Problem Set #10 Answer Key

Economics 435: Quantitative Methods

Fall 2011

1 Proxy variables

a)

$$\begin{aligned} \text{plim } \hat{\beta}_1^A &= \frac{cov(school, faminc)}{var(faminc)} \\ &= \frac{cov(\beta_0 + \beta_1 faminc + \beta_2 ability + u, faminc)}{var(faminc)} \\ &= \beta_1 + \beta_2 \frac{cov(ability, faminc)}{var(faminc)} \end{aligned}$$

b) I would imagine that $\beta_2 > 0$ (higher-ability individuals are more likely to acquire additional school) and $cov(ability, faminc) > 0$ (higher-ability individuals are more likely to come from high-income families). This implies that $\hat{\beta}_1^A$ is likely to overestimate β_1.

c)

$$\begin{aligned} E(school|faminc, test) &= E(\beta_0 + \beta_1 faminc + \beta_2 ability + u|faminc, test) \\ &= \beta_0 + \beta_1 faminc + \beta_2 E(ability|faminc, test) + E(u|faminc, test) \\ &= \beta_0 + \beta_1 faminc + \beta_2(\gamma_0 + \gamma_1 faminc + \gamma_2 test) \\ &= (\beta_0 + \beta_2\gamma_0) + (\beta_1 + \beta_2\gamma_1) faminc + \beta_2\gamma_2 test \end{aligned}$$

d)

$$\text{plim } \hat{\beta}_1^B = \beta_1 + \beta_2\gamma_1$$

e) Most likely, the relationship between family income and ability is much weaker if you control for test scores, i.e., $|\gamma_1| < |\gamma_2|$. But I wouldn't be very confident making an assertion about the sign of γ_1. Given two students with identical test scores, would you assume the student from the wealthier family has higher or lower ability? I'd guess "lower," though my guess wouldn't be a very confident one.

So my best guess is that:

- The bias in $\hat{\beta}_1^B$ is negative. That is, it will tend to understate the effect of family income.
- The absoulute size of the bias in $\hat{\beta}_1^B$ is probably smaller than the bias in $\hat{\beta}_1^A$.
- So, you might expect the true value of β_1 to be somewhere between $\hat{\beta}_1^B$ and $\hat{\beta}_1^A$, and somewhat closer to $\hat{\beta}_1^B$.

Your answer here may be different, without being wrong.

2 Regression with standardized variables

a)

$$\hat{\beta}_0 = 0$$

b)

$$R^2 = \hat{\beta}_1^2$$

c)

$$c\hat{o}rr(\tilde{y}, \tilde{x}) = \hat{\beta}_1$$

d) Well, since we have shown that $\hat{\beta}_1 = c\hat{o}rr(\tilde{x}, \tilde{y})$, and all correlations are between -1 and $+1$, the answer to the question is "no."