Problem Set #3 Answer Key

Economics 305: Macroeconomic Theory

Spring 2007

1 Chapter 4, Problem #2

a) To specify an indifference curve, we hold utility constant at $\bar{u}$. Next, rearrange in the form:

$$C = \frac{\bar{u}}{b} - \frac{a}{b} l$$

Indifference curves are, therefore, linear with slope, $-a/b$, which represents the marginal rate of substitution. There are two main cases, according to whether $(a/b) > w$ or $(a/b) < w$. The first panel of Figure 1 shows the case of $(a/b) < w$. In this case, the indifference curves are flatter than the budget line and the consumer picks point A, at which $l = 0$ and $C = wh + \pi - T$. The bottom panel of Figure 1 shows the case of $(a/b) > w$. In this case, the indifference curves are steeper than the budget line, and the consumer picks point B, at which $l = h$ and $C = \pi - T$. In the coincidental case in which $(a/b) = w$, the highest attainable indifference curve coincides with the budget line, and the consumer is indifferent among all possible amounts of leisure and hours worked.

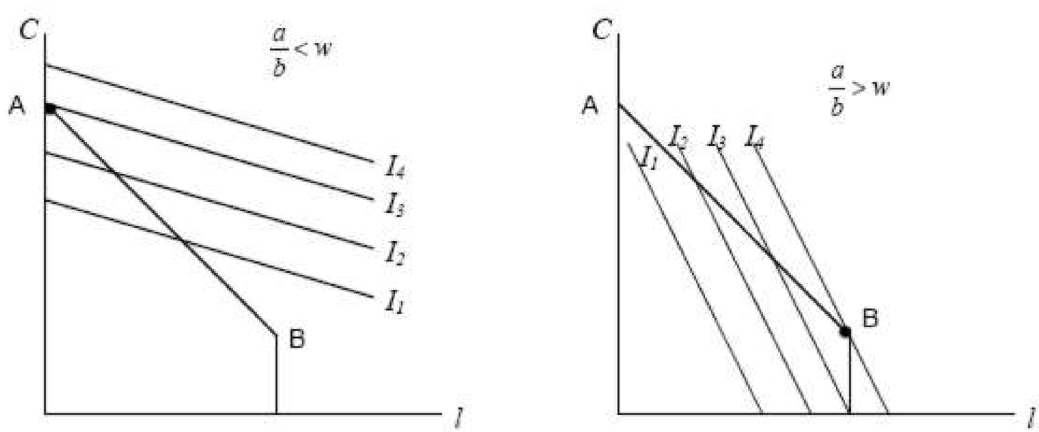


Figure 1: Perfect substitutes.

b) The utility function in this problem does not obey the property that the consumer prefers diversity, and is, therefore, not a likely possibility.

c) This utility function does have the property that more is preferred to less. However, the marginal rate of substitution is constant, and, therefore, this utility function does not satisfy the property of diminishing marginal rate of substitution.

2 Chapter 4, Problem #3

When the government imposes a proportional tax on wage income, the consumers budget constraint is now given by:

$$C = w(1 - t)(h - l) + \pi - T$$

where t is the tax rate on wage income. In Figure 2, the budget constraint for $t = 0$, is FGH. When $t > 0$, the budget constraint is EGH. The slope of the original budget line is w, while the slope of the new budget line is $(1t)w$. Initially, the consumer picks the point A on the original budget line. After the tax has been imposed, the consumer picks point B. The substitution effect of the imposition of the tax is to move the consumer from point A to point D on the original indifference curve. The point D is at the tangent point of indifference curve, I1, with a line segment that is parallel to EG. The pure substitution effect induces the consumer to reduce consumption and increase leisure (work less).

The tax also makes the consumer worse off, in that he or she can no longer be on indifference curve, I1, but must move to the less preferred indifference curve, I2. This pure income effect moves the consumer to point B, which has less consumption and less leisure than point D, because both consumption and leisure are normal goods. The net effect of the tax is to reduce consumption, but the direction of the net effect on leisure is ambiguous. Figure 4.3 shows the case in which the substitution effect on leisure dominates the income effect. In this case, leisure increases and hours worked fall. Although consumption must fall, hours worked may rise, fall, or remain the same.

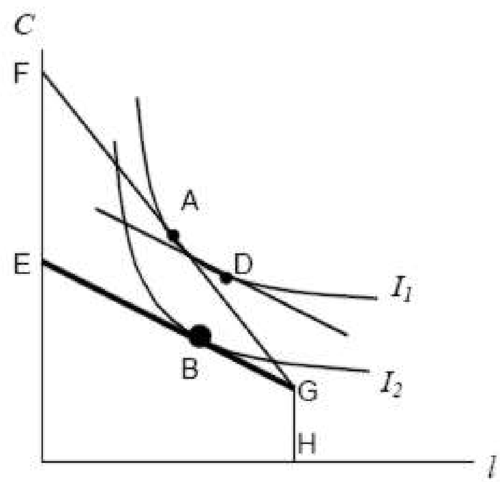


Figure 2: A proportional income tax.

3 Chapter 4, Problem #4

In Figure 3, with a proportional tax on wage income, the consumers budget constraint is

$$C = w(1 - t)(h - l) + \pi$$

where t is the tax rate. In Figure 3, the budget constraint is DEF, and the consumer chooses point A, where $C = C_1$ and $l = l_1$. Now, suppose that the government taxes the consumer lump-sum, and the total tax the consumer pays with the lump-sum tax is the same as it was with the proportional tax, so that the lump-sum tax is $T = wt(h - l_1)$. The consumers budget constraint is now

$$C = w(h - l) + \pi - wt(h - l_1).$$

Figure 3 shows the new budget constraint, which is DGH. Note that the new budget constraint is steeper than the old one, and that point A is on the new budget constraint, because if (C_1, l_1) satisfies the old budget constraint it must also satisfy the new budget constraint. Thus, the consumer will now choose point B, which must be on a higher indifference curve that A, so the consumer is better off with a lump-sum tax than with a proportional tax. The proportional tax distorts economic decisions and is therefore less efficient in extracting the same revenue that a lump-sum tax can generate.

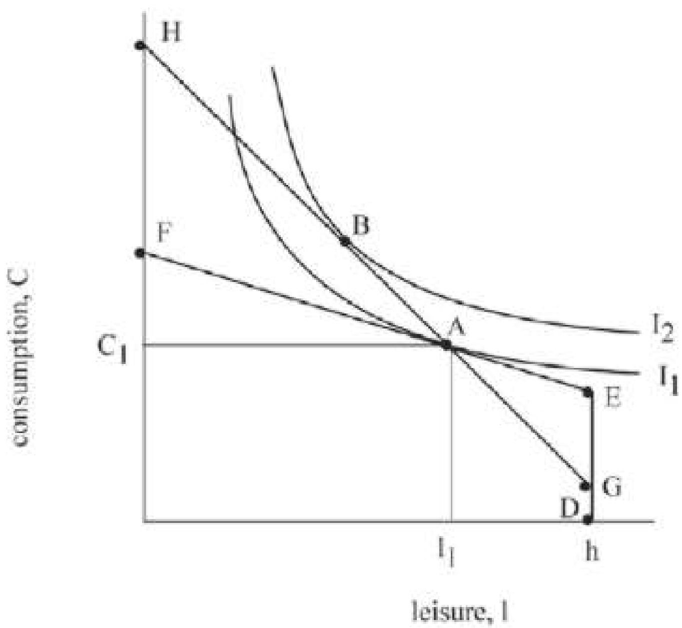
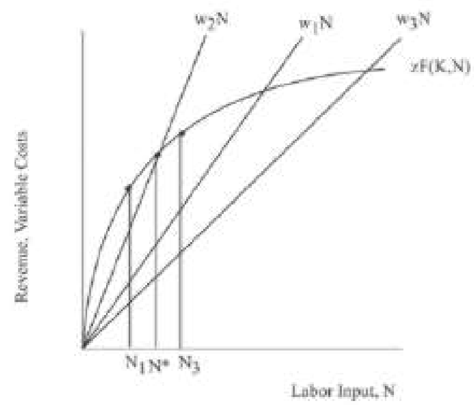


Figure 3: Proportional vs. lump-sum income tax.

4 Chapter 4, Problem #9

In the top panel of Figure 4, if there is a minimum quantity of employment, N^*, then suppose that the market real wage is w_1, so that the firm would choose labour input $N_1 < N^*$ without the minimum consumption requirement. However, given that the firm must choose employment to be greater than or equal to N^*, it will choose $N = N^*$. But, if the real wage is smaller than the marginal product of labour at $N = N^*$, for example, if the real wage is w_3 as in the top panel of Figure 4, then the firm optimizes by choosing $N > N^*$, that is, $N = N_3$. If the real wage is higher than w_2 as in the top panel of Figure 4, then, the firm will earn negative profits for any feasible level of the labour input, for any N that is greater than or equal to N^*. This implies that the labour demand curve for the firm is as depicted in the bottom panel of Figure 4. That is, for $w > w_2$, labour demand is zero. For a middle range of real wage rates, labour demand is N^*, and for low wage rates, the firm is unconstrained by the minimum quantity of employment and the labour demand curve is just the marginal product of labour curve.



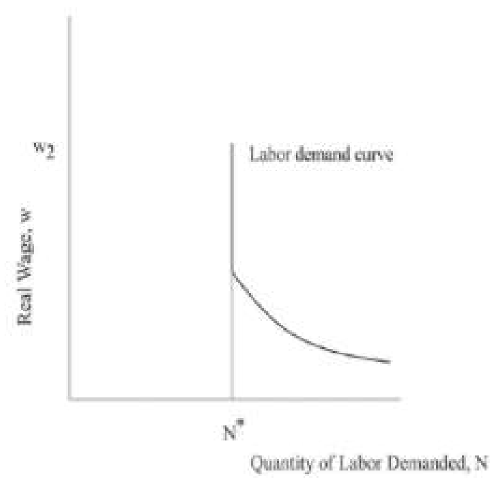


Figure 4: A minimum employment level.

5 Chapter 4, Working With the Data #2

As Figure 5 shows, the Solow residual tends to grow more slowly than does real GDP, but its growth rate is highly correlated with the growth rate of real GDP.

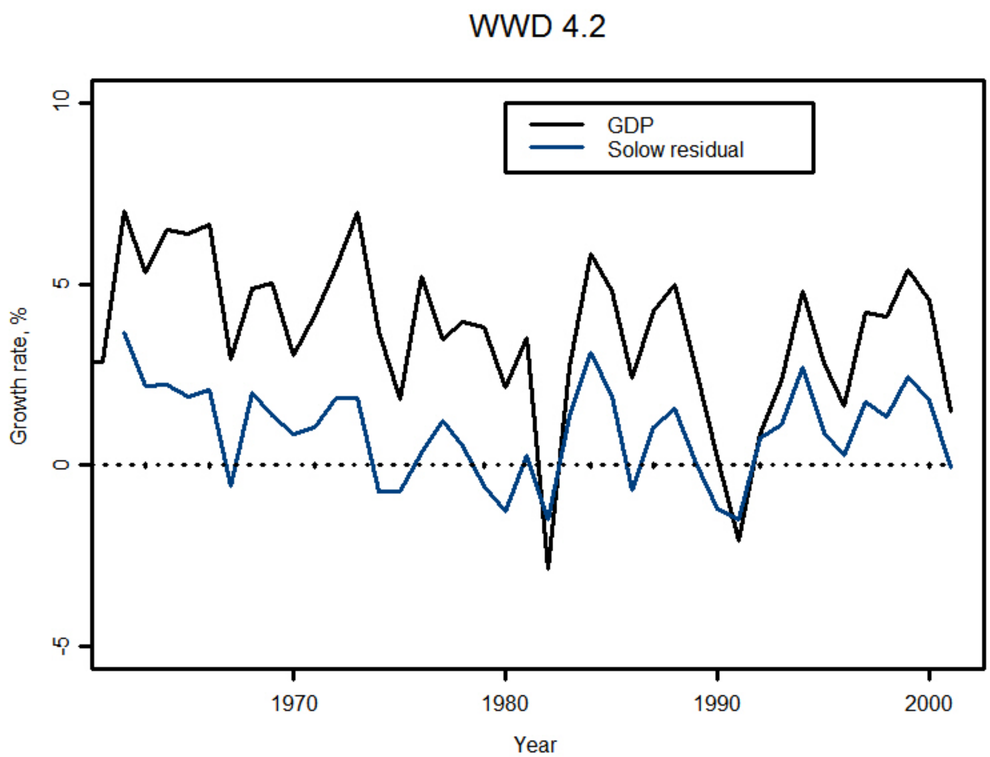


Figure 5: The Solow residual and GDP.