Tests of the CAPM and the Fama-French Methodology

Testing the CAPM: The Basics

- The most commonly quoted equation for the CAPM is

$$E(R_i) = R_f + \beta_i[E(R_m) - R_f]$$

- So the CAPM states that the expected return on any stock i is equal to the risk-free rate of interest, R_f, plus a risk premium.
- This risk premium is equal to the risk premium per unit of risk, also known as the market risk premium, $[E(R_m) - R_f]$, multiplied by the measure of how risky the stock is, known as 'beta', β_i
- Beta is not observable from the market and must be calculated, and hence tests of the CAPM are usually done in two steps:
  - Estimating the stock betas
  - Actually testing the model
- If the CAPM is a good model, then it should hold 'on average'.

Testing the CAPM: Calculating Betas

- A stock's beta can be calculated in two ways:
  1. Calculate it directly as the covariance between the stock's excess return and the excess return on the market portfolio, divided by the variance of the excess returns on the market portfolio:

     $$\beta_i = \frac{Cov(R_i^e, R_m^e)}{Var(R_m^e)}$$

     where the ^{e} superscript denotes excess return
  2. Equivalently, we can run a simple time-series regression of the excess stock returns on the excess returns to the market portfolio separately for each stock, and the slope estimate will be the beta:

     $$R_{i,t}^e = \alpha_i + \beta_i R_{m,t}^e + u_{i,t}$$

Testing the CAPM: The Second Stage Regression

- Example:
  - Suppose that we had a sample of 100 stocks (N=100) and their returns using five years of monthly data (T=60)
  - The first step would be to run 100 time-series regressions (one for each individual stock), the regressions being run with the 60 monthly data points
- The second stage involves a single cross-sectional regression of the average (over time) of the stock returns on a constant and the betas:

$$\bar{R}_i = \lambda_0 + \lambda_1 \beta_i + v_i$$

where $\bar{R}_i$ is the return for stock i averaged over the 60 months

Testing the CAPM: The Second Stage Regression (Cont'd)

- Essentially, the CAPM says that stocks with higher betas are more risky and therefore should command higher average returns to compensate investors for that risk

- If the CAPM is a valid model, two key predictions arise which can be tested using this second stage regression:
    1. $\lambda_0 = R_f$;
    2. $\lambda_1 = [R_m - R_f]$.

Testing the CAPM: Further Implications

- Two further implications of the CAPM being valid:
  - There is a linear relationship between a stock's return and its beta
  - No other variables should help to explain the cross-sectional variation in returns
- We could run the augmented regression:

$$\bar{R}_i = \lambda_0 + \lambda_1 \beta_i + \lambda_2 \beta_i^2 + \lambda_3 \sigma_i^2 + v_i$$

  where β_i^2 is the squared beta for stock i and σ_i^2 is the variance of the residuals from the first stage regression, a measure of idiosyncratic risk
- The squared beta can capture non-linearities in the relationship between systematic risk and return
- If the CAPM is a valid and complete model, then we should see that $\lambda_2 = 0$ and $\lambda_3 = 0$.

Testing the CAPM: A Different Second-Stage Regression

- It has been found that returns are systematically higher for small capitalisation stocks and are systematically higher for 'value' stocks than the CAPM would predict.

- We can test this directly using a different augmented second stage regression:

$$\bar{R}_i = \alpha + \lambda_1 \beta_i + \lambda_2 MV_i + \lambda_3 BTM_i + v_i$$

  where MV_i is the market capitalisation for stock i and BTM_i is the ratio of its book value to its market value of equity

- Again, if the CAPM is a valid and complete model, then we should see that $\lambda_2 = 0$ and $\lambda_3 = 0$.

Problems in Testing the CAPM

- Problems are numerous, and include:
  - Heteroscedasticity – some recent research has used GMM or another robust technique to deal with this
  - Measurement errors since the betas used as explanatory variables in the second stage are estimated – in order to minimise such measurement errors, the beta estimates can be based on portfolios rather than individual securities

Alternatives to Test the CAPM

- Alternative approaches:
  - Fama-MacBeth approach:
    - Fama, E. F. and MacBeth, J. D., 1973, "Risk, return and Equilibrium: Empirical Tests", *Journal of Political Economy*, 81(3), p607-636.
  - Fama-French approach:
    - Fama, E. F., and French, K. R., 1992, "The Cross-Section of Expected Stock Returns", *Journal of Finance*, 47, p427-465;
    - Fama, E. F., and French, K. R., 1993, "Common Risk Factors in the Returns on Stocks and Bonds", *Journal of Financial Economics*, 33, p3-53.
  - Carhart approach:
    - Carhart, M. 1997, "On Persistence of Mutual Fund Performance", *Journal of Finance*, 52, p57-82.

The Fama-MacBeth Approach

- Fama and MacBeth (1973) used the two stage approach to testing the CAPM outlined above, but using a time series of cross-sections
- Instead of running a single time-series regression for each stock and then a single cross-sectional one, the estimation is conducted with a rolling window
- They use five years of observations to estimate the CAPM betas and the other risk measures (the standard deviation and squared beta) and these are used as the explanatory variables in a set of cross-sectional regressions each month for the following four years

The Fama-MacBeth Approach (Cont'd)

- The estimation is then rolled forward four years and the process continues until the end of the sample is reached

- Since we will have one estimate of the lambdas for each time period, we can form a t-ratio as the average over t divided by its standard error (the standard deviation over time divided by the square root of the number of time-series estimates of the lambdas).

- The average value of each lambda over t can be calculated using:

$$\hat{\lambda}_j = \frac{1}{T_{FMB}} \sum_{t=1}^{T_{FMB}} \hat{\lambda}_{j,t}, \quad j = 1, 2, 3, 4$$

  where T_{FMB} is the number of cross-sectional regressions used in the second stage of the test, the j are the four different

The Fama-MacBeth Approach (Cont'd)

parameters (the intercept, the coefficient on beta, etc.) and the standard deviation is

$$\hat{\sigma}_j = \sqrt{\frac{1}{T_{FMB} - 1} \sum_{t=1}^{T_{FMB}} (\hat{\lambda}_{j,t} - \hat{\lambda}_j)^2}$$

- The test statistic is then simply $\sqrt{T_{FMB}}\hat{\lambda}_j / \hat{\sigma}_j$, which is asymptotically standard normal, or follows a t-distribution with $T_{FMB} - 1$ degrees of freedom in finite samples.

The Fama-French Methodology

- The 'Fama-French methodology' is a family of related approaches based on the notion that market risk is insufficient to explain the cross-section of stock returns
- The Fama-French and Carhart models seek to measure abnormal returns after allowing for the impact of the characteristics of the firms or portfolios under consideration
- It is widely believed that small stocks, value stocks, and momentum stocks, outperform the market as a whole
- If we wanted to evaluate the performance of a fund manager, it would be important to take the characteristics of these portfolios into account to avoid incorrectly labelling a manager as having stock-picking skills.

The Fama-French (1992) Approach

- The Fama-French (1992) approach, like Fama and MacBeth (1973), is based on a time-series of cross-sections model
- A set of cross-sectional regressions are run of the form

$$R_{i,t} = \alpha_{0,t} + \alpha_{1,t}\beta_{i,t} + \alpha_{2,t}MV_{i,t} + \alpha_{3,t}BTM_{i,t} + u_{i,t}$$

  where $R_{i,t}$ are again the monthly returns, $\beta_{i,t}$ are the CAPM betas, $MV_{i,t}$ are the market capitalisations, and $BTM_{i,t}$ are the book-to-price ratios, each for firm i and month t

- So the explanatory variables in the regressions are the firm characteristics themselves
- Fama and French show that size and book-to-market are highly significantly related to returns
- They also show that market beta is not significant in the regression (and has the wrong sign), providing very strong evidence against the CAPM.

The Fama-French (1993) Approach

- Fama and French (1993) use a factor-based model in the context of a time-series regression which is run separately on each portfolio i

$$R_{i,t} = \alpha_i + \beta_{i,M} RMRF_t + \beta_{i,S} SMB_t + \beta_{i,V} HML_t + \epsilon_{i,t}$$

  where $R\ i,t$ is the return on stock or portfolio i at time t, $RMRF$, SMB, and HML are the factor mimicking portfolio returns for market excess returns, firm size, and value respectively

- The excess market return is measured as the difference in returns between the S&P 500 index and the yield on Treasury bills ($RMRF$)

The Fama-French (1993) Approach (Cont'd)

- *SMB* is the difference in returns between a portfolio of small stocks and a portfolio of large stocks, termed 'Small Minus Big
- *HML* is the difference in returns between a portfolio of value stocks and a portfolio of growth stocks, termed 'High Minus Low'.
- Check Ken French webpage for more details on the "Fama-French factors".

The Fama-French (1993) Approach 2

- These time-series regressions are run on portfolios of stocks that have been two-way sorted according to their book-to-market ratios and their market capitalisations
- It is then possible to compare the parameter estimates qualitatively across the portfolios i
- The parameter estimates from these time-series regressions are factor loadings that measure the sensitivity of each individual portfolio to the factors
- The second stage in this approach is to use the factor loadings from the first stage as explanatory variables in a cross-sectional regression:

$$\bar{R}_i = \alpha + \lambda_M \beta_{i,M} + \lambda_S \beta_{i,S} + \lambda_V \beta_{i,V} + e_i$$

The Fama-French (1993) Approach 2 (Cont'd)

- We can interpret the second stage regression parameters as factor risk premia that show the amount of extra return generated from taking on an additional unit of that source of risk.

The Carhart (1997) Approach

- It has become customary to add a fourth factor to the equations above based on momentum
- This is measured as the difference between the returns on the best performing stocks over the past year and the worst performing stocks – this factor is known as UMD – 'up-minus-down'
- The first and second stage regressions then become respectively:

$$
\begin{aligned}
R_{i,t} &= \alpha_i + \beta_{i,M} RMRF_t + \beta_{i,S} SMB_t + \beta_{i,V} HML_t \\
&\quad + \beta_{i,U} UMD_t + \epsilon_{i,t} \\
\bar{R}_i &= \alpha + \lambda_M \beta_{i,M} + \lambda_S \beta_{i,S} + \lambda_V \beta_{i,V} + \lambda_U \beta_{i,U} + e_i
\end{aligned}
$$

The Carhart (1997) Approach (Cont'd)

- Carhart forms decile portfolios of mutual funds based on their one-year lagged performance and runs the time-series regression on each of them
- He finds that the mutual funds which performed best last year (in the top decile) also had positive exposure to the momentum factor (UMD) while those which performed worst had negative exposure.

More recently

- Researchers develop their own factors:
  - Lettau and Ludvigson's (2001) conditional version of the consumption CAPM (CC-CAY);
  - Lustig and Van Nieuwerburgh's (2005) conditional version of the consumption CAPM (CC-MY);
  - Santos and Veronesi's (2006) conditional version of the CAPM (C-SW);
  - Li, Vassalou, and Xing's (2006) investment growth model (IGM);
  - Petkova's (2006) empirical implementation of Merton's (1973) intertemporal extension of the CAPM (ICAPM) based on Campbell (1996);
  - Yogo's (2006) (D-CCAPM) is due to and highlights the cyclical role of durable consumption in asset pricing;
  - ...

More recently (Cont'd)

Model	Number of factors	List of factors
CC-CAY	4	$1,\ cay_{t-1},\ c_{nd,t},\ c_{nd,t} \cdot cay_{t-1}$
CC-MY	4	$1,\ my_{t-1},\ c_{nd,t},\ c_{nd,t} \cdot my_{t-1}$
C-SW	3	$1,\ r_{mkt,t},\ r_{mkt,t} \cdot s^{w}_{t-1}$
IGM	6	$1,\ i_{hh,t},\ i_{corp,t},\ i_{ncorp,t},\ i_{fcorp,t},\ i_{fm,t}$
ICAPM	6	$1,\ r_{mkt,t},\ term_t,\ def_t,\ div_t,\ rf_t$
CCAPM	4	$1,\ r_{mkt,t},\ c_{nd,t},\ c_{d,t}$

Description of the factors:

c_{nd}	growth rate in real per capita nondurable consumption (seasonally adjusted at annual rates) from BEA
cay	consumption-aggregate wealth ratio of Lettau and Ludvigson (2001) from Martin Lettau's website
my	housing collateral ratio
r_{mkt}	excess return (in excess of the one-month T-bill rate) on the value-weighted stock market index (NYSE-AMEX-NASDAQ) from Kenneth French's website
sw	labor income-consumption ratio
ihh	log (gross fixed) investment growth rates for households
icorp	log (gross fixed) investment growth rates for non-financial corporations
incorp	log (gross fixed) investment growth rates for non-corporate sector
ifcorp	log (gross fixed) investment growth rates for financial corporations
ifm	log (gross fixed) investment growth rates for farm sector
term	difference between the yields of ten-year and one-year government bonds (from the Board of Governors of the Federal Reserve System)
def	difference between the yields of long-term corporate Baa bonds (from the Board of Governors of the Federal Reserve System) and long-term government bonds (from Ibbotson Associates)
div	dividend yield on the Center for Research in Security Prices (CRSP) value-weighted stock market portfolio
rf	one-month T-bill yield (from CRSP, Fama Risk Free Rates)
cd	growth rate in real per capita durable consumption (seasonally adjusted at annual rates) from BEA

- New econometric issues arise:
    - identification
    - misspecification
    - ...