Equilibrium Security Design and Liquidity Creation by Privately Informed Issuers*

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Abstract

By supplying riskless claims ("liquidity") an issuer can insulate uninformed investors from adverse selection in competitive securities markets. We evaluate private incentives to supply riskless securities, analyzing equilibrium security design by a privately informed issuer selling asset-backed claims in competitive markets to an endogenously informed speculator and rational uninformed investors who demand liquidity. First-best social welfare is attained if pooling occurs at the low type's preferred structuring in which the full asset is securitized as riskless debt and levered equity. However, this structuring is not generally an equilibrium. There always exists an equilibrium in which the issuer sells riskless debt and retains all risk. However, this equilibrium fails to be unique precisely when uninformed investors have high liquidity demand. Specifically, when uninformed investors have high liquidity demand, pooling may occur at the preferred structure of the high type who maximizes informed trading by intead selling risky debt, with optimal information-sensitivity trading off per-unit speculator gains against endogenous declines in uninformed demand. The model thus shows the private supply of riskless debt can fall precisely when investors most value liquidity. However, a government supplying relatively small amounts of riskless debt potentially ensures sufficient aggregate liquidity. This is because, as is shown, public and private liquidity are strategic complements. Specifically, by siphoning-off uninformed demand for risky securities, public liquidity crowds-out informed speculation, and crowds-in private liquidity.

Keywords: Security design, adverse selection, welfare, liquidity, Kyle model.

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By supplying riskless claims ("liquidity") an issuer can insulate uninformed investors from adverse selection in competitive securities markets. We evaluate private incentives to supply riskless securities, analyzing equilibrium security design by a privately informed issuer selling asset-backed claims in competitive markets to an endogenously informed speculator and rational uninformed investors who demand liquidity. First-best social welfare is attained if pooling occurs at the low type's preferred structuring in which the full asset is securitized as riskless debt and levered equity. However, this structuring is not generally an equilibrium. There always exists an equilibrium in which the issuer sells riskless debt and retains all risk. However, this equilibrium fails to be unique precisely when uninformed investors have high liquidity demand. Specifically, when uninformed investors have high liquidity demand, pooling may occur at the preferred structure of the high type who maximizes informed trading by intead selling risky debt, with optimal information-sensitivity trading off per-unit speculator gains against endogenous declines in uninformed demand. The model thus shows the private supply of riskless debt can fall precisely when investors most value liquidity. However, a government supplying relatively small amounts of riskless debt potentially ensures sufficient aggregate liquidity. This is because, as is shown, public and private liquidity are strategic complements. Specifically, by siphoning-off uninformed demand for risky securities, public liquidity crowds-out informed speculation, and crowds-in private liquidity.

A fundamental question in securities markets, posed by Holmström and Tirole (1998), is whether there will be an adequate private supply of stores of value, or whether the government should augment this supply. For example, the U.K. government recently introduced "ultra gilts" in response to a perceived shortage of safe long-term assets preferred by pension funds and insurers. As argued by Tirole (2011), the recent financial crises, and resulting flights-to-quality, have refocused attention on the supply of such riskless stores of value at various investment horizons.

The supply of such riskless stores of value, i.e. *liquidity*, has important implications for social welfare. For example, if issuers instead supply risky securities, speculators will exert socially costly effort. Further, in the absence of sufficient liquidity, uninformed investors face adverse selection as they trade in competitive securities markets, as shown by Kyle (1985). Although Kyle and many subsequent papers have treated uninformed trading as exogenous, it is natural to expect such investors to distort their portfolios in order to reduce trading losses when facing adverse selection. As in Akerlof (1970), such distortions will reduce social welfare as some Pareto-improving trades are not exploited.

This paper analyzes the private incentive to supply riskless claims, and the potential welfare arguments for government-supplied public liquidity. We consider the following setting. An owner chooses the design of asset-backed securities (ABS) to be marketed in order to fund a scalable positive NPV investment. The payoff on the underlying asset-in-place is L or H and is privately known by the ABS issuer. There is a continuum of risk-averse uninformed investors facing a future endowment shock who would like to save using an information-insensitive store of value. They face a speculator who can exert costly effort to acquire a noisy signal regarding the asset payoff. The issuer can signal positive information via retention of cash flow rights or the two types can pool, with competitive market-makers then setting prices in a noisy rational expectations equilibrium à la Kyle (1985). Critically, we depart from Kyle and an extant market microstructure literature by treating security design and uninformed trade as endogenous. Endogenous uninformed trading is the key causal mechanism–and it is this mechanism that underpins novel implications of the model.

First-best social welfare would be achieved if the types were to pool at the structure preferred by the owner of a low value asset (the "low type"): full securitization of the underlying asset bifurcated (i.e. split) into riskless senior debt and junior levered equity. The uninformed investors would use the riskless debt to save and the speculator would not exert costly effort given the absence of uninformed trading to cover her orders in the risky equity market. However, this socially preferred structure is not generally in the set of perfect Bayesian equilibria (PBE), and never satisfies the Intuitive Criterion of Cho and Kreps (1987). Intuitively, the high type has a strong incentive to deviate from such a pooling equilibrium since the absence of informed trading implies severe underpricing of his marketed equity tranche.

The set of PBE always includes the *Low Information Intensity Optimum* (LIIO), and also includes those pooling allocations weakly Pareto dominating the LIIO from the perspective of the two issuer types.[1] The LIIO is the incentive compatible, profitable type-by-type, allocation maximizing the high type's utility.[2] In the LIIO, the two issuer types only sell riskless debt with face value L, retaining all risk on their own balance sheets. Consequently, the speculator does not exert costly effort and uninformed investors are supplied with the riskless store of value they prefer. The only deadweight loss in the LIIO is that high type investment is below first-best.

The high type can potentially improve upon his LIIO payoff. In his preferred pooling equilibrium, the high type chooses the structuring that maximizes speculator effort since this drives prices closer to fundamentals. This is accomplished by bifurcating the cash flow stream into a risky senior debt claim and a residual equity claim. The rational uninformed investors only buy the senior debt claim. The speculator hides behind uninformed demand in the risky debt market, and this is her only source of trading gains. To maximize speculator effort, the optimal information-sensitivity of the senior debt trades off higher per-unit speculator profit against endogenous decreases in uninformed demand.

The high type's preferred pooling equilibrium has an attractive feature socially in that expected

[1] This is an application of a general result from Maskin and Tirole (1992).

[2] See Tirole (2005), Chapter 6, for definitions.

investment is first-best since the entire asset is securitized. However, the speculator acquires costly information and uninformed investors distort their portfolios since they face adverse selection. Thus, from a social welfare perspective the LIIO dominates the high type's preferred pooling equilibrium when risk sharing is sufficiently important, e.g. when uninformed investors face large endowment shocks or when they have high risk-aversion. It is interesting that the LIIO can dominate pooling equilibria since in the traditional corporate finance signaling framework, e.g. Myers and Majluf (1984), it is socially preferable for issuers to invest at first-best, with transfers across types having no direct social cost. However, the traditional argument in favor of pooling abstracts from speculative markets and thus ignores the possibility for costly informed speculation and concomitant distortions in uninformed investors' portfolios.

A striking implication of the model is that the privately informed owner may fail to implement the LIIO, featuring riskless debt, precisely when uninformed investors have high liquidity demand. And this is precisely when a social planner would prefer the LIIO on risk-sharing grounds. To see this, suppose the high type is considering pooling. He knows that when uninformed investors face large endowment shocks (or are highly risk-averse) they are more willing to trade despite facing adverse selection. The higher level of uninformed demand allows the speculator to place larger non-revealing trades, raising her gains. This increases speculator effort and drives prices closer to fundamentals in pooling equilibria. It follows that the high type is better off in a pooling equilibrium cum risky debt precisely when uninformed investors have high liquidity demand. Essentially, the privately informed owner fails to internalize the negative externality he imposes on uninformed investors when he issues risky debt rather than safe debt. Strikingly, the larger this negative externality, the more willing the high type owner is to impose it.

The model also delivers a novel and subtle prediction regarding the role of government-supplied riskless debt, i.e. public liquidity, in increasing social welfare. We extend the baseline model by considering a setting in which the government can issue some riskless debt, but has insufficient debt capacity to satisfy uninformed demand for liquidity. Even in this setting, the government can potentially ensure adequate *aggregate* liquidity. By issuing riskless debt, the government siphons uninformed demand away from any risky security market. But in light of such an endogenous decline in uninformed demand for risky claims, the high type anticipates deeper underpricing if he were to market risky claims, implying he may be better off at the LIIO in which all types market only safe debt. Thus, the model shows that public and private liquidity are strategic complements, with public liquidity crowding-in private liquidity.

Holmström and Tirole (1998) analyze the social welfare benefits of public liquidity in a setting where limits on income verifiability constrain the private supply of stores of value.[3] They consider a setting with hidden action and potential production inefficiencies while we consider a setting with hidden information and potentially inefficient risk sharing. In their model the role of public liquidity is a direct one, in that it increases aggregate liquidity dollar for dollar. In our model public liquidity can have a disproportionate multiplier effect by crowding-in private liquidity.

Gorton and Pennacchi (1990) also analyze the equilibrium supply of riskless claims in a setting where uninformed investors prefer safe storage. However, in their model the issuer does not have private information and it is the uninformed investors who exercise effective control over an intermediary's financial structure. In their setting, uninformed investors carve out a safe debt claim for themselves. In contrast, we show a privately informed issuer has diametrically opposing incentives: He can switch from riskless to risky debt precisely when uninformed investors have high demand for safe storage. Further, while both papers posit a direct benefit to safe government debt as a store of value, we show public liquidity can actually change equilibrium security design and crowd-in private liquidity.

Dang, Gorton and Holmström (2010) also analyze the interplay between security design and risk sharing. They predict privately optimal security design minimizes incentives for information acquisition. In contrast, we show a privately-informed owner may have an incentive to promote information acquisition by speculators since this drives prices closer to fundamentals in noisy rational expectations equilibria.

[3] Tirole (2011) provides a recent survey.

Boot and Thakor (1993) also argue security design can be used to stimulate informed trading.[4] However, the models differ in key assumptions and predictions. First, they assume uninformed investors are pure noise traders, ruling out welfare analysis and our key causal mechanism. In our model, the issuer seeks to utilize *endogenous* changes in uninformed demand. And the liquidity-multiplier effect of public liquidity operates via this mechanism. Second, in their model bifurcation into riskless debt and levered equity stimulates informed speculation because noise traders do not optimize. In our model, such a bifurcation leads to zero information production since rational uninformed investors would boycott the equity market. Third, their equilibrium set is narrower since they rule out signaling via retentions.

In the model of DeMarzo and Duffie (1999), the issuer chooses security design before observing asset value. After the structure is locked-in, the issuer observes asset value and decides how much to sell. Under technical conditions, e.g. monotonicity, debt is an optimal security since its low information-sensitivity results in low price impact. In contrast, we consider a setting where the issuer knows the asset value before choosing security design and retentions. Further, we allow for a speculator to acquire information. Finally, the model of DeMarzo and Duffie is silent on risk sharing since they assume universal risk-neutrality.

Nachman and Noe (1994) analyze a setting, like ours, where the issuer is privately informed at the time the security is designed. In their setting, the scale of investment is fixed, and there is no possibility for separating equilibria. Further, they rule out informed speculation and assume risk-neutral investors. Under technical conditions, e.g. monotonicity, they show firms will pool at a debt contract, since debt minimizes cross-subsidies from high to low types.

Allen and Gale (1988) evaluate optimal security design in a setting with endogenously incomplete asset-backed securities markets. They assume symmetric information, but firms incur a cost for introducing a security. In their economy, equilibrium is constrained Pareto optimal. In the signaling game we consider, equilibrium need not be Pareto optimal.

Our model of price formation extends the tractable models of Maug (1998) and Faure-Grimaud

[4] See also Fulghieri and Lukin (2001).

and Gromb (2004), for example. However, these papers assume pure noise-trading of shares in an all-equity firm. Such setups preclude welfare analysis and rule out our central causal mechanism–endogenous changes in uninformed demand arising from tranching and/or public liquidity.

The remainder of the paper is as follows. Section I describes the economic setting. Section II analyzes the market-making process. Section III determines the structuring preferred by each type. Section IV describes the equilibrium set. Section V compares social welfare across equilibria. Section VI analyzes the effect of public liquidity.

I. Economic Setting

This section describes preferences, endowments, information sets, and timing of the game.

A. Preferences and Endowments

There are three periods, 1, 2, and 3, and a single nonstorable consumption good. Agents receive income endowments and consume in periods 2 and 3. The perishability of the consumption good creates the potential for liquidity scarcity as investors desire stores of value across periods 2 and 3. There is a tangible real asset with type $\tau \in \{L, H\}$, with its original owner ("Owner" below) being the only party endowed with perfect knowledge of the asset type. The asset delivers τ units of the consumption good in period 3 with probability one, with $L \in (0, H)$. The prior probability that $\tau = H$ is $q \in (0, 1)$.

Owner can sell asset-backed securities, since the asset payoff is verifiable. However, income endowments are not verifiable. Thus, the other agents cannot use their income endowments as collateral for borrowing, short-selling or securities flotations. Allen and Gale (1988) and Holmström and Tirole (1998) also rule out unsecured credit based on limited verifiability of income.

Section VI relaxes assumptions as in Holmström and Tirole (1998). There we analyze an extension in which the government has a unique capability to verify and tax a fraction of the final period income endowments. Under this alternative assumption, the treasury can borrow against future tax receipts, with government bonds serving as a public source of liquidity. Of particular interest will be how the introduction of public liquidity affects privately optimal security design and social welfare.

There is a continuum of uninformed investors (UI below) of measure one. By construction, UI are analogous to the "liquidity traders" in the model of Gorton and Pennacchi (1990). They are akin to pension funds and insurance companies in that they are risk-averse and prefer informationally-insensitive stores of value. UI are sufficiently wealthy to buy the underlying asset since each has a period 2 endowment $y_2^{UI} \geq H$. Each UI has an uncertain period 3 endowment $y_3^i \in \{\xi - \phi, \xi\}$ where $\xi \geq \phi$. The fraction of UI drawing $y_3^i = \xi - \phi$ is an independent random variable $\pi \in \{\underline{\pi}, \overline{\pi}\}$ with $0 \leq \underline{\pi} < \overline{\pi} \leq 1$. Each realization of π is equally likely, but the realized value is neither observable nor verifiable. Correlation in UI shocks, here modeled via π, results in uncertain aggregate UI demand. This is a standard feature in noisy rational expectations models since Kyle (1985).[5]

Just prior to the trading of securities taking place in period 2, each UI privately observes the size of his period 3 endowment. UI are risk-neutral over period 2 consumption (c_2) and risk-averse over period 3 consumption (c_3). We follow the tractable specification of risk-aversion employed by Dow (1998) in that final period utility is piecewise linear, with a concave kink.[6] UI are indexed by the intensity of risk-aversion as captured by a privately-known preference parameter θ. An UI with preference parameter θ has the utility function:

$$u(c_2, c_3; \theta) \equiv c_2 + \theta \min(0, c_3 - \xi). \tag{1}$$

Notice each UI is averse to period 3 consumption falling below the critical level ξ. This creates a storage motive whenever they face a low final period endowment. The degree of aversion to low consumption is captured by the idiosyncratic parameter θ. The θ have support $\Theta \equiv [1, \theta^{\max}]$, ensuring gains from trade in the absence of adverse selection. Throughout, $\theta^{\max}$ is assumed to be sufficiently high such that UI demand is positive for at least one security.[7] The θ parameters have density f with cumulative density F. This distribution is common knowledge. It has no atoms, with

[5] To see this, note i.i.d. shocks hitting a continuum of UI would result in constant noise trading in Kyle (1985).

[6] Other smooth utility functions could be assumed, with more complex aggregate UI demands.

[7] This avoids the need to continually check upper limits of integration when computing their demand.

f strictly positive and continuously differentiable.

Given the preferences described in equation (1), it is apparent that any UI facing a low terminal endowment would like to invest in a riskless asset delivering ϕ in period 3, bringing his final period consumption up to the critical level ξ. Initially, the main question addressed is whether the owner will market such a riskless asset in equilibrium. In an extension, we consider how the introduction of a riskless government bond affects equilibrium security design and social welfare.

In addition to the UI, there is a speculator S with utility $c_2 + c_3$. She is endowed with $y_2^S \geq H$ units of the numeraire and can afford to buy the entire asset. Her terminal endowment is normalized at zero. The speculator is unique in receiving a signal regarding τ and can exert costly effort to increase signal precision. Letting $s \in \{s_L, s_H\}$ denote the signal, S chooses $\sigma \equiv \Pr(s = s_\tau)$ from the feasible set $[1/2, 1]$. Iff $\sigma > 1/2$ the signal is informative. The speculator's non-pecuniary effort cost function e is twice continuously differentiable, strictly increasing and convex, with

$$\begin{aligned} \lim_{\sigma \downarrow \frac{1}{2}} e(\sigma) &= \lim_{\sigma \downarrow \frac{1}{2}} e'(\sigma) = 0 \\ \lim_{\sigma \uparrow 1} e'(\sigma) &= \infty. \end{aligned}$$

The final set of agents is a measure one continuum of market-makers (MM below) having utility $c_2 + c_3$. Their period 2 endowment is $y_2^{MM} \geq H$, so they too can afford to buy the entire asset. Their terminal endowment is $y_3^{MM} \geq 0$.

Owner has utility $c_2 + c_3$. Owner has access to a linear technology allowing him to convert each unit of the good received from investors in period 2 into $\beta > 1$ units of the good. In contrast to the original tangible real asset, the payoff on the new investment is not verifiable and cannot be pledged. The model is a standard ABS setup with securities backed by an asset-in-place, as in DeMarzo and Duffie (1999).

B. The Security Design Game

Since Owner is privately informed regarding τ, his choice of retention and security design in period 1 represents a signaling game. Maskin and Tirole (1992) and Tirole (2005) show the equilibrium set of signaling games can be refined and Pareto-improved (from the perspective of the privately informed party) by altering the fundraising mechanism in a way that expands the set of feasible initial actions. We modify their approach to allow for market-making, informed speculation, and endogenous uninformed security demand.

The game starting in period 1 is called the *Security Design Game.* It is a signaling game played between Owner and investors. This game begins with Owner registering a menu containing two allocations $\{\Sigma_L, \Sigma_H\}$ that he will choose from subsequently. To fix ideas, one may think of this menu registration as approximating a shelf-registration. In a shelf-registration, a prospective issuer undergoes a single compliance stage with a regulator, registering a number of securities. After the compliance stage, the issuer is free to market any of the registered securities within some time period. This procedure is commonly used in the U.S., for example. DeMarzo and Duffie (1999) also model shelf-registrations. However, they consider an issuer with interim private information, while we consider an issuer with ex ante private information.

Each *allocation* on the menu is a vector stipulating payoffs for all claimholders (including Owner) as a function of the verified asset payoff in period 3. In a *pooling equilibrium*, both types propose a trivial menu in which $\Sigma_L = \Sigma_H$, implying the allocation selection reveals nothing about type.[8] In a *separating equilibrium,* $\Sigma_L \neq \Sigma_H$ and type L weakly prefers Σ_L to Σ_H, and vice-versa, so the choice from the menu reveals the true type.

All agents must have a belief regarding the asset type in response to any menu registered by Owner, including those off the equilibrium path. Thus, the registered menu is itself a signal. Next, Owner chooses an allocation from the menu. After observing this choice, the other agents revise beliefs. The equilibrium concept is perfect Bayesian equilibrium. Robustness of alternative equilibria to the Intuitive Criterion will be discussed.

C. The Market-Making Game

In period 2, play passes to a continuation game, labeled the *Market-Making Game.*[9] This game is in the spirit of Kyle (1985) and Glosten and Milgrom (1985) in that atomistic market-

[8] Nontrivial menus where both types opt for the same selection are subsumed in trivial menus and thus ignored.

[9] This game is similar to that presented in Maug (1998), but we have endogenous security design and UI demand.

makers compete à la Bertrand in bidding for securities until a market-clearing price is determined. However, we depart from these canonical noise-trading models of price determination by considering endogenous trading decisions by all investors.

In any separating equilibrium, τ is revealed, so MM set prices equal to true payoffs and S does not exert costly effort. In any pooling equilibrium the Market-Making Game is a signaling game played between the privately informed speculator and the MM. Consistent with Bayes' rule, the UI and MM enter this continuation game holding their prior belief that $\tau = H$ with probability q.

The Market-Making Game begins with S choosing signal precision σ at cost $e(\sigma)$. Her choice of σ is not observed, but is correctly inferred in equilibrium. Then S privately observes her signal $s \in \{s_L, s_H\}$. Next, each UI privately observes whether he faces a low endowment in period 3. Simultaneous market orders are then submitted by S and each UI. The MM compete and set prices after having observed aggregate demands in all markets, with no market segmentation. At this information set, MM must have a belief about the speculator's signal for any aggregate demand configuration. Effectively, MM clear markets, buying any securities not purchased by S or the UI.

Since MM cannot short, we impose the following technical assumption:

$$A1 : \phi \leq \frac{L}{2}.$$

The role of Assumption 1 is as follows. Aggregate UI demand is weakly increasing in the size of the endowment shock ϕ. Therefore, to avoid the possibility of aggregate demand exceeding supply for any security, ϕ must be sufficiently small. By construction, even if the asset has the low final payoff L, this payoff is still sufficient to meet the storage demands of the UI. That is, we are considering a setting where the private owner has the ability to satisfy investor storage demand, but may be unwilling to do so.

To summarize, the sequence of events is as follows. At time 1 Owner privately observes τ, publicly registers a securities menu, and then publicly chooses an allocation from that menu. At time 2: S exerts effort at cost $e(\sigma)$ and privately observes her signal; each UI privately observes whether he faces a negative endowment shock; UI and S simultaneously submit market orders; MM observe aggregate demands and set prices competitively. In period 3 claimholders receive their contractually specified payments based upon the verified asset payoff.

II. Market-Making Game Equilibrium

In any separating equilibrium, the owner's choice of allocation from his registered menu reveals the asset type and the MM simply set the price of each marketed claim equal to its true payoff. Therefore, the focus of this section is price determination in pooling equilibria. Suppose Owner markets no more than two claims in equilibrium. Below this is shown to be without loss of generality. Denote the two marketed claims as A and B, with (A_L, A_H) and (B_L, B_H) denoting their respective period 3 payoffs as functions of the verified asset payoff. Each marketed payoff must be weakly positive since the other agents' endowments are not verifiable. Heuristically, this captures investor limited liability. Security B is treated as the default security if only one security is sold. Finally, define the total payoff on marketed securities as $l \equiv A_L + B_L$ and $h \equiv A_H + B_H$. For example, if the asset is fully securitized then the marketed bundle $(l, h) = (L, H)$.

Following Nachman and Noe (1994) and DeMarzo and Duffie (1999), attention is confined to marketed securities that are weakly increasing in the period 3 asset payoff. Monotonicity is assumed for three reasons. First, with monotone securities, the informed speculator's optimal strategy takes a simple and intuitive form: he buys when he receives a positive signal. Second, the majority of marketed claims are monotone. Finally, if one of the securities was decreasing, other claimants would benefit from making clandestine contributions to the asset. As argued by DeMarzo and Duffie, only securities with monotone payoffs will be issued if such hidden contributions are feasible.

The key measure of a security's information-sensitivity in this model is its relative payoff across the two total asset payoff states. Whenever Owner markets to outside investors two claims, our notational convention is to treat security A as the more information-sensitive claim with:

$$\frac{A_L}{A_H} \leq \frac{l}{h} \leq \frac{B_L}{B_H}. \tag{2}$$

To ensure the issuer faces a non-trivial problem in choosing an optimal structuring and to even allow for the possibility of the marketing of riskless claims, attention is confined to marketed payoff bundles (l, h) such that $l > 0$. In fact, $l = 0$ is not a feature of the separating equilibrium, nor the preferred pooling equilibrium of either type, so our characterization of the equilibrium set remains valid.

A. Price Setting and Expected Revenues

Table 1 depicts order flow on the equilibrium path. We conjecture and verify UI demand is confined to security B. Intuitively, UI buy only the least information-sensitive claim in order to minimize trading losses due to adverse selection. Consequently, S can only buy security B given the lack of any cover provided by UI demand in the market for A. As shown in Table 1, aggregate UI demand can be expressed as πX_B for either realization of π. This is because the individual UI cannot condition their own trades on the unobservable realized value of π. The UI demand variable X_B will be determined endogenously.

Since S cannot short-sell, her optimal strategy is to place a buy order for B if and only if she receives the signal s_H. In order to confound the MM, the speculator must set her buy order equal to the difference between high and low aggregate UI demands, or $(\overline{\pi} - \underline{\pi})X_B$. As shown in Table 1, this ensures the MM will be confounded when they observe the aggregate demand $\overline{\pi}X_B$ being unsure of whether this arises from $(\pi, s) = (\overline{\pi}, s_L)$ or $(\underline{\pi}, s_H)$. In contrast, the MM will be able to infer that the speculator has received a positive signal when the observed aggregate demand is $\overline{\pi}X_B$, and a negative signal when the observed demand is $\underline{\pi}X_B$.

Table 1 depicts the possible equilibrium aggregate demands for security B, with demand for security A equal to 0. Since there is no market segmentation, MM use the demand for security B in setting prices for securities A and B. On the equilibrium path, MM set prices as follows:

$$P_A(D_B) = A_L + (A_H - A_L)\Pr[\tau = H|D_B] \quad \forall \quad D_B \in \{\underline{\pi}X_B, \overline{\pi}X_B, (2\overline{\pi} - \underline{\pi})X_B\} \tag{3}$$

$$P_B(D_B) = B_L + (B_H - B_L)\Pr[\tau = H|D_B] \quad \forall \quad D_B \in \{\underline{\pi}X_B, \overline{\pi}X_B, (2\overline{\pi} - \underline{\pi})X_B\}.$$

Using Table 1, MM beliefs regarding the signal received by S can be mapped directly to beliefs regarding τ:

$$\begin{aligned}
\Pr[\tau &= H|D_B = (2\overline{\pi} - \underline{\pi})X_B] = \frac{q\sigma}{1 - q - \sigma + 2q\sigma} \\
\Pr[\tau &= H|D_B = \overline{\pi}X_B] = q \\
\Pr[\tau &= H|D_B = \underline{\pi}X_B] = \frac{q(1-\sigma)}{q + \sigma - 2q\sigma}.
\end{aligned} \tag{4}$$

Prices increase monotonically in aggregate demand with:

$$P_i[(2\overline{\pi} - \underline{\pi})X_B] \geq P_i(\overline{\pi}X_B) \geq P_i(\underline{\pi}X_B) \quad \forall i \in (A, B). \tag{5}$$

To support the PBE conjectured in Table 1 it suffices to verify S has no incentive to deviate. To that end, off the equilibrium path MM form adverse beliefs from the perspective of S, inferring that any deviation arises from S receiving signal s_H. Such beliefs imply the following pricing:

$$(D_A, D_B) \notin 0 \times \{\underline{\pi}X_B, \overline{\pi}X_B, (2\overline{\pi} - \underline{\pi})X_B\} \Rightarrow P_i = P_i[(2\overline{\pi} - \underline{\pi})X_B] \quad \forall i \in (A, B). \tag{6}$$

S has no incentive to change her trading strategy when confronted with such beliefs.

While such beliefs are sufficient to support the conjectured PBE of the market-making game, it is worthwhile to briefly discuss their plausibility. Note that any deviation must be due to S placing a strictly positive order. The chosen specification of beliefs is predicated on the notion that MM should view such an order as being placed by S after having observed s_H. After all, if a negative signal is received, S incurs a loss from buying securities unless MM form the most favorable beliefs from her perspective, which would entail $\Pr(s = s_H) = 0$. For this reason, the posited PBE of the market-making game survives the Intuitive Criterion.

The expected revenue received by the high type Owner is:

$$E[P_A + P_B|\tau = H] = l + (h - l)\left[\frac{\sigma \Pr[\tau = H|(2\overline{\pi} - \underline{\pi})X]}{2} + \frac{(1-\sigma)\Pr[\tau = H|\underline{\pi}X]}{2} + \frac{\Pr[\tau = H|\overline{\pi}X]}{2}\right]. \tag{7}$$

Equation (7) can be rewritten as:

$$\begin{aligned} E[P_A + P_B|\tau &= H] = hZ(\sigma) + l[1 - Z(\sigma)] \\ Z(\sigma) &\equiv \frac{1}{2}\left[\frac{q\sigma^2}{1-q-\sigma+2q\sigma} + \frac{q(1-\sigma)^2}{q+\sigma-2q\sigma} + q\right]. \end{aligned} \tag{8}$$

The endogenous variable Z plays a critical role, proxying for informational efficiency. Specifically, the wedge between the high type's expected revenue and the fundamental value of his marketed securities (h) is equal to $(1 - Z)(h - l)$.

The expected revenue of the low type is:

$$\begin{aligned} E[P_A + P_B|\tau &= L] = hz(\sigma) + l[1 - z(\sigma)] \\ z(\sigma) &\equiv \left(\frac{q}{1-q}\right)[1 - Z(\sigma)]. \end{aligned} \tag{9}$$

Lemma 1 shows high type benefits from higher speculator signal precision since this drives prices closer to fundamentals. All but the most important proofs are presented in the appendix.

Lemma 1 *The expected revenue of the owner of a high (low) value asset is increasing (decreasing) in the speculator's signal precision.*

From Lemma 1 it follows that Z is increasing in σ, with

$$\begin{aligned} Z\left(\frac{1}{2}\right) &= q \\ Z(1) &= \frac{1+q}{2}. \end{aligned} \tag{10}$$

If the speculator does not exert effort ($\sigma = 1/2$), $Z = q$ and both types have the same expected revenue equal to $qh + (1 - q)l$.

B. Speculator Effort and Uninformed Demand in Equilibrium

From Table 1 it follows the expected trading gain of the speculator is:

$$\begin{aligned} G(\sigma, X_B) &= \left[\begin{array}{c} \frac{q\sigma}{2}\left\{[B_H - P_B((2\overline{\pi} - \underline{\pi})X)] + [B_H - P_B(\overline{\pi}X)]\right\} \\ + \frac{(1-q)(1-\sigma)}{2}\left\{[B_L - P_B((2\overline{\pi} - \underline{\pi})X)] + [B_L - P_B(\overline{\pi}X)]\right\} \end{array}\right](\overline{\pi} - \underline{\pi})X_B \\ &= \frac{1}{2}q(1-q)(2\sigma - 1)(B_H - B_L)(\overline{\pi} - \underline{\pi})X_B. \end{aligned} \tag{11}$$

Her incentive compatible signal precision (σ_{ic}) satisfies:

$$e'(\sigma_{ic}) = G_\sigma(\sigma, X) = q(1-q)(B_H - B_L)(\overline{\pi} - \underline{\pi})X_B. \tag{12}$$

Letting $\Psi : \Re_+ \rightarrow [1/2, 1)$ denote the inverse of e', we have the following lemma.

Lemma 2 *The incentive compatible signal precision of the speculator is*

$$\sigma_{ic} = \Psi[q(1-q)(B_H - B_L)(\overline{\pi} - \underline{\pi})X_B].$$

The next step is to determine UI security demand. To do so, we analyze the optimal portfolio of each UI and then aggregate. If $y_3^i = \xi$, investor i has no need for intertemporal storage and has zero security demand. So we focus on the demand of those UI facing the low terminal endowment of $\xi - \phi$. To this end, let $x_j^*(\theta)$ denote demand for security j by an individual UI of type θ conditional upon his facing $y_3^i = \xi - \phi$. Note that each individual UI bases demand on his own terminal endowment, but not the actual realized value of π since he cannot observe π. It follows that, consistent with Table 1, aggregate UI demand for security j can be written as

$$\pi \int_1^{\theta^{\max}} x_j^*(\theta) f(\theta) d\theta \equiv \pi X_j. \tag{13}$$

Using Table 1 we arrive at the following expected prices computed by individual UI facing negative endowment shocks:

$$\begin{aligned} E[P_A | y_3^i &= \xi - \phi] = qA_H + (1-q)A_L + q(1-q)(2\sigma - 1)(A_H - A_L)(\overline{\pi} - \underline{\pi})/(\overline{\pi} + \underline{\pi}) \quad (14) \\ E[P_B | y_3^i &= \xi - \phi] = qB_H + (1-q)B_L + q(1-q)(2\sigma - 1)(B_H - B_L)(\overline{\pi} - \underline{\pi})/(\overline{\pi} + \underline{\pi}) \end{aligned}$$

Equation (14) shows UI face adverse selection, with expected prices exceeding expected payoffs, unless the security is riskless or the speculator exerts no effort. Further, consistent with intuition, the severity of adverse selection increases in the precision of the speculator's signal.

The intuition for UI demand is straightforward. If θ for an individual UI is sufficiently low, his demand for both securities is zero, with adverse selection dominating his storage motive. For intermediate values of θ, the UI partially insures in the sense of buying enough units of security B

such that c_3 is ξ if $\tau = H$, implying $c_3 < \xi$ if $\tau = L$. Finally, if θ is sufficiently high, the investor completely insures in the sense of purchasing enough units of security B such that c_3 is ξ even if $\tau = L$, implying $c_3 > \xi$ if $\tau = H$. Formally, in the appendix it is shown that:

$$\begin{aligned}
\theta &\leq \frac{E[P_B|y_3^i = \xi - \phi]}{qB_H + (1-q)B_L} \equiv \theta_1 \Leftrightarrow (x_A^*, x_B^*) = (0,0) \\
\theta &\geq \frac{E[P_B|y_3^i = \xi - \phi]}{(1-q)B_L} \equiv \theta_2 \Leftrightarrow (x_A^*, x_B^*) = \left(0, \frac{\phi}{B_L}\right) \\
\theta &\in (\theta_1, \theta_2) \Rightarrow (x_A^*, x_B^*) = \left(0, \frac{\phi}{B_H}\right).
\end{aligned} \tag{15}$$

Letting $\kappa \equiv B_H/B_L$, and integrating over θ we have the following lemma pinning down UI demand.

Lemma 3 *There is zero uninformed demand for the more information-sensitive claim A. Aggregate uninformed demand for security B is* πX_B *where*

$$\begin{aligned}
X_B &= \frac{\phi}{B_L}\left[1 - \frac{F(\theta_1)}{\kappa} - \frac{(\kappa-1)F(\theta_2)}{\kappa}\right] \\
\theta_1 &= 1 + \frac{q(1-q)(\kappa-1)(2\sigma-1)(\overline{\pi} - \underline{\pi})/(\overline{\pi} + \underline{\pi})}{1 + q(\kappa-1)} \\
\theta_2 &= 1 + \frac{q\kappa + q(1-q)(\kappa-1)(2\sigma-1)(\overline{\pi} - \underline{\pi})/(\overline{\pi} + \underline{\pi})}{1-q}.
\end{aligned} \tag{16}$$

Lemma 3 can be contrasted with a result obtained by Boot and Thakor (1993). In their model, speculators make trading gains in the information-sensitive levered equity claim. In our model, UI optimally save using only the least information-sensitive claim, so the speculator is unable to make profits in that market. A similar effect is operative in the model of Gorton and Pennacchi (1990), since they too predict UI will buy the claim with low information-sensitivity.

We can pull together the incentive compatible signal precision from Lemma 2 and the UI demand from Lemma 3 to verify existence of a unique PBE in the market-making game.

Proposition 1 *In the market-making game following the registration of a pooling menu, if* $B_H = B_L$ *the speculator exerts zero effort* ($\sigma = 1/2$) *and all uninformed investors with a low endowment* *purchase ϕ/B_i units of this riskless claim. For $B_H > B_L$, there exists a unique equilibrium pair (σ^{eq}, X_B^{eq}) satisfying*

$$\begin{aligned} \sigma_{ic}(X_B^{eq}) &= \sigma^{eq} \in \left(\frac{1}{2}, 1\right) \\ X_B(\sigma^{eq}) &= X_B^{eq} \in \left(0, \frac{\phi}{B_L}\right). \end{aligned}$$

III. Preferred Security Designs by Type

Section IV pins down the set of equilibrium security designs. As a precursor, this section considers which security designs would be preferred by each type in the event that types pool and market an arbitrary payoff bundle (l, h).

Lemma 4 is a useful simplifying result stating that we may confine attention to one or two marketed securities. Intuitively, all UI demand migrates to the marketed security with the lowest information-sensitivity, so any remaining securities can be priced as if they were rolled into a single claim.

Lemma 4 *Any payoff outcome attainable with three or more marketed securities is attainable with two marketed securities.*

Consider first the low type pondering his preference for the bifurcation of the payoff bundle (l, h) in the event of pooling. Lemma 1 shows the low type prefers zero speculator effort. And Proposition 1 shows this occurs if and only if the bifurcation results in the marketing of a riskless security (regardless of the riskiness of the remaining security). Thus, the low type's preferred pooling equilibrium entails any structuring in which a riskless claim is marketed.

Consider next the high type's preference for the bifurcation of the bundle (l, h) in the event of pooling. Lemma 1 shows his expected revenue is increasing in the equilibrium signal precision of the speculator. Substituting the UI demand function from Lemma 3 into the IC signal precision from Lemma 2 yields the following equation implicitly defining σ^{eq}:

$$\sigma^{eq} = \Psi[(\overline{\pi} - \underline{\pi})q(1-q)\phi M(\kappa, \sigma^{eq})] \tag{17}$$

$$M(\kappa, \sigma) \equiv (\kappa - 1)\left[1 - \frac{F[\theta_1(\kappa, \sigma)]}{\kappa} - \frac{(\kappa-1)F[\theta_2(\kappa, \sigma)]}{\kappa}\right]. \tag{18}$$

Equation (17) shows that the variable M is the key endogenous variable determining equilibrium speculator effort. And equation (18) shows that M is simply the product of the speculator's per-unit trading gain and the endogenous UI demand variable X_B. As intuition would suggest, endogenous changes in UI demand leads to an inherent tradeoff between these two. In particular, increasing per-unit profits for the speculator reduces UI trading volume.

From the implicit function theorem it follows:

$$\frac{d\sigma^{eq}}{d\kappa} = \frac{\Psi'(\cdot)(\overline{\pi} - \underline{\pi})q(1-q)\phi\frac{\partial M}{\partial \kappa}}{1 - \Psi'(\cdot)(\overline{\pi} - \underline{\pi})q(1-q)\phi\frac{\partial M}{\partial \sigma}}. \tag{19}$$

Since Ψ is an increasing function and M is decreasing in its second argument it follows σ^{eq} increases (decreases) with κ iff M increases (decreases) with κ. This yields the following lemma.

Lemma 5 *Given a marketed bundle (l, h), the maximum pooling equilibrium signal precision $\sigma^*(h/l)$ is attained with $B_L^* = l$ and $B_H^* = l\kappa^*(h/l)$ where*

$$\kappa^*(h/l) \in \arg\max_{\kappa \leq h/l} \quad M[\kappa, \sigma^*(h/l)].$$

In the preceding lemma, the constraint $\kappa \leq h/l$ ensures claim B is, in fact, the low information-sensitivity claim that will be traded by the UI. Lemma 6 presents a sufficient condition for global concavity of M and the existence of a unique preferred informational sensitivity for the high type in pooling equilibria.

Lemma 6 *If the cumulative distribution function (F) for uninformed investors' preference parameter θ is weakly convex, then the speculator gain function $M(\cdot, \sigma)$ is strictly concave for all $\sigma \in (1/2, 1]$ and the high type has a unique preferred informational sensitivity $\kappa^*(h/l)$ for all $h/l \in (1, \infty)$. If*

$h/l \geq \kappa^*(\infty)$, *then* $\kappa^*(h/l) = \kappa^*(\infty)$ *and* $\sigma^*(h/l) = \sigma^*(\infty)$. *If* $h/l < \kappa^*(\infty)$, *then* $\kappa^*(h/l) = h/l$ *and* $\sigma^*(h/l) < \sigma^*(\infty)$.

The intuition behind Lemma 6 is as follows. For F convex, marginal increases in κ result in ever larger reductions in UI demand. Further, the benefit to the speculator of the increase in per-unit profits stemming from an increase in κ is spread over a progressively smaller trading base. Consequently, the maximand (the product of per-unit speculator gains and UI trading volume) is strictly concave. The remainder of the paper adopts two technical assumptions:

$$A2 \; : \; F \text{ is weakly convex.}$$

$$A3 \; : \; \frac{H}{L} > \kappa^*(\infty).$$

Assumption 2 ensures a concave programme and Assumption 3 ensures the high type's preferred informational sensitivity is interior if the underlying asset is fully securitized.

To better understand the factors determining the high type's preferred informational sensitivity, we differentiate M to obtain:

$$\begin{aligned} M_\kappa(\kappa, \sigma) &= \left[1 - \frac{F(\theta_1)}{\kappa} - \frac{(\kappa-1)F(\theta_2)}{\kappa}\right] \\ &\quad - \left[\frac{\kappa - 1}{\kappa}\right]\left[\frac{F(\theta_2) - F(\theta_1)}{\kappa} + f(\theta_1)\frac{\partial \theta_1}{\partial \kappa} + (\kappa - 1) f(\theta_2)\frac{\partial \theta_1}{\partial \kappa}\right] \\ \frac{\partial \theta_1}{\partial \kappa} &= \frac{q(1-q)(2\sigma - 1)(\overline{\pi} - \underline{\pi})/(\overline{\pi} + \underline{\pi})}{[1 + q(\kappa - 1)]^2} > 0 \\ \frac{\partial \theta_2}{\partial \kappa} &= \left[\frac{q}{1-q} + q(2\sigma - 1)(\overline{\pi} - \underline{\pi})/(\overline{\pi} + \underline{\pi})\right] > 0. \end{aligned} \tag{20}$$

The first term in (20) captures the incentive benefit from increasing informational sensitivity (via κ), as it increases the speculator's per-unit trading gain. The negative term captures the cost of increasing informational sensitivity in terms of reducing equilibrium uninformed demand, behind which the speculator hopes to hide her trading. Canceling terms one obtains the following first-order condition capturing the optimal trade-off between per-unit speculator gains and endogenous changes in UI demand:

$$1 - \frac{F(\theta_1)}{\kappa^{**}} - \frac{(\kappa^{**} - 1)F(\theta_2)}{\kappa^{**}} = \left[\frac{\kappa^{**} - 1}{\kappa^{**}}\right]\left[\frac{F(\theta_2) - F(\theta_1)}{\kappa^{**}} + f(\theta_1)\frac{\partial \theta_1}{\partial \kappa} + (\kappa^{**} - 1) f(\theta_2)\frac{\partial \theta_2}{\partial \kappa}\right]. \tag{21}$$

This establishes the following proposition characterizing the high type's preferred bifurcation of marketed claims in a pooling equilibrium with full securitization.

Proposition 2 *If the entire asset is securitized, the high type's preferred pooling equilibrium bifurcation consists of a low-information-sensitivity claim such that* $B_L^* = L$ *and* $B_H^* = L\kappa^{**}$*, where* $\kappa^{**} > 1$ *is the unique solution to equation (21). The second residual claim attracts zero aggregate uninformed demand. All informed trading gains are derived in the market for the low information-sensitivity claim.*

The following corollary shows each type's preferred pooling structuring can be achieved by combining standard securities.

Corollary *(Full Securitization) The high type's preferred pooling equilibrium bifurcation maximizes speculator effort and consists of a risky senior tranche with face value* $L\kappa^{**}$*, where* κ^{**} *is the unique solution to equation* (21)*, and a residual junior tranche. The low type's preferred pooling equilibrium bifurcation reduces speculator effort to zero via the sale of a riskless senior claim with face value* L *and a residual junior tranche.*

When analyzing social welfare, we will be particularly interested in whether and how the privately informed asset owner will respond to increases in the size of endowment shocks hitting the UI. Foreshadowing that analysis, consider again the high type's preferred pooling equilibrium bifurcation. Applying the implicit function theorem to the equilibrium condition (17) we have

$$\frac{d\sigma^{eq}}{d\phi} = \frac{\Psi'(\cdot)(\overline{\pi} - \underline{\pi})q(1-q)M}{1 - \Psi'(\cdot)(\overline{\pi} - \underline{\pi})q(1-q)\phi\frac{\partial M}{\partial \sigma}} > 0. \tag{22}$$

Importantly, equilibrium signal precision is increasing in the size of UI endowment shocks. Intuitively, individual and aggregate UI demand for security B increases linearly in ϕ. In turn, increases in UI demand allow the speculator to make larger profits given her ability to place larger masking trades. Anticipating, the comparative static in (22) will have important consequences for understanding potential conflicts between private and public objectives in security design.

Figure 1 provides further intuition regarding the high type's preferred bifurcation. Recall, aggregate UI demand is πX_B where $\pi \in \{\underline{\pi}, \overline{\pi}\}$. When she receives a positive signal, the speculator places a buy order of size $(\overline{\pi} - \underline{\pi})X_B$. Figure 1 plots the endogenous UI demand factor X_B.[10] UI demand for security B declines monotonically in informational sensitivity (κ) since increases in κ induce marginal UI to either forego purchase of the security (θ_1 increasing) or to purchase less units (θ_2 increasing). The figure also shows that increases in the size of UI endowment shocks (ϕ) induce rightward shifts in aggregate UI demand.

Figure 2 plots ϕM, the key term in equation (17) pinning down equilibrium signal precision. As shown in the figure, the high type's preferred κ value is independent of ϕ. However, higher ϕ values result directly in higher values of ϕM, which implies higher equilibrium speculator effort (σ^{eq}). Again, the intuition is that larger UI endowment shocks stimulate informed trading by allowing the speculator to place larger buy orders.

IV. Equilibrium Security Designs

This section characterizes the set of perfect Bayesian equilibria. We begin with an analysis of the Low Information Intensity Optimum.

A. The Low Information Intensity Optimum

The LIIO is that pair of incentive compatible, profitable type-by-type, allocations (Σ_L, Σ_H) maximizing high type utility. The LIIO minimizes the low type's incentive to mimic the high type by giving him his first-best allocation in which he retains zero interest in the asset. This allows the low type to invest at first-best and achieve utility βL.

In the LIIO, there is no need for the high type to sell more than one public security since marketed securities are priced at fundamental value given type is revealed. In the LIIO, the high

[10] The figure assumes the θ are uniformly distributed on $[1, 6]$, $q = 1/2$, $\underline{\pi}= .20$, and $\overline{\pi} = .80$.

type owner retains on his own balance sheet a security R with state-contingent payoffs solving:

$$\begin{aligned}
& \max_{(R_L,R_H)} \quad R_H + \beta(H - R_H) \\
& s.t. \\
NM \quad : \quad & \beta L \geq R_L + \beta(H - R_H) \\
& Limited\ Liability \\
& Monotonicity.
\end{aligned}$$

To determine the LIIO, we first ignore monotonicity constraints and then verify they are slack. Clearly, in this relaxed program the optimal policy is to loosen the non-mimicry constraint to the maximum extent by setting $R_L = 0$. Intuitively, if the low type were to mimic, he would receive zero payoff on his retained security. Clearly, NM must bind at the optimum, implying $R_H = H - L$. Thus, in the LIIO both types only market safe debt with face value L, with the high type holding a residual junior claim on his own balance sheet.

Proposition 3 *In the Low Information Intensity Optimum, the only marketed security for both types is a safe debt claim with face value L.*

As a purely technical matter keeping our terminology consistent with standard terminology and also allowing us to draw directly upon standard results in signaling games, e.g. Tirole (2005), we note the LIIO is properly understood as a separating menu since the payoff vector $\Sigma_L \neq \Sigma_H$. It is thus distinct from pooling menus, which have $\Sigma_L = \Sigma_H$. To see this, note that under Σ_L the low type surrenders any claim to residual cash flow net of L, and is willing to do so since he knows any such claim to be worthless. That is, the low type's retained security has state-contingent payoffs $(0, 0)$. In contrast, under Σ_H the final period payoff on the owner's retained security is $(0, H - L)$. As is standard, in the LIIO the low type is just indifferent between Σ_L and Σ_H, while the high type strictly prefers Σ_H to Σ_L. The LIIO is the best possible separating allocation for both types, subject to the constraint that market-makers break even on a type-by-type basis.

The intuition behind Proposition 3 is as follows. In the LIIO the low type would always mimic if the high type were to sell risky debt with face value greater than L, since then the low type would benefit from overpricing. Therefore, the best the high type can do is to get the maximum funding possible subject to zero informational-sensitivity. Debt with face value L achieves this objective. In the LIIO, the high type experiences a loss relative to symmetric information equal to $(\beta - 1)(H - L)$. As in the model of Myers and Majluf (1984), in the LIIO the high type invests less than first-best. The socially attractive feature of the LIIO is that it achieves first-best insurance for the UI who can use the marketed riskless debt claim, issued by either type, as a safe savings vehicle. And further, the speculator does not exert socially wasteful effort in the LIIO.

B. The Equilibrium Set

The following lemma, an application of a general signaling game result due to Maskin and Tirole (1992), provides a general characterization of the set of PBE.

Lemma 7 *The set of perfect Bayesian equilibria of the Security Design Game includes the Low Information Intensity Optimum (LIIO) and any pooling menu that weakly Pareto dominates the LIIO from the perspective of both owner types.*

Proof. Consider first supporting the LIIO. If beliefs were set to $\Pr[\tau = H] = 0$ in response to any deviating menu, then no such deviation is profitable. Suppose next there is a contract menu weakly Pareto dominating the LIIO. If beliefs were set to $\Pr[\tau = H] = 0$ in response to any deviating menu, the deviator would get weakly less than his LIIO payoff and the deviation is not profitable. ■

From Lemma 7 it follows the LIIO is actually the unique PBE if there is no menu that makes both owner types weakly better off. By construction, no separating contract can make both types better off than the LIIO. Thus, we need only consider pooling equilibria.[11] The low type will often be better off in pooling equilibria since he benefits from overpricing, so the first step in verifying whether pooling can be supported is to determine the high type's preferred pooling contract.

[11] Tirole (2005) allows pre-commitments to cross-subsidies which are impossible in competitive securities markets.

We characterize the high type's preferred pooling contract in two steps. First, Lemma 5 characterized his preferred bifurcation of arbitrary marketed cash flows (l, h) respecting monotonicity and limited liability. Given any such marketed pair (l, h), let $\sigma^*(h/l)$ denote the resulting maximum feasible σ. Next, (l, h) are optimized in light of their effect on speculator effort incentives. The following lemma shows the low payoff is fully securitized in the high type's preferred pooling equilibrium.

Lemma 8 *In the high type's preferred pooling contract, the security retained by the original asset owner has a payoff of zero if the realized asset payoff is low.*

The intuition for Lemma 8 is that the high type places zero value on cash flow rights in the event of a low realized asset payoff since he knows this is a probability zero event for him. For this same reason he also knows that the market will overvalue any marketed claim delivering positive payoffs in the low state. Thus, the high type prefers to market low state payoffs.

Let $\overline{U}(l, h)$ denote the high type's expected utility when the bundle (l, h) is marketed in a pooling equilibrium with the marketed payoffs bifurcated according to Lemma 5. From Lemma 8 it follows the high type finds it optimal to set $l = L$. The optimal value of the high state marketed payoff in a pooling equilibrium, call it h^*, solves:

$$h^* \in \max_{h \in [L,H].} \quad \overline{U}(L, h) \equiv H - h + \beta[L + (h - L)Z(\sigma^*(h/L))]. \tag{23}$$

If the high type opts to pool at a structuring in which only safe debt is securitized, his utility $\overline{U}(L, L)$ is just equal to what he obtains in the the LIIO. Further, if $\beta Z[\sigma^*(H/L)] \leq 1$ the preferred pooling contract for the high type is to pool at a structuring in which only safe debt is marketed. To see this, note:

$$\beta Z[\sigma^*(H/L)] \leq 1 \Rightarrow \overline{U}(L, h) \leq \overline{U}(L, L) = H - L + \beta L \quad \forall \quad h \in (L, H]. \tag{24}$$

That is, if $\beta Z \leq 1$, the high type cannot achieve higher utility from pooling than he achieves at the LIIO. And it follows from Lemma (7) that in such cases the LIIO is the unique PBE. We have the following proposition.

Proposition 4 *If feasible speculator effort and corresponding market informational efficiency are low, with $Z[\sigma^*(H/L)] \leq 1/\beta$, the unique perfect Bayesian equilibrium of the Security Design Game is the Low Information Intensity Optimum in which both types market only safe debt with face value L.*

The intuition for Proposition 4 is as follows. If the speculator cannot be incentivized to produce sufficiently precise signals, then the costs of underpriced securities exceed the value of immediate funding and the owner of a high value asset prefers not to issue any risky security.

Consider next the preferred pooling contract for the high type when it is possible to induce high levels of speculator effort and informational efficiency, in the sense that $\beta Z > 1$. To analyze this case, note that $\sigma^*(h/L) = \sigma^*(H/L)$ for all $h > L\kappa^{**}$, where κ^{**} is the unique solution to equation (21). It follows that:

$$\forall \quad h \in (L\kappa^{**}, H), \quad \overline{U}_h(L, h) = \beta Z(\sigma^*(H/L)) - 1 \tag{25}$$

which is a strictly positive constant in the posited setting. Thus

$$\beta Z[\sigma^*(H/L)] > 1 \Rightarrow \overline{U}(L, H) > \overline{U}(L, h) \quad \forall \quad h \in [L\kappa^{**}, H). \tag{26}$$

That is, when informational efficiency (Z) is sufficiently high, the high type prefers marketing $h = H$ to marketing $h \in [L\kappa^{**}, H)$.

Using the fact that the objective function $\overline{U}(L, \cdot)$ is linear for $h > L\kappa^{**}$ we also know:

$$\begin{aligned}
\beta Z[\sigma^*(H/L)] \quad &> \quad 1 \Rightarrow \overline{U}(L, L\kappa^{**}) = \overline{U}(L, H) - [H - L\kappa^{**}][\beta Z(\sigma^*(H/L)) - 1] \qquad (27) \\
&\Downarrow \\
\overline{U}(L, L\kappa^{**}) \quad &= \quad H - L + \beta L + (L\kappa^{**} - L)[\beta Z(\sigma^*(H/L)) - 1] \geq \overline{U}(L, h) \quad \forall h \in [L, L\kappa^{**}).
\end{aligned}$$

It follows from the inequalities in (26) and (27) that:

$$\beta Z[\sigma^*(H/L)] > 1 \Rightarrow \overline{U}(L, H) > \overline{U}(L, h) \quad \forall \quad h \in [L, H). \tag{28}$$

This last inequality implies that, with sufficient speculator effort and market informational efficiency ($\beta Z > 1$), the high type can achieve higher utility at his preferred pooling contract than at the LIIO since the LIIO provides the high type with utility $\overline{U}(L, L)$. And the low type is clearly better off with this pooling outcome that at the LIIO since his LIIO payoff is only βL. From Lemma (7) it then follows the high type's preferred pooling contract is in the set of PBE iff $\beta Z[\sigma^*(H/L)] > 1$.

Having determined necessary and sufficient conditions for the high type's preferred pooling contract, with maximum information production, to be in the set of PBE, we turn next to the low type's preferred pooling contract. As discussed in the previous section, this contract entails full securitization of the asset cum issuance of riskless debt to destroy incentives for the speculator to exert effort. From Lemma 7 it follows this pooling contract is in the set of PBE iff the high type is better off under it than under the LIIO, or:

$$\beta[qH + (1-q)L] > H - L + \beta L \Leftrightarrow q > 1/\beta. \tag{29}$$

That is, iff $q > 1/\beta$, the low type's preferred pooling contract is in the set of PBE. Since $Z > q$, this is a more restrictive condition than that needed to support the high type's preferred pooling contract as a PBE. Intuitively, the high type has relatively strong incentives to deviate from the low type's preferred pooling outcome since it entails large deviations of prices from fundamentals.

We can summarize these results as follows.

Proposition 5 *Iff informational efficiency is high, with $Z[\sigma^*(H/L)] > 1/\beta$, the high type's preferred pooling contract is a perfect Bayesian equilibrium. This contract entails full securitization with the asset bifurcated into risky senior debt with face value $L\kappa^{**}$, where κ^{**} is the unique solution to equation (21), and a residual junior tranche. Iff $q > 1/\beta$, the low type's preferred pooling contract is in the set of perfect Bayesian equilibria. This contract entails full securitization with the asset bifurcated into riskless senior debt with face value L and a residual junior tranche.*

Finally, we evaluate whether the three potential PBE discussed above satisfy the Intuitive Criterion of Cho and Kreps (1987). In the interest of brevity, the high type's preferred pooling equilibrium is labeled *HPOOL* and that of the low type is labeled *LPOOL.* Recall, from the proof of Lemma 7 that the various PBE are supported by imputing any off-equilibrium menu registration to the low type. As one might suspect, LPOOL is fragile inasmuch as such beliefs can be viewed as implausible given that the high type has a strong incentive to deviate from LPOOL, while the low type finds LPOOL very attractive. Consistent with this intuition, in the appendix we prove the following proposition.

Proposition 6 *The LIIO satisfies the Intuitive Criterion. LPOOL does not satisfy the Intuitive Criterion. HPOOL satisfies the Intuitive Criterion iff speculator effort and informational efficiency are sufficiently high such that*

$$\frac{1 - z[\sigma^*(H/L)]}{1 - Z[\sigma^*(H/L)]} \geq \frac{\beta}{\beta - 1}.$$

V. Private versus Public Incentives in Securitization

The model allows us to determine whether the private sector will implement socially preferred equilibria. In comparing across equilibria in which informed speculation does and does not occur, Pareto improvements are impossible since uninformed investors suffer when informed speculation occurs while the speculator benefits. So we take the perspective of a utilitarian social planner placing equal weight on all agents.

Interestingly, there is one case in which we can say that one PBE Pareto dominates another. Specifically, suppose $\beta q > 1$ so that LPOOL (defined in Section IV) is a potential PBE. This PBE Pareto dominates the LIIO. To see this, note that the UI, S and the low type are equally well off in the two equilibria. However, since $\beta q > 1$ the high type is strictly better off pooling at full securitization of the asset than at the LIIO. Thus, if $\beta q > 1$ LPOOL Pareto dominates the LIIO.

To set a benchmark, consider social welfare under symmetric information. Here, the owner would sell the entire asset, regardless of type, converting each unit of funds raised into $\beta > 1$ units of consumption. The speculator and market-makers would consume their endowments. Each UI facing a low endowment would spend ϕ units of period 2 goods to insure against negative consumption in period 3. The implied ex ante social welfare in the case of first-best is:

$$W^{FB} = \beta[qH + (1-q)L] + y_2^S + y_2^{MM} + y_3^{MM} + y_2^{UI} - \frac{\phi(\underline{\pi} + \overline{\pi})}{2}. \tag{30}$$

Consider now social welfare under LPOOL, recalling it is a PBE iff $\beta q > 1$. *LPOOL achieves the first-best social welfare level W^{FB}*. To see this, note that the low type's preferred pooling contract carves out a safe debt claim. This safe debt claim will be used by the UI to save across periods 2 and 3, resulting in first-best risk sharing. Further, the marketing of safe debt destroys incentives for socially wasteful speculator effort. In LPOOL there is a transfer from the high to low type due to mispricing of the risky equity tranche, but this transfer is zero-sum.

Consider next the LIIO. Ex ante, the social planner computes the following expectation (over types) of social welfare loss in the LIIO relative to first-best:

$$DWL^{LIIO} = q(\beta - 1)(H - L). \tag{31}$$

The only deadweight loss in the LIIO is the loss in NPV resulting from the high type operating the new investment below first-best scale. From a risk sharing perspective the LIIO is attractive, since all investors achieve their first-best consumption profiles. The UI save using the safe marketed claim and the speculator does not exert costly effort.

Consider finally HPOOL, the high type's preferred pooling equilibrium in which the asset is fully securitized under the corresponding optimal structuring described in Proposition 2. The expected level of investment in the pooling equilibrium equals first-best. However, this equilibrium entails costly speculator effort and results in inefficient risk sharing as the UI distort their portfolios. The calculation of social welfare in the pooling equilibrium is a bit more involved. As a first step it can be computed as:

$$\begin{aligned} W^{HPOOL} &= \beta[qH + (1-q)L] + y_2^S + y_2^{MM} + y_3^{MM} + y_2^{UI} + G - e \\ &\quad - \frac{\phi(\underline{\pi} + \overline{\pi})}{2}\left[\int_1^{\theta_1} \theta f(\theta) d\theta + (1-q)\left(1 - \kappa^{-1}\right) \int_{\theta_1}^{\theta_2} \theta f(\theta) d\theta + \frac{X_B E[P_B | y_3^i = \xi - \phi]}{\phi}\right]. \end{aligned} \tag{32}$$

The first line in equation (32) measures the expected value of aggregate investment, plus the income endowments plus the net trading gain to the speculator. The second line measures the costs incurred by UI from $c_3 < \xi$ in addition to the expected cost of the UI portfolios. This simplifies to:

$$\begin{aligned} W^{HPOOL} &= \beta[qH + (1-q)L] + y_2^S + y_2^{MM} + y_3^{MM} + y_2^{UI} - e \\ &\quad -\frac{\phi(\underline{\pi} + \overline{\pi})}{2}\left[[1 - F(\theta_1)] + \int_1^{\theta_1} \theta f(d\theta) + (1-q)\left(1 - \kappa^{-1}\right)\int_{\theta_1}^{\theta_2} (\theta - 1) f(d\theta) + q(\kappa - 1)\left[1 - F(\theta_2)\right]\right] \end{aligned} \tag{33}$$

We then obtain the following expression for the deadweight loss in the high type's preferred pooling equilibrium:

$$DWL^{HPOOL} = e[\sigma(\phi)] + \frac{\phi(\underline{\pi} + \overline{\pi})}{2}\left[\int_1^{\theta_1} (\theta - 1) f(d\theta) + (1-q)\left(1 - \kappa^{-1}\right)\int_{\theta_1}^{\theta_2} (\theta - 1) f(d\theta) + q(\kappa - 1)[1 - F(\theta_2)]\right]. \tag{34}$$

Equation (34) has the following intuition. The first term reflects the fact that speculator effort is socially costly. The term in large square brackets reflects the fact that the existence of asymmetric information in this pooling equilibrium leads to distortions in UI portfolios. The first term in the large brackets captures the fact that a socially inefficient number of UI forego saving altogether. The second term in the large brackets reflects the fact that adverse selection induces a socially inefficient number of UI to only partially insure against low consumption. And the final term represents the social cost associated with overinsurance ($c_3 > \xi$) by extremely risk-averse UI.

From equation (34) it is readily verified that the deadweight loss in HPOOL is increasing in the size of endowment shocks as follows:

$$\frac{\partial DWL}{\partial \phi} = \frac{\partial \sigma}{\partial \phi} e' + \frac{\underline{\pi} + \overline{\pi}}{2}\left[\int_1^{\theta_1} (\theta - 1) f(\theta) d\theta + (1-q)\left(1 - \kappa^{-1}\right)\int_{\theta_1}^{\theta_2} (\theta - 1) f(\theta) d\theta + q(\kappa - 1)[1 - F(\theta_2)]\right]. \tag{35}$$

Note that the deadweight loss in the LIIO is independent of ϕ, but increasing in β. Conversely, the deadweight loss under the high type's preferred pooling contract is independent of β, but increasing in ϕ. It follows that by equating the deadweight losses across these two equilibria we may pin down a critical value of β, call it β_{public}, at which the social planner would be just indifferent between

LIIO and HPOOL. Specifically:

$$\beta_{public}(\phi) - 1 = \frac{e[\sigma(\phi)] + \frac{\phi(\pi+\overline{\pi})}{2}\left[\int_1^{\theta_1}(\theta-1)f(d\theta) + (1-q)\left(1-\frac{1}{\kappa}\right)\int_{\theta_1}^{\theta_2}(\theta-1)f(d\theta) + q(\kappa-1)[1-F(\theta_2)]\right]}{q(H-L)}. \tag{36}$$

It is readily verified that β_{public} is increasing in ϕ. Intuitively, an increase in ϕ raises the risk sharing cost associated with HPOOL, so the only way to maintain social planner indifference across the equilibria is to have a compensating increase in β, which raises the deadweight cost of the underinvestment associated with the LIIO.

Similarly, we may pin down a critical value of β, call it $\beta_{private}$ at which the high type would be just indifferent between HPOOL and the LIIO. From Proposition 5 we know that the indifference region is determined by:

$$\beta_{private}(\phi) = [Z(\sigma(\phi)]^{-1} \Rightarrow \frac{d\beta_{private}}{d\phi} = -[Z(\sigma(\phi)]^{-2}\left[\frac{\partial\sigma}{\partial\phi}\right] < 0. \tag{37}$$

In contrast to the social planner, the high type is more attracted to HPOOL for higher values of ϕ since large endowment shocks stimulate uninformed demand and speculator effort, resulting in less underpricing in this pooling equilibrium. Hence, to maintain indifference for the high type, a compensating decrease in β is required in response to a marginal increase in ϕ.

Figure 3 compares private and public preferences over three potential equilibria: the LIIO, HPOOL, and LPOOL while excluding any of the three that are not PBE on particular regions. For example, LPOOL is always preferred by the utilitarian social planner, but LPOOL is only in the set of PBE on Region 5, when $\beta q > 1$. Consequently, LPOOL is listed as the publicly preferred PBE in Region 5 only. Similarly, HPOOL is not at a PBE on Regions 1 and 2 since these regions are to the left of the downward sloping line where $\beta Z = 1$. On each region, the private preference is from the perspective of the high type since the low type's preference (LPOOL) is not generally a PBE and never satisfies the Intuitive Criterion.

On Regions 1 and 2, the LIIO is the unique equilibrium so there is no conflict between public and private preferences over the PBE. Here riskless debt will be supplied by either type, with the high type owner retaining all risk. In the LIIO there will be efficient risk sharing across investors, with no informed speculation taking place.

On Region 3 both the LIIO and HPOOL are potential equilibria. Here both the high type and the social planner prefer HPOOL to the LIIO. The high type prefers HPOOL to the LIIO on this region because the high value he places on funding (high β) dominates the severe underpricing he will face (low ϕ). The planner prefers HPOOL to the LIIO on this region despite the fact that risky debt will replace riskless debt. The planner is willing to accept vanishing liquidity because efficient risk sharing is less socially important than high investment on this region.

Private and public preferences over PBE conflict on Regions 4 and 5. On Region 4 the planner prefers the LIIO, with high ϕ values raising the risk sharing and speculator effort costs inherent in HPOOL. In other words, liquidity is particularly socially valuable on Region 4 given the large storage demands (high ϕ) of the uninformed investors. However, on this same region the high type prefers HPOOL, where liquidity vanishes and is replaced by risky debt. The high type recognizes that high ϕ values stimulate speculator effort and mitigate the extent of underpricing. This makes HPOOL more attractive to him. Similarly, an increase in UI risk-aversion via a first-order stochastic dominant shift in θ would also increase the social welfare loss associated with HPOOL, while simultaneously making that equilibrium more attractive to the owner of a high quality asset.

The analysis of Region 4 illustrates starkly that the private sector can prefer the pooling equilibrium cum risky debt, with high volumes of securitized asset sales and inefficient risk sharing, precisely when liquidity and efficient risk sharing have high social value. Also, the analysis shows that it is not simply a matter of the high type failing to account for the negative externality he imposes on uninformed investors. Rather, the larger the negative externality (higher ϕ or UI risk-aversion), the greater the relative attractiveness of HPOOL for the high type. Paradoxically, the negative externality is more likely to be imposed when it is larger.

On Region 5, LPOOL can be supported as a PBE. The planner prefers LPOOL here since it results in first-best social welfare, with the asset being fully securitized in the form of riskless debt and risky equity. However, the high type would prefer to deviate from this PBE, instead issuing risky debt in order to stimulate speculator effort and drive prices closer to fundamentals. And further, HPOOL is arguably a more plausible outcome inasmuch as LPOOL relies upon beliefs that do not satisfy the Intuitive Criterion. So once again we see a conflict between public and private preferences can emerge in security design.

VI. Publicly Supplied Liquidity

In recent years governments have explicitly recognized the demand of uninformed investors for liquidity. For example, during the consultation period leading up to its introduction of long-dated gilts, the Debt Management Office (DMO) of the United Kingdom stated: "Excess demand for high quality inflation-linked bonds and very long-dated bonds in the formats desired by long-term investors has featured strongly in the DMO's informal discussions. It seems likely that both Her Majesty's Government and investors may benefit from the issuance of bonds in maturity ranges or formats where there currently exists either no or insufficient supply."

The analysis up to this point has assumed the income endowments of the various agents are not verifiable, precluding the use of these endowments to back the sale of securities in period 2. Thus, in the baseline model only asset-backed securities could be issued. We now relax this assumption, examining the welfare implications of publicly supplied liquidity backed by income endowments.

Holmström and Tirole (1998) argue governments are more capable of providing liquidity. After all, governments generally have superior infrastructure for verifying incomes. Further, governments generally have at their disposal harsher methods for compelling delivery of required payments (e.g. jail). We assume now that a fraction of the final period income endowment (y_3^{MM}) of the MM is verifiable by the public treasury. In particular, assume the treasury has the ability to levy a non-distorting tax equal to a fraction t of the period 3 endowment of the MM. To fix ideas, one may think of the MM as having high incomes that are impossible to completely hide from the treasury. Alternatively, one may think of MM as deriving a portion of their incomes from a transparent source.

The government can supply liquidity as follows. In period 2 it can sell riskless bonds carrying face value ty_3^{MM}. The government bond will be used as a safe store of value by the UI. The proceeds raised by the government bond flotation (ty_3^{MM}) can be used to finance a period 2 transfer to the MM to avoid income redistribution. In terms of its effect on social welfare, such a government policy is equivalent to the MM having the ability to credibly pledge to the UI a fraction t of their period 3 endowment.

To illustrate the potential welfare benefits of publicly supplied liquidity, return to Figure 3 and assume the pair (ϕ, β) falls within in Region 4. In the absence of government intervention, the private sector may implement HPOOL, implying no riskless securities are issued and resulting in distorted risk sharing. Here, the social planner would prefer the LIIO to HPOOL given the high social value of efficient risk sharing in this region. If $ty_3^{MM} \geq \overline{\pi}\phi$ the government can easily achieve the same social welfare as the LIIO via the public liquidity scheme. By selling safe debt with face value ty_3^{MM} the government would fully satisfy UI demand for safe storage regardless of whether this demand is high or low. And absent UI demand for risky securities, the private sector would implement the LIIO securitization structure.

A more subtle and powerful role for publicly supplied liquidity is revealed if one considers a government with lower debt capacity. To see this, return to Region 4 of Figure 3 but now assume $ty_3^{MM} < \underline{\pi}\phi$. In this case public liquidity is insufficient to meet the storage demands of the UI even when their demand is low. In particular, each UI facing a negative endowment shock can now buy only ty_3^{MM}/π units of the riskless government bond in period 2. This leaves each UI facing a negative endowment shock with a residual storage demand equal to

$$\underline{\phi} \equiv \phi - \frac{ty_3^{MM}}{\pi} > 0. \tag{38}$$

It follows that the resulting equilibrium security design is equivalent to what one would obtain in the baseline model, but with the parameter ϕ being replaced by the smaller quantity $\underline{\phi}$. Further, we know from equation (22) that lower UI storage demand results in lower speculator effort and informational efficiency as measured by Z. If the reduction in the incentive compatible σ were sufficiently small, HPOOL would still remain viable as a PBE. However, social welfare would still be higher since risk sharing distortions would be smaller and speculator effort costs would be lower.

If instead the reduction in the incentive compatible σ were sufficiently large, the introduction of even this relatively small amount of public liquidity could potentially cause LIIO to become the unique PBE (on Region 4). This illustrates the following novel result: *The introduction of public liquidity can crowd-in private liquidity and crowd-out informed speculation.* Intuitively, public liquidity siphons off UI demand for information-sensitive claims, reducing the potential gains to informed speculation and, with it, the informational efficiency of prices. This makes it less likely that an issuer with positive information would be willing to issue information-sensitive claims.

As an interesting illustration of this effect at work, suppose:

$$ty_3^{MM} < \underline{\pi}\phi < \overline{\pi}\phi < ty_3^{MM} + L.$$

In the above case, the public supply of liquidity is insufficient to satisfy even low UI demand for safe assets. However, by crowding in L units of private liquidity, the introduction of public liquidity could result in sufficient aggregate liquidity to satisfy even high UI demand for safe assets, resulting in first-best risk sharing.

The beneficial effect of public liquidity on social welfare is not confined to Region 4. To see this, consider next (ϕ, β) pairs falling within Region 5. Here the introduction of public liquidity does not rule out HPOOL as a PBE. However, public liquidity would reduce speculator effort and risk sharing distortions taking place under HPOOL. Further, the public supply of liquidity could crowd-out all informed speculation on this region so that HPOOL vanished, with the only pooling equilibrium then being LPOOL. Under LPOOL first-best social welfare would be achieved.

However, the analysis also shows that excessive publicly supplied liquidity can reduce social welfare. To see this, consider (ϕ, β) pairs falling within Region 3. On this region, with relatively small UI endowment shocks, efficient risk sharing is of second order concern from a social welfare perspective. Here the social planner would view HPOOL as preferable to the LIIO, and it would not be socially optimal to supply public liquidity in an amount sufficient to destroy HPOOL as a viable equilibrium. Rather, the social optimum would entail supplying just enough public liquidity so that $\beta Z = 1$, with speculator effort just sufficient to keep HPOOL viable.

The above arguments imply the following characterization of the socially optimal supply of public liquidity.

Proposition 7 *If uninformed investors face large endowment shocks (Region 4), government should supply maximum public liquidity to crowd-in private liquidity and implement the LIIO. If the owner places high value on funding (Region 5), government should supply maximum public liquidity to eliminate informed speculation. For intermediate endowment shocks and funding values (Region 3), government should supply limited public liquidity to curtail informed speculation.*

Conclusions

This paper analyzes security design, the private equilibrium supply of liquidity, and the role of public liquidity. The model bridges two distinct literatures: the corporate finance literature on security design and the microstructure literature on rational expectations equilibria. We extend the former by allowing for information production by the market driving prices closer to fundamentals in the event of pooling. We extend the latter by modeling rational trading of ABS by optimizing uninformed investors. Endogenous uninformed trading is the key causal mechanism in the model responsible for the most interesting results. Additionally, endogenous uninformed trading allows us to assess social welfare.

First-best social welfare would be attained under pooling at the low type's preferred outcome in which the full asset is marketed as riskless debt and levered equity. However, this structuring is not generally an equilibrium and fails to survive weak refinements. Intuitively, the high type has an obvious incentive to deviate from a pooling equilibrium with zero informed trading and severe underpricing.

The equilibrium set always includes the low information intensity optimum (LIIO) in which only riskless debt is marketed. In the LIIO, the high type retains all risk on his own balance sheet and underinvests. However, this equilibrium achieves first-best risk sharing across investors since there is no speculator effort and uninsured investors are immune from adverse selection as they save using the riskless debt issued by either type. From a social perspective, the LIIO is particularly attractive when risk sharing is important.

The high type may be better off than under the LIIO by securitizing the entire asset in a pooling equilibrium. In his preferred pooling equilibrium, the asset is split into a risky senior tranche and a junior equity tranche. The absence of riskless claims then leaves uninformed investors exposed to adverse selection. In contrast to canonical signaling models, we find the (separating) LIIO may actually be socially preferred to this pooling equilibrium. This is because pooling gives rise to speculative markets with costly speculator information acquisition and distortions in the portfolios of uninformed investors.

The model highlights the following fundamental conflict between private and public preferences: Private incentives to implement the pooling equilibrium cum risky debt, adverse selection and portfolio distortions, can be strongest precisely when risk sharing is most socially valuable. Specifically, increases in the size of endowment shocks hitting uninformed investors, or their risk-aversion, encourage asset owners to rely upon speculative activity, since higher uninformed trading volume subsidizes information acquisition by speculators, which reduces the extent of mispricing in pooling equilibria.

An important problem highlighted by the model is that issuers fail to internalize the negative externality they impose on uninformed investors when they sell information-sensitive securities in speculative markets. In fact, our analysis shows that even when privately-informed owners have the ability to fully insulate uninformed investors from adverse selection, they have an incentive to issue securities with nonzero information-sensitivity in order to promote information production by speculators. Worse still, the larger the negative externality imposed on uninformed investors when risky debt is marketed instead of safe debt, the stronger the incentive of a high type to impose it.

The model offers a novel rationale for publicly-supplied liquidity. Public liquidity has an obvious direct benefit in providing uninformed investors with some amount of safe storage. However, we show public liquidity has a subtle indirect multiplier-effect. In particular, public liquidity siphons off the demand of uninformed investors for risky assets, reducing the profitability of informed speculation in markets for information-sensitive claims. This will increase the extent of underpricing anticipated by issuers with private information, potentially causing them to switch to marketing safe debt instead. Thus, public liquidity can have a disproportionate effect on aggregate liquidity and social welfare by crowding-out informed trading and crowding-in private liquidity.

Appendix: Proofs

Lemma 1

We can write

$$E[P_A + P_B|\tau = H] = l + \frac{1}{2}q\left[\frac{\sigma^2}{1-q-\sigma+2\sigma q} + \frac{(1-\sigma)^2}{q+\sigma-2\sigma q} + 1\right](h-l). \tag{39}$$

We need only verify the square bracketed term is increasing in σ. To this end let

$$\begin{aligned} a(\sigma) &\equiv q + \sigma - 2\sigma q \\ \Omega(\sigma) &\equiv 1 + \frac{\sigma^2}{(1-a)} + \frac{(1-\sigma)^2}{a}. \end{aligned}$$

We need only verify Ω is increasing. Differentiating we obtain:

$$\begin{aligned} \Omega'(\sigma) &= \frac{2(1-a)\sigma + (1-2q)\sigma^2}{(1-a)^2} - \frac{2a(1-\sigma) + (1-2q)(1-\sigma)^2}{a^2} \qquad (40) \\ &= \frac{[2(1-a) + (1-2q)\sigma]\sigma a^2 - (1-a)^2(1-\sigma)[2a + (1-2q)(1-\sigma)]}{(1-a)^2 a^2} \end{aligned}$$

This is strictly positive iff:

$$\begin{aligned} [2(1-a) + \sigma(1-2q)]\sigma a^2 &> (1-a)^2(1-\sigma)[2a + (1-2q) - \sigma(1-2q)] \\ &\Updownarrow \\ [(1-a) + (1-q)]\sigma a^2 &> (1-a)^2(1-\sigma)[a + (1-q)] \\ &\Updownarrow \\ (1-q)\sigma a^2 &> (1-a)\left[(1-\sigma)(1-a)a + (1-\sigma)(1-q)(1-a) - \sigma a^2\right] \\ &\Updownarrow \\ (1-q)\sigma a^2 &> (1-a)[a(1-a) - a\sigma + (1-\sigma)(1-q)(1-a)] \\ &\Updownarrow \\ [(1-q)a + 1 - a]\sigma a &> (1-a)^2[a + (1-\sigma)(1-q)] \\ &\Updownarrow \end{aligned}$$

$$
\begin{aligned}
[1-qa]\,\sigma a &> (1-a)^2(1-q\sigma) \\
&\Updownarrow \\
\sigma a - \sigma q a^2 &> (1-a)^2 - q\sigma(1-a)^2 \\
&\Updownarrow \\
q\sigma\left[(1-a)^2 - a^2\right] + \sigma a &> (1-a)^2 \\
&\Updownarrow \\
q\sigma + \sigma a(1-2q) &> (1-a)^2 \\
&\Updownarrow \\
a^2 + q(\sigma - a) &> (1-a)^2 \\
&\Updownarrow \\
q^2(2\sigma-1) + 2\left[\sigma - q(2\sigma-1)\right] &> 1 \\
&\Updownarrow \\
(q-1)^2(2\sigma-1) - (2\sigma-1) + 2\sigma &> 1 \\
&\Updownarrow \\
(q-1)^2(2\sigma-1) &> 0.\blacksquare
\end{aligned}
$$

Proposition 1

Consider a graph with X on the vertical axis and σ on the horizontal axis. We know $X(1/2) \in (\phi/B_H, \phi/B_L)$ and that X is strictly decreasing in σ on $(1/2, 1)$, yet strictly positive for $\theta^{\max}$ sufficiently high. Plotting the IC signal precision, we know σ_{ic} is strictly increasing in X with $\sigma_{ic}^{-1}(1/2) = 0$ and the limit as σ_1 converges to one of $\sigma_{ic}^{-1}(\sigma_1) = \infty$. Thus, the two curves intersect once, and only once, implying a unique equilibrium.$\blacksquare$

Lemma 3

Since security A is more information-sensitive it follows:

$$\frac{qA_H + (1-q)A_L}{E[P_A|y_3^i = \xi - \phi]} \leq \frac{qB_H + (1-q)B_L}{E[P_B|y_3^i = \xi - \phi]} \quad \text{and} \quad \frac{(1-q)A_L}{E[P_A|y_3^i = \xi - \phi]} \leq \frac{(1-q)B_L}{E[P_B|y_3^i = \xi - \phi]}. \tag{41}$$

The two inequalities in (41) indicate security A is viewed by UI as having higher expected cost per unit of c_3.

Consider some UI facing $y_3^i = \xi - \phi$. His terminal consumption is:

$$c_3(x_A, x_B, \tau) = x_A A_\tau + x_B B_\tau + \xi - \phi \quad \forall \quad \tau \in \{L, H\}. \tag{42}$$

Attention is confined to portfolios satisfying $c_3(x_A, x_B, L) \leq \xi$ since the marginal utility of terminal consumption is zero for all portfolios such that $c_3(x_A, x_B, L) > \xi$.

Consider first an arbitrary portfolio such that $c_3(x_A, x_B, H) < \xi$ and evaluate a local perturbation. We have:

$$\begin{aligned} \frac{\partial E[u]}{\partial x_A} &= \theta\left[qA_H + (1-q)A_L\right] - E[P_A|y_3^i = \xi - \phi] \\ \frac{\partial E[u]}{\partial x_B} &= \theta\left[qB_H + (1-q)B_L\right] - E[P_B|y_3^i = \xi - \phi]. \end{aligned} \tag{43}$$

If θ is sufficiently low, both perturbation gains listed above are negative and optimal UI demand is zero. Specifically:

$$\theta \leq \frac{E[P_B|y_3^i = \xi - \phi]}{qB_H + (1-q)B_L} \equiv \theta_1 \Leftrightarrow (x_A^*, x_B^*) = (0, 0). \tag{44}$$

Next, consider an arbitrary portfolio (x_A, x_B) such that $c_3(x_A, x_B, H) = \xi + \varepsilon$, where ε is arbitrarily small. At such points:

$$\begin{aligned} \frac{\partial E[u]}{\partial x_A} &= \theta(1-q)A_L - E[P_A|y_3^i = \xi - \phi] \\ \frac{\partial E[u]}{\partial x_B} &= \theta(1-q)B_L - E[P_B|y_3^i = \xi - \phi]. \end{aligned} \tag{45}$$

If θ is sufficiently high, the second perturbation gain listed above is positive. Further, since the maximand is piece-wise linear, it would then be optimal to fully insure by achieving $c_3(x_A^*, x_B^*, L) = \xi$. From (41) it follows the minimal cost means of achieving this full insurance is to purchase only security B. Thus,

$$\theta \geq \frac{E[P_B|y_3^i = \xi - \phi]}{(1-q)B_L} \equiv \theta_2 \Leftrightarrow (x_A^*, x_B^*) = \left(0, \frac{\phi}{B_L}\right). \tag{46}$$

The final case to consider is $\theta \in (\theta_1, \theta_2)$. From the perturbation arguments given above, we know such UI partially insure, with $c_3(x_A^*, x_B^*, H) = \xi$. From the first inequality in (41) we know security B provides higher marginal utility per unit spent. It follows that security B yields the highest marginal utility on the region of partial insurance, so that

$$\theta \in (\theta_1, \theta_2) \Rightarrow (x_A^*, x_B^*) = \left(0, \frac{\phi}{B_H}\right). \blacksquare \tag{47}$$

Lemma 4

Suppose Owner sells $N \geq 3$ securities. Rank these securities in descending order in terms of the ratio of their payoff if value is low relative to their payoff if value is high. We established UI trading will be concentrated in security 1, and security 1 will be the only source of informed trading gains. Aggregate demand of UI and S will then be zero in securities 2 to N. Therefore, one may roll up these securities into a single security having no effect on the equilibrium σ or expected revenues.$\blacksquare$

Lemma 6

We first establish concavity of $M(\cdot, \sigma)$. Differentiation yields:

$$\begin{aligned}
M_\kappa(\cdot,\sigma) &= 1 - F(\theta_2) + \kappa^{-2}[F(\theta_2) - F(\theta_1)] - (1-\kappa^{-1})\left[f(\theta_1)\frac{\partial\theta_1}{\partial\kappa} + f(\theta_2)\frac{\partial\theta_2}{\partial\kappa}(\kappa - 1)\right] \\
M_{\kappa\kappa}(\cdot,\sigma) &= -2\kappa^{-2} f(\theta_1)\frac{\partial\theta_1}{\partial\kappa} - 2\kappa^{-3}[F(\theta_2) - F(\theta_1)] \\
&\quad -(1-\kappa^{-1})\left[f'(\theta_1)\left(\frac{\partial\theta_1}{\partial\kappa}\right)^2 + f'(\theta_2)\left(\frac{\partial\theta_2}{\partial\kappa}\right)^2\right] \\
&\quad -(1-\kappa^{-1})\left[2(1+\kappa^{-1})f(\theta_2)\frac{\partial\theta_2}{\partial\kappa} - f(\theta_1)\left|\frac{\partial^2\theta_1}{\partial\kappa^2}\right|\right].
\end{aligned}$$

Since F is convex, a sufficient condition for strict concavity is

$$\left|\frac{\partial^2\theta_1}{\partial\kappa^2}\right| \leq \frac{\partial\theta_2}{\partial\kappa},$$

which always holds.

We next prove existence of a unique interior solution when κ is unconstrained, denoted $\kappa^*(\infty)$. For arbitrary $\sigma > 1/2$

$$\lim_{\kappa \uparrow \infty} \quad M(\kappa, \sigma) = 1 - F[1 + (1-q)(2\sigma - 1)(\overline{\pi} - \underline{\pi})/(\overline{\pi} + \underline{\pi})].$$

For $\widehat{\kappa}$ arbitrarily large there exists ε arbitrarily small such that

$$M(\widehat{\kappa}, \sigma) = \lim_{\kappa \uparrow \infty} \quad M(\kappa, \sigma) \pm \varepsilon.$$

Now choose $\widetilde{\kappa}$ such that $M(\widetilde{\kappa}, \sigma) = M(\widehat{\kappa}, \sigma)$. We know $\widetilde{\kappa} << \widehat{\kappa}$ since

$$\widetilde{\kappa} \approx 1 + \frac{1 - F[1 + (1-q)(2\sigma - 1)(\overline{\pi} - \underline{\pi})/(\overline{\pi} + \underline{\pi})]}{1 - \frac{F[\theta_1(\widetilde{\kappa},\sigma)]}{\widetilde{\kappa}} - \frac{(\widetilde{\kappa}-1)F[\theta_2(\widetilde{\kappa},\sigma)]}{\widetilde{\kappa}}} < 2.$$

From the strict concavity of $M(\cdot, \sigma)$ and the intermediate value theorem there exists unique $\kappa^*(\infty) \in (\widetilde{\kappa}, \widehat{\kappa})$ such that $M_\kappa[\kappa^*(\infty), \sigma] = 0$ with $M(\cdot, \sigma)$ increasing (decreasing) for κ less (greater) than $\kappa^*(\infty)$.■

Lemma 8

The following program characterizes the preferred pooling contract for the high type.

$$\begin{aligned} \max_{l,h} \quad \overline{U}(l,h) &\equiv H - h + \beta[l + (h-l)Z(\sigma^*(h/l))] \\ & s.t. \\ Monotonicity &: \quad h \geq l \\ Limited\ Liability &: \quad h \in [0, H] \text{ and } l \in [0, L]. \end{aligned}$$

We pin down the optimal policy via perturbation and dominance arguments. First, we claim

$$h^* = H \Rightarrow l^* = L.$$

and thus

$$l^* < L \Rightarrow h^* < H.$$

To demonstrate this, note

$$h^* = H \Rightarrow \forall l \in (0, L), \quad h^*/l > H/L \Rightarrow \overline{U}_1(l, h^*) = \beta(1 - Z) > 0.$$

Now let $\lambda^*(l,h)$ denote the multiplier on the low-information sensitivity constraint ($\kappa \leq h/l$) in the residual structuring problem for a given marketed bundle (l,h). Consider the two possible values for this multiplier λ. We claim first:

$$\lambda^*(l^*,h^*) = 0 \Rightarrow l^* = L.$$

To demonstrate this, suppose to the contrary that (l_0,h_0) are optimal with $\lambda^*(l_0,h_0) = 0$ but $l_0 < L$. Then consider increasing l by ε arbitrarily small, noting that such an increase meets all constraints including monotonicity since $\lambda^*(l_0,h_0) = 0$ implies $h_0 > l_0$. The gain is $\varepsilon\beta(1-Z) > 0$, contradicting the initial conjecture.

Next we claim

$$\lambda^*(l^*,h^*) > 0 \Rightarrow l^* = L.$$

To demonstrate this claim, suppose to the contrary that (l_0,h_0) are optimal with $\lambda^*(l_0,h_0) > 0$ but $l_0 < L$. Then let $\kappa_0 \equiv h_0/l_0$ and consider all pairs $(l,\kappa_0 l)$. By construction, all such pairs keep Z fixed at $Z[\sigma^*(h_0/l_0)] \equiv Z_0$. Then consider

$$\frac{d}{dl}\ \overline{U}[l,\kappa_0 l] = \beta(1-Z_0) + \kappa_0[\beta Z_0 - 1] \quad \forall \quad l \in (0,L).$$

Note that the value of this derivative is constant by construction. We next claim the derivative must be weakly positive. For if it is not, the optimal policy is to decrease both l and h to zero leaving the owner to collect $\overline{U} = H$ which is strictly dominated by $l = h = L$. Finally, since the derivative is weakly positive $l^* = L$.■

Proposition 6

We prove this proposition with three lemmas.

Lemma: LIIO satisfies the Intuitive Criterion.

Proof. LIIO imputes any deviating menu registration to the low type. We verify these beliefs satisfy the Intuitive Criterion (IC). Note first no separating menu is preferred by the high type, so registration of any other separating menu can be imputed to the low type. Within the set of pooling menus, consider an arbitrary pair of marketed payoffs (l,h) such that the high type benefits from deviating provided beliefs would be focused on the high type. It must be that $h > L$. But with beliefs focused on the high type, the low type would also gain by making this same deviating proposal, capturing $L - l + \beta h > \beta L$. Thus, IC admits beliefs imputing any such deviation to the low type.

Lemma: LPOOL does not satisfy the Intuitive Criterion.

Proof. Consider LPOOL and a deviating offer of marketed payoffs (L, h) with h such that:

$$\begin{aligned} H - h + \beta h &> \beta[qH + (1-q)L] > \beta h \\ &\Updownarrow \\ h &\in \left(H - \frac{\beta(H-L)(1-q)}{\beta - 1}, qH + (1-q)L\right). \end{aligned}$$

By construction, the high (low) type strictly gains (loses) from deviating if beliefs are focused on the high type. Thus, IC demands beliefs imputing this deviation to the high type, and so the high type would in fact deviate from LIIO.

Lemma: HPOOL satisfies the Intuitive Criterion iff $(1-z)/(1-Z) \geq \beta/(\beta-1)$.

Proof. We begin first with necessity. Suppose instead $(1-z)/(1-Z) < \beta/(\beta-1)$. Now consider HPOOL and a deviating offer of marketed payoffs (L, h) with h such that:

$$\begin{aligned} H - h + \beta h &> \beta[ZH + (1-Z)L] > \beta[zH + (1-z)L] > \beta h. \\ &\Updownarrow \\ h &\in \left(H - \frac{\beta(H-L)(1-Z)}{\beta - 1}, zH + (1-z)L\right). \end{aligned}$$

By construction, the low type is always worse off making this deviation while the high type is better off provided beliefs were to be focused on the high type. Thus, IC demands imputing this contract to the high type, causing him to deviate.

Consider next sufficiency of the stated $\geq$ condition. To show IC is satisfied we need only establish that if the high type were to make a strict gain from some deviation, so too would the low type (provided beliefs were focused on the high type). Consider then that the high type gains by deviating to marketed cash flows (l, h) iff $h > H - \beta(H - L)(1 - Z)/(\beta - 1)$. But if the stated $\geq$ condition holds, the low type gains from any such deviation.■

References

[1] Akerlof, G.A, 1970, "The market for lemons: Quality uncertainty and the market mechanism," *Quarterly Journal of Economics* 84, 488–500.

[2] Allen, F., and D. Gale, 1988, "Optimal Security Design," *Review of Financial Studies* 89, 229-63.

[3] Boot, A., and A. Thakor, 1993, "Security Design," *Journal of Finance* 48 , 1349-78.

[4] Cho, I. and D. Kreps, 1987, "Signaling Games and Stable Equilibria," *Quarterly Journal of Economics*, 179-221.

[5] Dang,T.V., Gorton, G., and B. Holmström, 2010, "Opacity and the Optimality of Debt for Liquidity Provision," working paper, MIT.

[6] Debt Management Office of the United Kingdom, 2004, "Issuance of Ultra-Long Gilt Instruments: Consultation Document."

[7] DeMarzo, P., 2005, "The Pooling and Tranching of Securities: A Model of Informed Intermediation," *Review of Financial Studies* 15, 100-150.

[8] DeMarzo, P., and D. Duffie, 1999, "A Liquidity-Based Model of Security Design," *Econometrica* 67, 65-99.

[9] Dow, J., 1998, "Arbitrage, Hedging, and Financial Innovation," *Review of Financial Studies* 11, 739-755.

[10] Faure-Grimaud, A. and D. Gromb, 2004, "Public Trading and Private Incentives," *Review of Financial Studies* 17, 985-1014.

[11] Fulghieri, P., and D. Lukin, 2001, "Information Production, Dilution Costs, and Optimal Security Design", *Journal of Financial Economics* 61, 3-42.

[12] Glosten, L., and P. Milgrom, 1985, "Bid, Ask and Transactions Prices in a Specialist Market with Heterogeneously Informed Traders," *Journal of Financial Economics* 14, 71-100.

[13] Gorton, G., and G. Pennacchi, 1990, "Financial Intermediaries and Liquidity Creation," *Journal of Finance* 45, 49-71.

[14] Holmström, B., and J. Tirole, 1998, "Private and Public Supply of Liquidity," *Journal of Political Economy* 106, 1-40.

[15] Kyle, A. S., 1985, "Continuous Auctions with Insider Trading," *Econometrica*, 1315-1335.

[16] Maskin, E., and J. Tirole, 1992, "The Principal-Agent Relationship with an Informed Principal, II: Common Values," *Econometrica* 60, 1-42.

[17] Maug, E., 1998, "Large Shareholders as Monitors: Is There a Trade-off Between Liquidity and Control?," *Journal of Finance* 53, 65-98.

[18] Myers, S., and N. Majluf, 1984, "Corporate Financing and Investment When Firms Have Information Shareholders Do Not Have," *Journal of Financial Economics* 13, 187-221.

[19] Nachman, D., and T. Noe, 1994, "Optimal Design of Securities Under Asymmetric Information," *Review of Financial Studies* 7, 1-44.

[20] Tirole, J., 2005, *The Theory of Corporate Finance*, Princeton University Press.

[21] Tirole, J., 2011, "Illiquidity and All its Friends," *Journal of Economic Literature* 49, 287-325.

Table 1: Aggregate Demand Outcomes

Type	Signal	Fraction UI Low Endow	Speculator Demand	UI Demand	Aggregate Demand	Probability
H	s_H	$\overline{\pi}$	$(\overline{\pi} - \underline{\pi})X_B$	$\overline{\pi}X_B$	$(2\overline{\pi} - \underline{\pi})X_B$	$\frac{q\sigma}{2}$
H	s_H	$\underline{\pi}$	$(\overline{\pi} - \underline{\pi})X_B$	$\underline{\pi}X_B$	$\overline{\pi}X_B$	$\frac{q\sigma}{2}$
H	s_L	$\overline{\pi}$	0	$\overline{\pi}X_B$	$\overline{\pi}X_B$	$\frac{q(1-\sigma)}{2}$
H	s_L	$\underline{\pi}$	0	$\underline{\pi}X_B$	$\underline{\pi}X_B$	$\frac{q(1-\sigma)}{2}$
L	s_L	$\overline{\pi}$	0	$\overline{\pi}X_B$	$\overline{\pi}X_B$	$\frac{(1-q)\sigma}{2}$
L	s_L	$\underline{\pi}$	0	$\underline{\pi}X_B$	$\underline{\pi}X_B$	$\frac{(1-q)\sigma}{2}$
L	s_H	$\overline{\pi}$	$(\overline{\pi} - \underline{\pi})X_B$	$\overline{\pi}X_B$	$(2\overline{\pi} - \underline{\pi})X_B$	$\frac{(1-q)(1-\sigma)}{2}$
L	s_H	$\underline{\pi}$	$(\overline{\pi} - \underline{\pi})X_B$	$\underline{\pi}X_B$	$\overline{\pi}X_B$	$\frac{(1-q)(1-\sigma)}{2}$

Figure 1: Aggregate Uninformed Demand

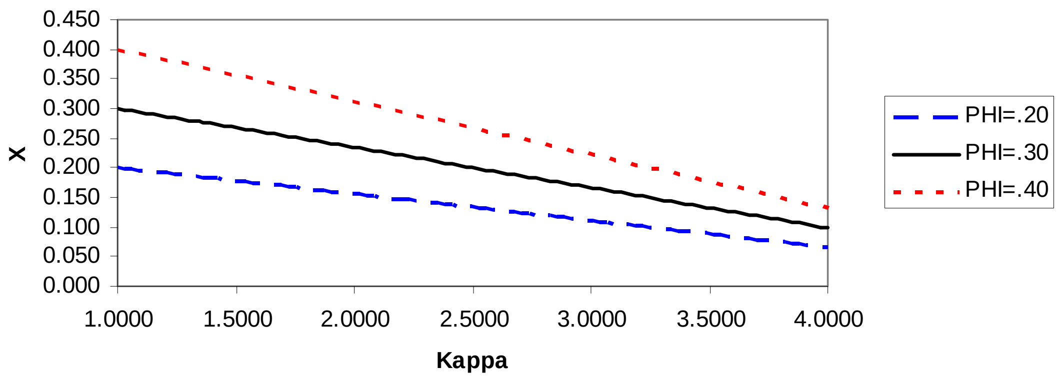

Figure 2: High Type Preferred Informational Sensitivity

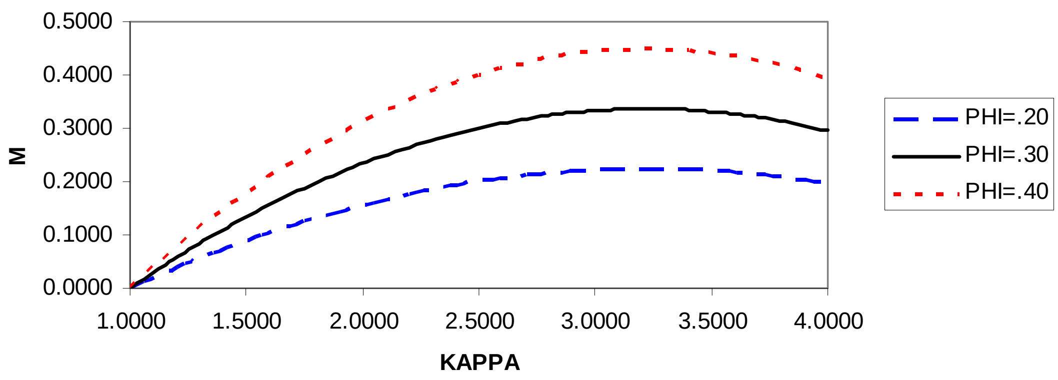

Figure 3: Public and Private Preferences over Equilibria