Valuation using the Certainty Equivalent Approach. International Investment Projects

Lecture 5

Section 9.4

9.4. Risk, DCF and CEQ

$$PV = \sum_{t=1}^{n} \frac{E[C_t]}{(1+r)^t}$$

where, for example, $E[C_t] = p_1 C_{1t} + p_2 C_{2t}$
and p_i 's are probabilities.

There are two methods to account for risk in the PV formula:

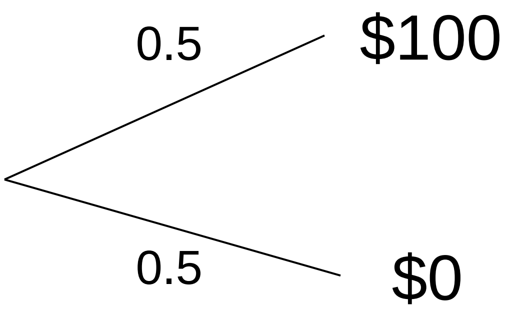
- Discount rate
- Expected cash flows

In the expected CF calculation, we can adjust either probabilities *p* or payoff cash flows *C*.

Changing the Probabilities: Example

Let the COC be at 25%, and the risk-free rate at 10%.

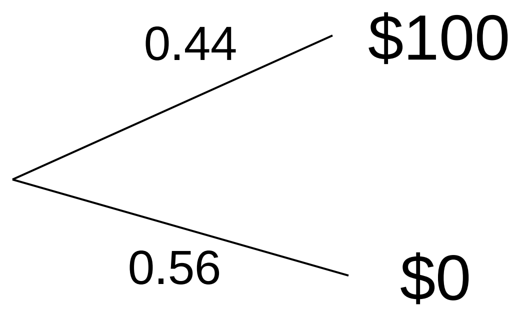
Risky ("physical", our) world



$$PV = \frac{E[C_1]}{1+r} = \frac{0.5*100+0.5*0}{1+r}$$

$$= \frac{\$50}{1.25} = \$40$$

Risk-free (risk-neutral) world



$$PV = \frac{E^Q[C_1]}{1+r_f} = \frac{0.44*100+0.56*0}{1+r_f}$$

$$= \frac{\$44}{1.10} = \$40$$

Superscript Q means risk-neutral probabilities are used to take the expectation.

Certainty Equivalent vs. Risky Cash Flows

Certainty Equivalent (CEQ):

The smallest riskless CF for which you would exchange the risky CF.

Risky expected C_t

$$PV = \frac{C_t}{(1+r)^t}$$

is equivalent to (i.e., has the same PV) as *riskless* cash flow CEQ

$$PV = \frac{CEQ_t}{(1+r_f)^t}$$

Note: in the previous example, the CEQ of $E[C_1]$ is …

Risk, DCF and CEQ

$$PV = \frac{C_t}{(1+r)^t} = \frac{CEQ_t}{(1+r_f)^t}$$

Example

Project A is expected to produce CF = $100 mil for each of the next three years. Given the risk free rate of 6%, market premium of 8%, and beta of .75, what is the PV of the project?

Project B expected cash flows are 94.6, 89.6, 84.8 in years 1-3 respectively. However, these cash flows are RISK FREE. What is Project B's PV?

Risk, DCF and CEQ

Example

Project A is expected to produce CF = $100 mil for each of three years. Given a risk free rate of 6%, a market premium of 8%, and beta of .75, what is the PV of the project?

Step 2. Calculate the NPV

Project A

Year	Cash Flow	PV @ 12%
1	100	89.3
2	100	79.7
3	100	71.2
	Total PV	240.2

Step 1. Calculate the Cost of Capital

$$r = r_f + \beta(r_m - r_f)$$
$$= 6 + .75(8)$$
$$= 12\%$$

Risk, DCF and CEQ

Example

Project B expected cash flows are 94.6, 89.6, 84.8 in years 1-3 respectively. However, these cash flows are RISK FREE. What is Project B's PV?

Project A

Year	Cash Flow	PV @ 12%
1	100	89.3
2	100	79.7
3	100	71.2
	Total PV	240.2

Project B

Year	Cash Flow	PV @ r_f (6%)
1	94.6	89.3
2	89.6	79.7
3	84.8	71.2
	Total PV	240.2

Risk, DCF and CEQ

Project A		
Year	Cash Flow	PV @ 12%
1	100	89.3
2	100	79.7
3	100	71.2
	Total PV	240.2

Project B		
Year	Cash Flow	PV @ 6%
1	94.6	89.3
2	89.6	79.7
3	84.8	71.2
	Total PV	240.2

Since the 94.6 is risk free, we call it a ***Certainty Equivalent*** of the 100 in year 1.

Risk, DCF and CEQ

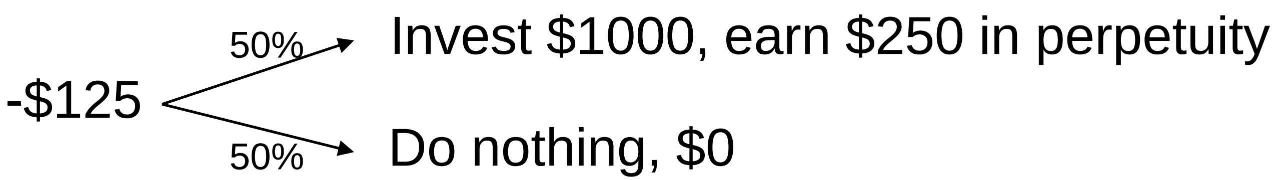
The Risk Premium (*in $*) is the difference between expected risky CF ($100) and certain CF (CEQ). Said differently, it is the amount we are willing to forego to avoid risk and achieve certainty about our CFs.

Year	Cash Flow	CEQ	Deduction for risk
1	100	94.6	5.4
2	100	89.6	10.4
3	100	84.8	15.2

The deduction for risk is greater for later cash flows – we have to give up more $ to assure the certainty of CFs occurring further in the future.

Long lived assets and the need to use several discount rates

Example (from 9.4 p.230): The scientists at Vegetron have come up with an electric mop and are ready to go ahead with pilot production. The preliminary phase will take one year and costs $125k. Management feels that there is only a 50% chance that the pilot production will be successful. If the project fails, the project will be dropped. If the project succeeds, Vegetron will build a $1M plant that would generate an expected annual cash flow of $250k in perpetuity.



r_f=7%, Risk Premium=9%. Regular projects of the firm have a beta of 0.33; however, due to the 50% probability of failure, management assumes a beta of 2 for the project. (An assumption about beta is actually better than assuming a fudge discount factor, in contradiction to footnote on p. 230).

1. What is the project's NPV?
2. Is management correct about its approach for the NPV calculation?

Long lived assets and the need to use several discount rates

- **Solution**.
- Regular projects' Cost of Capital = 7% + 0.33 * 9% = 10%.
- According to the manager's assumed beta, the Cost of Capital of this project is 7+2*9 = 25%.

  Cash flows: C0=-$125;

  C1= 50% prob of (-$1000 + $250 in perpetuity @25% risk)

  + 50% prob of ($0) = $0;
- NPV = C0 + C1 / (1.25) = - $125. Negative NPV: the manager does not invest

- **Correct Solution:**
- Discount the perpetuity at the regular COC of 10% because these CFs take place after the uncertainty has been resolved.
- Thus, expected CF in year 1 is

  C1= 50% of (-$1000 + $250 in perpetuity @**10%** risk) + (50% of $0) =$750.
- NPV = C0 + C1 / (???) = -125 + 750/ (???).

Long lived assets and the need to use several discount rates

- At t=1, the high risk is present, but we don't know what number to use as the risky discount rate.
- We can use the CEQ approach:
  - Replace CF1 with its certainty equivalent and discount it with the risk-free rate, in order to avoid assuming a number for the risky discount rate.
  - NPV = -125 + CEQ / 1.07.

    But: we don't know the CEQ of risky $750 at t=1.

  - Do we really need to know it accurately to make a decision whether to invest?
  - Sometimes, but not in this example. Here is why:
- Here, the risky CF_1 is so large ($750), that even for very risk-averse managers, who would be OK with small CEQs (such as $300 or even $150) replacing the uncertain $750, we still obtain a positive NPV and <u>*invest*</u>.

Beta Estimation with an Index Choice Problem

- **How much is the marginal investor diversified?**
- **Local Solution:** Estimate the beta relative to a local index.
- **U.S. Solution:** If the stock has an ADR listed on the U.S. exchanges, estimate the beta relative to the S&P 500.
- **Global Solution:** Use a global index to estimate the beta for a globally diversified investor.

  Example. For Aracruz Celulose,

Index	**Beta**
Brazil I-Senn	0.69
S & P 500 (with ADR)	0.46
Morgan Stanley Capital Index (with ADR)	0.35

MSCI EAFE is a stock market index of foreign stocks, from the perspective of a North American investor, maintained by MSCI/Barra.

ADR (American Depository Receipts) are certificates of ownership in foreign companies' stock listed in the U.S.

Summary and Conclusions (ch. 8 and 9)

- A project's required return and cost of capital depend on the *project's* β (systematic risk).
- A project's β can be estimated by considering comparable projects, industries, or firms, i.e. firms having similar risk and financial structure
- The cyclicality of project revenues and the project's operating leverage (proportion of fixed costs) can increase beta.

Cost of Capital Summary (no taxes)

Use the CAPM to estimate the cost of equity capital:

$$r_{equity} = r_f + \beta_{equity} (r_m - r_f)$$

This is the cost for the part of a project financed by issuing stock.

Then, for the whole firm or project, the cost of capital is

$$r_{assets} = \frac{D}{V} r_{debt} + \frac{E}{V} r_{equity}$$