Problem Set #1: Math Review

Economics 435: Quantitative Methods

Fall 2011

1 Sets

Let A, B, C be sets, let $A \cap B$ denote the intersection of A and B, let $A \cup B$ denote the union of A and B. Let A^c be the complement of A (i.e., $A^c = \{x : x \notin A\}$), and let $\emptyset$ be the empty set.

1. One of the following statements is true, and the other is false. Which one is true?
   (a) $(A \cap B) \cap C = (A \cap B) \cap (A \cap C)$.
   (b) $A \cap B = B \cup A$.
2. Which one of the following two statements is true?
   (a) $(A \cap B)^c = A^c \cup B^c$
   (b) $(A \cap B)^c = A^c \cap B^c$
3. Which one of the following two statements is true?
   (a) $(A \cup B)^c = A^c \cup B^c$
   (b) $(A \cup B)^c = A^c \cap B^c$
4. Which one of the following two statements is true?
   (a) $A \cap \emptyset = A$
   (b) $(A \cup B) \cap (A \cup B^c) = A$

2 Functions: Limits

The limit of $f(x)$ as x approaches infinity (written $\lim_{x\to\infty} f(x)$) equals k if and only if for any $\delta > 0$, there exists an N such that $|f(x) - k| < \delta$ for all $x > N$.

Prove that:

$$\lim_{x\to\infty} \frac{1}{x} = 0$$

3 Summations

Let $a_1, a_2, \ldots, a_n$ and $b_1, b_2, \ldots, b_n$ be sequences of length n and let c be a constant.

1. Which one of the following two statements is true?
   (a) $\sum_{i=1}^{n} c = nc$.
   (b) $\sum_{i=1}^{n} c = c$.
2. Which one of the following two statements is true?
   (a) $\sum_{i=1}^{n} ca_i = nc \sum_{i=1}^{n} a_i$
   (b) $\sum_{i=1}^{n} ca_i = c \sum_{i=1}^{n} a_i$
3. Which one of the following two statements is true?
   (a) $\sum_{i=1}^{n} f(a_i) = f\left(\sum_{i=1}^{n} a_i\right)$.
   (b) $\sum_{i=1}^{n} \left(f(a_i) + g(a_i)\right) = \sum_{i=1}^{n} f(a_i) + \sum_{i=1}^{n} g(a_i)$.
4. Which one of the following two statements is true?
   (a) $\left(\sum_{i=1}^{n} a_i\right)^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} a_i a_j$.
   (b) $\left(\sum_{i=1}^{n} a_i\right)^2 = \sum_{i=1}^{n} a_i^2$.
5. Which one of the following two statements is true?
   (a) $\left(\sum_{i=1}^{n} a_i\right) \times \left(\sum_{i=1}^{n} b_i\right) = \sum_{i=1}^{n} \sum_{i=1}^{n} a_i b_i$.
   (b) $\left(\sum_{i=1}^{n} a_i\right) \times \left(\sum_{i=1}^{n} b_i\right) = \sum_{i=1}^{n} \sum_{j=1}^{n} a_i b_j$.
6. Which one of the following two statements is true?
   (a) $\sum_{j=1}^{n} \sum_{i=1}^{n} a_i b_j = \sum_{i=1}^{n} \sum_{j=1}^{n} a_i b_j$.
   (b) $\sum_{j=1}^{n} \sum_{i=1}^{n} a_i b_j = \sum_{j=1}^{n} a_i \sum_{i=1}^{n} b_j$.

4 Functions: Slopes and elasticities

Find the slope ($\frac{dy}{dx}$) and elasticity ($\frac{dy}{dx}\frac{x}{y}$) in terms of x (i.e., "y" should not appear in your answer) for each of the following relationships:

1. $y = \beta_0 + \beta_1 x$.
2. $y = \beta_0 + \beta_1 x + \beta_2 x^2$.
3. $\ln y = \beta_0 + \beta_1 \ln x$.
4. $y = (\beta_0 + \beta_1 x)^{\beta_2}$

5 An introduction to R

Write an R script that does the following:

1. Print out your first name, 50 times. The R function `rep` will be useful for this.
2. Print out the first 50 odd numbers, beginning with 1. The R function `seq` will be useful for this.
3. Create a function called sqrtOfMyID.
   - It should take two arguments: the first should be a string and the second should be a number.
   - It should print out the following sentence:

     My name is [the string]. My student ID is [the number], and its square root is [the square root of the number].

     The R function `cat` will be useful for this.
4. Call that function in such a way as to print out your name, your student ID, and the square root of your student ID.

Attach the printed output from your script to this assignment, and turn in the script itself by WebCT.