The Effect of Short Sale Constraint Removal on Volatility in the Presence of Heterogeneous Beliefs

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ABSTRACT

We evaluate the effect of short sale constraint removal on a stock market. The intuition is derived from simple geometry. We show that the price curve as a function of the uncertain future payoff changes when investors are able to act on the belief that the price of the share is relatively high. In a very simple model we show that volatility can either increase or decrease, depending on the variability of news about final payoffs. As an empirical illustration, we consider data from the Israeli stock market. The data show that volatility increased following the initiation of index options, consistent with the fact that short sales were prohibited in Israel when index options were introduced.

I. INTRODUCTION

The Black and Scholes (1973) model for option pricing relies on the assumption that trading in the option has no effect on the underlying asset. Black and Scholes assume complete markets in which the traded underlying securities span the distribution of final payoffs. In such a market, options are redundant securities and do not affect the underlying asset. However, we know that markets are not always complete and options are not always redundant. For example, the emerging markets that are now engaging in adopting new derivatives are probably much less then complete. Regulators often seem concerned about the effect that options have on the underlying asset.1 This means that they implicitly reject the assumption of completeness.

This paper examines the incompleteness that arises from short sale constraints. We study how the elimination of short sale constraints affects the volatility of the underlying asset. In many emerging markets the ability to short sell is

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nonexistent. Even in developed markets such as the US, many investors face constraints when trying to short sell (e.g., Asquith and Meulbroek 1995; Danielsen and Sorescu 1999). On the other hand, the ability to trade derivatives effectively removes short sale constraints and allows investors to bet against the market. Therefore, an understanding of short sale constraints enhances our knowledge of an important effect that derivative trading has on financial markets.

The paper is aimed at readers who appreciate a simplified approach for understanding the change in volatility. We examine a market that contains as many assets as future states but which is incomplete in the absence of short sales. The intuition is derived from simple geometry: we show that the price function of the risky asset in a constrained economy is a concave function of the uncertain future payoff. When constraints are eliminated, the price function becomes less concave (closer to linear). The reduction in concavity allows us to derive predictions concerning the change in volatility. We show that volatility can increase or decrease depending on the variability of news and the other exogenous variables. Although elimination of constraints provides benefits to the market (i.e., increased welfare), it may increase price volatility. However, we show that volatility can also decrease when moving to an unconstrained economy. Thus, even though negative information can be more effectively revealed when short sale constraints are introduced, this does not mean that volatility will necessarily increase.

Our results are derived from a repetition of a static framework model, where investors have different beliefs about the future state of the economy, making some of the investors want to short sell. On the other hand, the idea that people will have different beliefs also in the long run seems realistic to us. If one agrees with this assessment, a static framework is sufficient to derive results on volatility.

As an empirical illustration, we study the effect of index option initiation in Israel. Options were introduced in Israel at a time when short sales were prohibited, presenting an opportunity to examine the removal of a short sale constraint. We find that option introduction was accompanied by an increase in volatility of the underlying index. This is of interest particularly because previous empirical results have found that introduction of options was accompanied by either reduced return volatility of the underlying asset or no change in volatility.2

Our approach is related to three strands of the economics and finance literature. The first is the literature that deals with short sale constraints starting with Miller (1977), who argues that in a market with short sale restrictions, the price of risky assets is higher than in a market with no restrictions due to a supply effect. Jarrow (1980) examines the influence of a short sale restriction on the prices of risky assets with two assets, while Diamond and Verrecchia (1987) conclude that relaxing the short sale constraint increases the speed of adjustment

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to private information, especially to bad news. They also claim that short selling makes excess returns smaller in absolute value and makes the distribution of excess returns on public information announcement days less skewed to the left. These studies do not deal with volatility changes of the underlying asset. Second, there exist a considerable number of theoretical papers that study the effect of financial innovation in general. Our study is related to a subset of this literature that considers the effect of additional derivative securities on the informational and allocational efficiency of the market (Grossman 1988; Kraus and Smith 1996; Huang and Wang 1997; Cao 1999). However, none of these studies focuses on the informational effect of relaxing the short sale constraint. Thus, we differentiate our analysis from these two strands of literature by studying the particular case of short sale constraints and volatility effects.

Finally, in the dynamic theoretical asset pricing literature there are a few papers that deal with volatility, all showing that volatility tends to increase with the elimination of the short sale constraints. Basak and Croitoru (2000) study short sale constraints and volatility in an economy that has a derivative in zero net supply. Thus, it is not clear whether the elimination of short sale constraints allows the pessimistic investors to do something that they were not able to do in the constrained economy. In Hollifield and Gallmeyer (2002) the pessimistic investor may hold the risky security even if its price is above his valuation, again the pessimistic investor does not necessarily want to short sell in the constrained economy. Finally, in Kogan and Uppal (2002) the constraint is not only on short sales but also on any borrowing in the economy, which affects the interest rate. Thus, these papers do not isolate the effect of allowing betting against the market. Moreover, while these papers use either numerical methods or approximation for volatility changes, our results are analytical.

The rest of the study proceeds as follows. In Section II, the model is developed and the differences between a constrained economy and a non-constrained economy are formulated. In Section III, we study how price volatility changes upon option introduction. Our empirical application is in Section IV. In Section V, we summarize and conclude.

II. THE MODEL

The specific assumptions describing the investors and the capital markets are chosen in the simplest setting possible:

1. The two date \((t = 0, t = 1)\) economy contains a riskless asset (‘cash’) in zero net supply and a risky asset (‘stock’) in a net supply of 1 share.
2. Investors have logarithmic utility functions\(^3\) and are initially endowed only with stock.

\(^3\) The model was also formulated for the more general power utility \(\frac{1}{p}(w)^p\), where \(|p| > 1\). The results are qualitatively the same.
3. The two aggregate investors agree that the $t = 1$ value of the stock will be either $A$ or $B$ but they differ in the probabilities they assign to each payoff.

4. Investors act as atomistic price takers maximizing the expected utility of terminal wealth.

5. There are no transaction costs or taxes, and asset shares are infinitely divisible.

6. The riskless rate is exogenous, and there is unrestricted borrowing and lending at the riskless rate. For simplicity, we assume without loss of generality that the riskless rate is zero.

7. Investors hold dogmatic beliefs and do not attempt to infer each other’s beliefs from prices.

As shown in Figure 1, the price of the stock is $P$ in a short sale constrained economy, and $P^u$ in an unconstrained economy. The two investors believe that the stock’s terminal value will be either $A$ or $B$. The only difference is that the optimistic investor assigns a higher probability to the state in which the stock’s value goes up than does the pessimistic investor (i.e., $\pi_o > \pi_p$). The optimistic and pessimistic aggregate investors’ endowments are made up of stock $(X_o, X_p)$.

**Figure 1** Schematic Description of the Two Aggregate Investors’ Beliefs about Future Stock Payoff ($\pi_o > \pi_p$). The Price in a Constrained Economy is $P$, while the Price in an Unconstrained Economy is $P^u$.

### A. A short sale constrained economy

We assume that the short sale constraint is binding for the pessimistic investor. Both the optimistic and pessimistic investors maximize the expected utility associated with their time $t = 1$ wealth. At time 0, each investor decides on the amounts of cash and stock in their portfolio.

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4 That is, investors give infinite weight to their prior probabilities.
The optimistic investor’s maximization problem is

\[
\max_{X_o \geq 0} \left[ \ln (\bar{W}) \right] = \max \pi_o \ln [P\bar{X}_o + X_o(A - P)] + (1 - \pi_o) \ln [P\bar{X}_o + X_o(B - P)]
\]

and similarly for the pessimistic investor’s problem.

The first-order condition for the optimistic investor is

\[
\frac{\pi_o(A - P)}{PX_o + (A - P)X_o} + \frac{1 - \pi_o)(B - P)}{PX_o + (B - P)X_o} = 0
\]  

(2)

The pessimistic investor would like to hold a negative amount of the stock but cannot do so because of the short sale constraint. That is, the first-order condition of the pessimistic investor is negative at \(X_p = 0\).

\[
\frac{\pi_p(A - P)}{PX_p} + \frac{1 - \pi_p)(B - P)}{PX_p} < 0
\]  

(3)

The condition for a binding short sale constraint can be written as

\[
\pi_p \leq \frac{P - B}{A - B}
\]  

(4)

The optimistic investor holds the total supply of stock, meaning \(X_o = 1\). In equilibrium, the supply of stock equals the demand, so the equilibrium price is derived from the first-order condition of the optimistic investor (equation (2)).

Equilibrium requires the stock price to be in the range \(B < P < A\). In this range the unique equilibrium price is given by

\[
P = T - \sqrt{T^2 - F}
\]  

(5)

where

\[
T = \frac{A(1 - \pi_o\bar{X}_o) + B(1 - \bar{X}_o(1 - \pi_o))}{2(1 - \bar{X}_o)}
\]  

(6)

\[
F = \frac{BA}{(1 - X_o)}
\]  

(7)

with the condition that

\[
\pi_o > \frac{P - B}{A - B}
\]  

(8)

**Proposition 1.** The price of the stock in a constrained economy \(P\) is a concave function of the two possible payoffs, \(A\) and \(B\).

**Proof.** Note two important properties of the price function:

1. It must be that \(T^2 > F\) in order that we obtain a viable real solution for the price in the economy.
Both $T$ and $F$ are increasing linear functions of $A$ and $B$ as the endowments and probabilities are not dependent on $A$ or $B$.

Let $x$ represent $A$ or $B$. Differentiating the price function, we have

$$\frac{\partial P}{\partial x} = \frac{\partial T}{\partial x} - \frac{2T\frac{\partial T}{\partial x} - \frac{\partial F}{\partial x}}{2\sqrt{T^2 - F}}$$  \quad (9)$$

Since the price is increasing in $A$ and $B$ then we must have

$$\frac{\partial T}{\partial x} > \frac{2T\frac{\partial T}{\partial x} - \frac{\partial F}{\partial x}}{2\sqrt{T^2 - F}}$$ \quad (10)$$

Note that

$$2T\frac{\partial T}{\partial x} = \frac{\partial(T)^2}{\partial x} > \frac{\partial F}{\partial x}$$ \quad (11)$$

This is because $T^2 > F$ for all $x$ while both $T$ and $F$ are increasing linear functions of $x$. Thus, $T^2(x)$ is a parabola, and $F(x)$ is a straight line, and the difference between $T^2$ and $F$ increases with $x$. Since both sides of equation (10) are positive

$$\left(\frac{\partial T}{\partial x}\right)^2 > \frac{\left(2T\frac{\partial T}{\partial x} - \frac{\partial F}{\partial x}\right)^2}{4(T^2 - F)} > 0$$ \quad (12)$$

The second derivative of $P$ with regard to $x$ is given by:

$$\frac{\partial^2 P}{\partial x^2} = \frac{\partial^2 T}{\partial x^2} - \left(\frac{1}{2}\right) \left[2\left(\frac{\partial T}{\partial x}\right)^2 + 2T\frac{\partial^2 T}{\partial x^2} - \frac{\partial^2 F}{\partial x^2}\right] \sqrt{T^2 - F} - \left(2T\frac{\partial T}{\partial x} - \frac{\partial F}{\partial x}\right) \frac{2T\frac{\partial T}{\partial x} - \frac{\partial F}{\partial x}}{2\sqrt{T^2 - F}}$$ \quad (13)$$

Considering that $T$ and $F$ are linear functions of $x$, resulting in second derivatives vanishing, this expression yields

$$\frac{\partial^2 P}{\partial x^2} = -\left(\frac{\partial T}{\partial x}\right)^2 + \frac{\left(2T\frac{\partial T}{\partial x} - \frac{\partial F}{\partial x}\right)^2}{4(T^2 - F)}$$ \quad (14)$$

By equation (12) this is negative. \qed

For the more general case of $u(w) = \frac{1}{\gamma}w^\gamma$, there are pathological cases where for high risk aversion $\gamma$, an increase in $A$ actually reduces the price of the underlying asset (see Fishburn and Porter 1976). Throughout the analysis, we ignore these cases.
B. Relaxing the short sale constraint

In an economy where short sales are allowed, both aggregate investors would be able to choose their most preferred portfolios. The first-order conditions for the stock holdings chosen by the two aggregate investors are

w.r.t. $X_o$:

\[
\frac{\pi_o (A - P^u)}{P^u X_o + (A - P^u) X_o} + \frac{(1 - \pi_o) (B - P^u)}{P^u X_o + (B - P^u) X_o} = 0
\]  

(15)

w.r.t. $X_p$:

\[
\frac{\pi_p (A - P^u)}{P^u X_p + (A - P^u) X_p} + \frac{(1 - \pi_p) (B - P^u)}{P^u X_p + (B - P^u) X_p} = 0
\]  

(16)

with the condition that

\[
\pi_p < \frac{P^u - B}{A - B}
\]  

(17)

The market clearing condition is

\[
X_o + X_p = 1
\]  

(18)

Solving for the equilibrium price, we have

\[
P^u = \frac{AB}{A(1 - X_o \pi_o - X_p \pi_p) + B(X_o \pi_o + X_p \pi_p)}
\]

Note that for the case of $\pi_p = \frac{P^u - B}{A - B}$, the pessimistic investor does not short sell, but simply sells all of the endowment $X_p$ in the market. In this case the constrained and unconstrained economies yield the same price by definition.

**Proposition 2.** The price in an unconstrained economy is lower than the price in a constrained economy, i.e., $P > P^u$.

**Proof.** To show that the price in the unconstrained economy is lower, note that

\[
\frac{\partial P^u}{\partial \pi_p} = \frac{AB (A - B) X_p}{(A(1 - X_o \pi_o - X_p \pi_p) + B(X_o \pi_o + X_p \pi_p))^2} > 0
\]

Thus, the price monotonically increases with $\pi_p$. Since $P = P^u$ for $\pi_p = \frac{P^u - B}{A - B}$, for $\pi_p < \frac{P^u - B}{A - B}$ the price in the unconstrained economy is lower. □

**Proposition 3.** The price in an unconstrained economy $P^u$ is a concave function of the two possible payoffs, $A$ and $B$.

**Proof.** We first note that $0 < \tilde{X}_o \pi_o + \tilde{X}_p \pi_p < 1$. This follows from the fact that $X_o + X_p = 1$, while $\pi_o \leq 1$ and $\pi_p < 1$. Thus, we can write the expression for the price as

\[
P^u = \frac{AB}{Ak_1 + Bk_2}
\]  

(19)
where

\[ 1 > k_1 > 0, \quad 1 > k_2 > 0 \]

Taking first and second derivatives:

\[ \frac{\partial P_u}{\partial A} = \frac{k_2 B^2}{(Ak_1 + Bk_2)^2} > 0, \quad \frac{\partial^2 P_u}{\partial A^2} = \frac{-2B^2 k_2 k_1}{(Ak_1 + Bk_2)^3} < 0 \]

\[ \frac{\partial P_u}{\partial B} = \frac{k_1 A^2}{(Ak_1 + Bk_2)^2} > 0, \quad \frac{\partial^2 P_u}{\partial B^2} = \frac{-2A^2 k_1 k_2}{(Ak_1 + Bk_2)^3} < 0 \]

**III. PRICE VARIANCE ACROSS ECONOMIES**

In this section, our aim is to gain an understanding of how the removal of the short sale constraint affects the price volatility of the risky asset. Because our object is to examine the qualitative results that are consistent with the static model in the previous section, we make some simplifying assumptions and study the time 0 price differences across different economies.

Investors choose portfolios based on beliefs about the end-of-period payoff (price plus dividend) of the risky asset. We assume that investors’ beliefs about the payoff have support on some positive interval \([a, \infty)\). To apply our static model framework, we assume that there are two types of ‘worlds’, a constrained world and an unconstrained world.

**Proposition 4.** The price as a function of the future \(A\) and \(B\) values is more concave in a constrained world than in the unconstrained world.

**Proof.** Hold \(B\) constant and analyze the price as a function of a varying \(A\) value. From Proposition 2 we know that the price is lower in the unconstrained world. We also know that the two price functions (constrained and unconstrained) must intersect at two extreme values. First, if \(B = A\) then there is no uncertainty and the price in both worlds is \(P = A = B\). Second, when \(A\) is high enough, even the pessimistic investor would not want to short sell, i.e., \(\pi_p \geq \frac{P - B}{A - B}\) so a constrained and unconstrained world yield the same price. Since by Propositions 1 and 3, both price functions are concave, it must be that the constrained world is more concave. A similar argument applies to varying \(B\). #

We now analyze what this difference in concavity means. Assume within each world there are two economies, which are similar in all aspects except the final payoffs (i.e., same endowments and probabilities for optimistic and pessimistic investors). We analyze the difference in \(t = 0\) prices between the two economies in their two respective worlds, the constrained and the

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6 If we normalize the prices the analysis also holds for the return volatility.
unconstrained world. As a first step in understanding how the two worlds differ, we assume that $B$ is constant in both economies, while $A$ takes on two different values in the two economies. Looking at only two economies simplifies the analysis, but is without loss of generality. Figure 2 illustrates how the different $A$ values affect the stock price, where $P$ is the price of stock in the constrained world and $P^u$ is the stock price in the unconstrained world, when short selling is prohibited. In both worlds the stock price is an increasing concave function of $A$ (and of $B$) and the crucial difference, as stated in Proposition 4, is that the concavity is greater in the constrained world than in the unconstrained world. For further simplification we have drawn Figure 2 so that the price function in the unconstrained world is linear. This assumption leads to easier understanding of the phenomenon we describe below, but the situation is qualitatively similar in the general case. The important point is that the concavity of the stock price as a function of $A$ in the unconstrained world is less than the concavity in the constrained world.

To the right of the intersection between the worlds’ price curves at $A_{\text{max}}$, the short sale constraint is not binding, and equation (4) does not hold.\(^7\) Denote by $A_c$ the value of $A$ for which the two price functions have the same slope. Figure 2 illustrates two possible cases where $A$ takes on two different values in the two

\(^7\) For values of $A$ above $A_{\text{max}}$, holding $B$ fixed, even the pessimistic investor is not prepared to sell short.
economies. The first case concerns \( A \) values to the left of \( A_c \), namely \( A_{d1} \) and \( A_{d2} \), and the second case concerns \( A \) values to the right of \( A_c \), namely \( A_{u1} \) and \( A_{u2} \). The tangent passing through point \( C \) is parallel to the price curve of the unconstrained world. In the first case, \( A \) takes a value of \( A_{d1} \) in one economy and \( A_{d2} \) in the second economy. It is apparent from Figure 2 that in this case the constrained world exhibits greater cross-sectional price differences across economies. The slope of the price function is greater in the constrained world at the two values of \( A \), resulting in a larger price difference. Therefore, relaxation of the short sales constraint leads to a reduction in cross-sectional price difference in this world. Even without the assumption of two economies, we observe that the differences across economies unambiguously decrease when all possible values of \( A \) are smaller then \( A_c \). To the right of point \( A_c \), the slope of the price function of the constrained world is smaller than that of the unconstrained world. Therefore, when the \( A \) values in the two economies are \( A_{u1} \) and \( A_{u2} \) respectively, or alternatively in a world where the \( A \) values are always above \( A_c \), then the unconstrained world entails larger cross-sectional price differences across economies. These results indicate that when \( B \) is constant the behavior of the price difference depends on the range over which \( A \) varies across economies. Depending on the distribution of \( A \), the relaxation of short sales constraints can either increase or decrease the cross-sectional variance in prices. If \( A \) takes on relatively high values (and \( B \) is constant) then the unconstrained world has larger price differences across economies. On the other hand, if the \( A \) values are relatively low (and \( B \) is constant) then the unconstrained world has smaller price differences across economies. Finally, if the variation of \( A \) is similar in magnitude on both sides of the tangency point, then the price differences across economies in both worlds would be similar in magnitude. The case of a varying \( B \) with a constant \( A \) is illustrated in Figure 3 and is similar to the previous case. The price function curves in the two worlds behave in the same manner as with variation in \( A \). Both price curves are increasing functions of \( B \). However, the price curve in the constrained world is more concave. For lower values of \( B \) (and a constant \( A \)) the stock price difference across different economies in the constrained world is larger, whereas for higher values of \( B \) (and a constant \( A \)) the price difference across different economies in the constrained world is smaller.

When we allow \( A \) to vary while keeping \( B \) constant, we also vary the magnitude of the difference \( A - B \). This difference determines whether relaxation of the short sale constraint increases or decreases the differences between stock prices across economies. When \( A - B \) is large it implies that \( A \) variation tends to be at higher values and/or that \( B \) variation tends to be at lower values. Therefore, when \( A - B \) is large and \( A \) differs across economies while \( B \) does not differ across economies, then the variability in time 0 prices across different economies in the unconstrained world is greater than the variability in time 0 prices across economies with a prohibition on short sales. On the other hand, a variation in \( B \) and a constant \( A \) results in smaller price differences across economies in the unconstrained world compared to the
constrained world when \( A - B \) is large. These comparisons are reversed when \( A - B \) is small. Thus, when allowing both \( A \) and \( B \) to vary, we have to look at two aspects of the variability of the news. First, which variation dominates, that in \( A \) or that in \( B \)? Is the news concerning the good state of the economy more uncertain or the news concerning the bad state of the economy more uncertain? Second, what is the difference between the low \( A \) and the high \( B \) values, i.e., the minimum \( A - B \) difference? If we know this information, we can determine whether relaxation of the short sale constraint will increase or decrease the price variance across economies.\(^8\) Note that since we derive the price difference implications from the two price curves of the risky asset, different assumptions about the specific beliefs or endowments across different economies would not result in qualitatively different results. As long as there are some investors in each economy whose beliefs are such that they want to short sell the stock, there is a difference in the concavity between the constrained and unconstrained economies, which implies a change in price differences across the two worlds.

\(^8\) We have assumed for simplicity that only one of \( A \) or \( B \) is varying. If both vary, the net effect could be in either direction, depending on the magnitude of \( A - B \) and the covariance between \( A \) and \( B \).
The empirical work is based on the assumption that differences across economies are descriptive of difference through time in a specific economy. One can view the prediction as a derivation from a repetition of a static framework model, where investors have different beliefs about the future state of the economy, making some of the investors want to short sell.

If one knows the pattern of uncertainty that prevails in the economy as reflected by the difference $A - B$, the variability of $A$, and the variability of $B$, then one can make predictions on how the stock price volatility will change. The theoretical model perceives the $A$ and $B$ values to be the realization of a ‘good’ and ‘bad’ state respectively. Therefore, the difference $A - B$ could be regarded as the uncertainty in the economy prior to the realization of an important event that has an effect on the market (example of events could be the passage of an important tax bill, a major change in the Federal Reserve interest rate policy, a possibility of a war, etc.). Thus, the $A - B$ difference is the unconditional uncertainty that prevails in the economy. If $A - B$ is large, then the realization of the event is expected to have a large influence on the financial markets. The second relevant aspect of uncertainty is the relative uncertainty conditional on the event. That is, conditional on the good (or bad) state occurring, there is some remaining uncertainty about the events effect on the market. If the conditional uncertainty in the good state is higher than that in the bad state, e.g., we can interpret this as a case of $\Delta A > \Delta B$. We realize that the conditional uncertainty may be hard to identify in practice; however, we believe that one could come up with reasonable comparisons on a case-by-case basis. For example, one might expect that the signing of a tax bill that increases taxes on dividends would have a more uncertain effect in the bad state (the signing of the bill). This is because the no-signing state (the good state) is the status quo, which investors are familiar with.

We now describe an empirical case study analyzing how the introduction of options affected stock return volatility in a specific market with a short sale constraint, the Israeli stock market. Index options were introduced in the Israeli market in August 1993. This market is a good case in which to test the volatility effect because of the situation that prevailed in the Israeli economy prior to option introduction. First, short selling was prohibited in the Israeli
market at the time of option introduction.\textsuperscript{10} The introduction of options effectively eliminated the short sale constraint, providing a unique opportunity to test our model. Second, during the time of option initiation, the Oslo agreements between the Israelis and the Palestinians were signed (on September 13, 1993). This was a major event in Middle East politics since it was the first time that an overall solution to the century-long Arab/Israeli conflict was put on the table. In our terminology, event $A$ is a successful implementation of the peace process, while event $B$ is a failure of the peace process implementation (arguably this occurred with the breakdown of the whole peace process in September 2000). We believe that one can also say, since a long string of previous peace efforts had failed, that while the effect of an unsuccessful outcome was relatively clear, the effect of a successful outcome was much more uncertain since the long-run economic effects of peace were unclear. Therefore, we perceive that the prevailing priors in the Israeli economy at the time of option initiation were that of large ($A - B$) difference and greater variability in $A$ than in $B$. Under such circumstances, our model predicts that the initiation of index options should be associated with an increase in the volatility of the index. This is in contrast to other theoretical and empirical studies that look at the effect on volatility due to option initiation alone (unrelated to the elimination of short sales constraints) and typically find a reduction in volatility.\textsuperscript{11}

In January 1, 1992 a new stock index was introduced in the Tel-Aviv Stock Market. The new index (MAOF Index) is a capped index\textsuperscript{12} weighted by market capitalization of the 25 largest firms in the Israeli economy. The index represents approximately one third of the total market capitalization. Trading in options started 19 months after the introduction of the index (on August 1, 1993). Daily prices on the MAOF Index and the broad Tel-Aviv Stock Exchange Index were obtained from the Israel Securities Authority (ISA). The data are daily closing prices for the period 1/1/92–31/7/96.

Index return volatility is calculated over periods starting 19 months prior to index option initiation (i.e., at the date the MAOF Index was launched) and ending at various dates after the initiation of the index options (1 year, 1.5 years, 2 years, and 3 years). The longer the interval, the more observations used

\textsuperscript{10} On October 1995 the Tel-Aviv Stock Exchange (TASE) tried to launch futures contract on the MAOF. This was more than two years after the option initiation so it does not affect our empirical analysis. These futures contracts had very thin trades and the TASE eventually decided to stop trading them.

\textsuperscript{11} Most of the previous research has suggested that option initiation is likely to reduce the volatility of the underlying asset. Options may have a stabilizing effect on the underlying asset volatility (Gjerde and Saettem 1995). Interaction between the spot and the derivative markets could result in more efficient risk allocation, which would increase the demand for the underlying security and reduce its volatility. Options may cause a diversion of trading away from the underlying asset market to the option market. However, an increase in the size of the opportunity set makes risky investments more attractive, resulting in an increase in liquidity of the underlying market. Empirically, option listing is associated with an increase in trading volume (e.g., Skinner 1989). This leads to lower bid–ask spreads, which reduces volatility.

\textsuperscript{12} Capped at 9.5\% for any single stock in the index.
in the estimation procedure and the sharper are the results. On the other hand, the longer the time frame, the higher the probability that other factors may influence the results. The short sale restriction is effectively relaxed when the options are initiated but additional effects may arise over time. For example, liquidity may increase with time, as investors become familiar with the new instrument. The result is that the power of the test is likely to be reduced with a longer time frame.

To determine whether the introduction of index options affected index return volatility, it is necessary to correct for other Israeli market forces, which influence the MAOF Index. The Tel-Aviv Stock Market Index reflects other economic developments that influence the Israeli market. After removing the broad Tel-Aviv Stock Market Index return, various specifications for conditional errors were tested on the residuals. First, the entire sample was checked for day of the week and month of the year effects.\textsuperscript{13} Second, each time frame was checked for ARMA model specification up to \( p = 2, q = 2 \). In an attempt to maintain a balance between models, a common criterion was used: the Schwarz criterion. The Schwarz criterion was used because it delivers relatively parsimonious specifications and is widely used in the literature.\textsuperscript{14} After jointly removing the Tel-Aviv Stock Market and ARMA effects, an ARCH test was conducted. In all cases the test suggested that there was heteroskedasticity in the errors. A joint estimation procedure was conducted including the Tel-Aviv Stock Market Index return, the ARMA(1, 1), and different variance models. Estimation was done by maximizing the likelihood function with the Schwarz criterion employed for selection. In all cases, the simple GARCH(1, 1) model was chosen. Finally, in order to check for the option initiation effect, a dummy variable was introduced in the variance equation for the time following the option initiation date. Thus, the tested specification was

\[
R_t = \alpha_1 TR_t + \alpha_2 R_{t-1} + \alpha_3 e_{t-1} + e_t
\]

\[
\sigma_t^2 = \beta_0 + \beta_1 D_{op} + \beta_2 e_{t-1}^2 + \beta_3 \sigma_{t-1}^2
\]

\( R_t \) – MAOF Index return.

\( \sigma_t^2 \) – the variance of the \( e_t \) component of the MAOF Index return.

\( TR_t \) – Tel-Aviv Stock Index return.

\( D_{op} = \begin{cases} 
1 & t \text{ subsequent to } 30/7/93 \\
0 & t \text{ preceding } 30/7/93 
\end{cases} \)

Table 1 reports the estimation results for the coefficients, the \( p \) values for the coefficient \( \beta_1 \) of \( D_{op} \), and the results of different tests on the residuals. The table.

\textsuperscript{13} It was found that February has some significant positive effect on the return. However, this significance disappeared after adding the ARMA specifications.

\textsuperscript{14} In all time frames the ARMA(1, 1) specification was chosen.
Table 1  Changes in volatility due to option initiation – regression results

<table>
<thead>
<tr>
<th></th>
<th>(1) 1 year</th>
<th></th>
<th>(2) 1.5 years</th>
<th></th>
<th>(3) 2 years</th>
<th></th>
<th>(4) 3 years</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_0$</td>
<td>0.479E-6</td>
<td></td>
<td>0.103E-5</td>
<td></td>
<td>0.724E-6</td>
<td></td>
<td>0.191E-6</td>
<td></td>
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<tr>
<td></td>
<td>(0.250E-6)</td>
<td></td>
<td>(0.043E-5)</td>
<td></td>
<td>(0.267E-6)</td>
<td></td>
<td>(0.091E-6)</td>
<td></td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.172E-5</td>
<td></td>
<td>0.316E-5</td>
<td></td>
<td>0.115E-5</td>
<td></td>
<td>0.886E-7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.096E-5)</td>
<td></td>
<td>(0.139E-5)</td>
<td></td>
<td>(0.053E-5)</td>
<td></td>
<td>(0.850E-7)</td>
<td></td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>0.085</td>
<td></td>
<td>0.124</td>
<td></td>
<td>0.137</td>
<td></td>
<td>0.073</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.025)</td>
<td></td>
<td>(0.032)</td>
<td></td>
<td>(0.028)</td>
<td></td>
<td>(0.014)</td>
<td></td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>0.870</td>
<td></td>
<td>0.784</td>
<td></td>
<td>0.808</td>
<td></td>
<td>0.0916</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.040)</td>
<td></td>
<td>(0.059)</td>
<td></td>
<td>(0.039)</td>
<td></td>
<td>(0.016)</td>
<td></td>
</tr>
</tbody>
</table>

Residuals test
- Box Pierce
  - If Yes – can reject hypothesis that residuals are normal
- Goodness of fit
- Jarque-Bera

Significance of $\beta_1$
- $p$ value
- $p$ value with robust standard errors

The results from estimation of the variance equation $\sigma_t^2 = \beta_0 + \beta_1 D_{op} + \beta_2\epsilon_{t-1}^2 + \beta_3\sigma_{t-1}^2$, where $\epsilon_{t-1}$ is the previous period unexplained MAOF Index return $\sigma_t^2$ is the variance of the $\epsilon_t$ component of the MAOF Index, and $D_{op}$ is a dummy variable that equals 1 if $t$ is subsequent to July 30, 1993 (date of option initiation). Index options (1 year, 1.5 years, 2 years, and 3 years). The table provides standard errors in parentheses and shows the results of different normality tests on the residuals. Since the residual tests are somewhat inconclusive, the last two rows provide two $p$-value for $\beta_1$, which are calculated with regular and robust standard deviation respectively.
also includes the standard deviation and $p$ values when using robust standard errors.\textsuperscript{15}

The values of $\beta_1$ indicate that option initiation had a positive influence on the volatility for time frames less than three years.\textsuperscript{16} With a time frame of three years, $\beta_1$ becomes insignificant. In tests using simple ARCH models the significance of the option dummy variable was extremely high, suggesting that the GARCH ($\sigma^2_{t-1}$) component captures much of the effect of option initiation. Our conclusion is that option initiation increased index return volatility, at least in the short run.

V. SUMMARY AND CONCLUSIONS

This study examines the influence of short sale restrictions on asset prices. Considering a market subject to a short sale restriction, so long as investors disagree on the probabilities assigned to final payoffs and so long as some investors would like to short sell the risky asset, the equilibrium price of the risky asset is higher and more concave as a function of possible payoffs than in an unconstrained economy.

We focus on the effect of relaxation of the short sale constraint on the price volatility of the risky asset. We show that generally there is a range of increasing volatility and a range of decreasing volatility, depending on the variability of information and the other values of the exogenous variables of the economy. These results contradict the belief that relaxing the short sale constraint should always increase volatility due to the increasing effect of bad news.

As an empirical case study, we looked at the behavior of the Tel-Aviv Stock Market, in which short sales are prohibited, when index options were introduced in August, 1993. The data indicate that the return volatility of the index increased after options were introduced.

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\textsuperscript{15} The standard errors are robust in the sense that conditional normality of the errors is not assumed. See Bollerslev (1986).

\textsuperscript{16} Results for time frames greater than three years were also insignificant.
REFERENCES


