

Problem Set #7: Causality

Economics 435: Quantitative Methods

Fall 2011

1 Public sector unionization and the size of government: Part IV

- a) Fix any errors in the Tables 1-3 and Figures 1-3.
- b) Complete Section 3 and Section 4.2 of the paper. Things to keep in mind:
 1. Our primary underlying goal is to measure the *effect* of public sector unionization on the size of government. The results should be discussed in the context of that goal.
 2. Your discussion should not focus on:
 - Statistical significance of the parameter estimates.
 - The fit of the model, as measured by the R^2 .
 3. Your discussion should focus on:
 - The magnitudes of the parameter estimates.
 - In your discussion you should discuss these magnitudes with the intention of helping your reader determine whether they are big or small.
 - A good way to do that is to have a statement in there like:

Our results imply that a one percentage point increase in the union membership rate is associated with a 0.2 percent increase in the size of per capita government payroll. A change in the membership rate from the 25th percentile of the distribution (Colorado, 24%) to the 75th percentile (Illinois, 28%) would imply a change in the predicted government payroll of 0.8 percent.
 - Your discussion of magnitudes should address inference issues by reporting confidence intervals for key parameter estimates.
 - The extent to which the coefficient estimates provide a plausible measure of the causal effect of interest.
 - Section 3 should describe the model you would like to estimate, as well as the model you are estimating.
 - This discussion should also identify the most important omitted variables from the model you are estimating (relative to the model you would like to estimate), and it should characterize the likely direction of bias in our estimates of the parameter of interest.
 4. When discussing results, try to stay on as much common ground with your readers as possible. I'll explain what I mean by that in class.

2 Project STAR

Project STAR was a famous experiment designed to measure the effect of class size on academic achievement. In this experiment both students and teachers were randomly assigned to classrooms of different sizes. The control group were assigned to classrooms with 22 to 25 students, the typical number in Tennessee public schools at the time. The treatment group were assigned to classrooms with 13 to 17 students. The experiment continued for 4 years, with the students taking a standardized test at the end of each year.

Within each school, students and teachers were assigned to the treatment group or control group by a random mechanism. However, students and teachers were not randomly assigned to schools. In addition, the assignment rates differed systematically across schools: one school might have 2 small classes and 1 regular-sized class, while another school might have 1 small class and 2 regular-sized class.

We will try to figure out how to estimate the treatment effect of smaller classes on test scores in this context.

We will simplify the model in a few ways to avoid complications. Individual students are indexed by $i = 1, 2, 3, \dots, N$. Schools are indexed by $s = 1, 2, 3, \dots, S$, and each school contains n students.

First, we will suppose a simple fixed treatment effect specification:

$$y = \beta_0 + \beta_1 x + u$$

where β_1 is the treatment effect of interest and β_0 is defined such that $E(u) = 0$.

Randomization within the school implies that:

$$E(u|x, s) = E(u|s)$$

However, we will make no further restrictions on the variables.

a) Let y^T be the treated outcome of a given individual, Let y^C be the untreated outcome of the individual, and let TE be the treatment effect for that individual. Find (y^T, y^C, TE) in terms of (β_0, β_1, u) .

b) Find the average treatment effect (ATE) in terms of model parameters:

c) Let $\bar{y}^T$ be the average test score among treated students, and let $\bar{y}^C$ be the average test score among control-group students. Without further assumptions, is $\bar{y}^T - \bar{y}^C$ an unbiased estimator of the average treatment effect? Prove it.

d) Let $\bar{y}_s^T$ be the average test score among treated students in school s , and let $\bar{y}_s^C$ be the average test score among control-group students in school s . Is $(\bar{y}_s^T - \bar{y}_s^C)$ an unbiased estimator of the average treatment effect? Prove it.

e) Use these results to construct an estimator of the average treatment effect that is consistent as S goes to infinity. Prove¹ that it is consistent

f) Suppose that you have been asked by a government commission to comment on a recently-released research paper using the Project STAR results to measure the effect of class sizes. They find a large benefit to smaller classes. Unfortunately, that paper used $\bar{y}^T - \bar{y}^C$ to measure the effect. You have just explained to the commission that this estimate is potentially biased, and they would like to get an idea of the direction of the bias.

Using plain language, describe for the committee the primary real-world mechanisms that would lead to bias in the estimates, and explain the likely direction of the bias.

¹For this you need the following generalization of the law of large numbers: Let x_i be an independent (but not necessarily identically distributed) sequence of random variables with common mean $E(x_i) = \mu$ and bounded variance $\text{var}(x_i) \leq \sigma^2 < \infty$. Then $\text{plim } \bar{x}_n = \mu$.