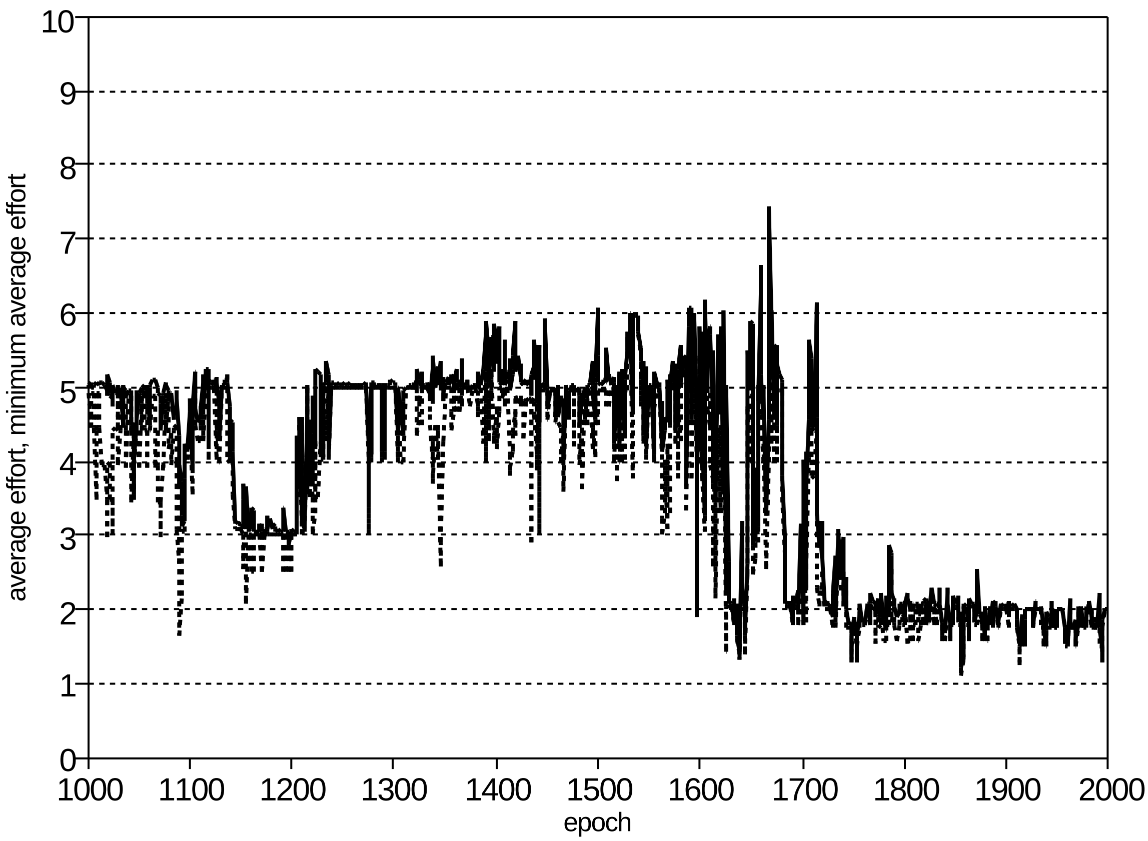

Figure 4 - Average and average minimum effort; group size = 14