Problem Set #7 Answer Key

Economics 435: Quantitative Methods

Fall 2011

1 Public sector unionization and the size of government: Part IV

Every student will do this question differently, so there are many correct "answers" here. If I see any common mistakes or have any suggestions I'll provide them in class.

2 Project STAR

a)

$$
\begin{aligned}
y^T &= \beta_0 + \beta_1 + u \\
y^C &= \beta_0 + u \\
TE &= \beta_1
\end{aligned}
$$

b)

$$
ATE = E(TE) = E(\beta_1) = \beta_1
$$

c) No.

$$
\begin{aligned}
E(\bar{y}^T - \bar{y}^C) &= E(y|x=1) - E(y|x=0) \\
&= E(\beta_0 + \beta_1 x + u | x = 1) - E(\beta_0 + \beta_1 x + u | x = 0) \\
&= \beta_0 + \beta_1 + E(u|x=1) - \beta_0 - E(u|x=0) \\
&= \beta_1 + E(u|x=1) - E(u|x=0)
\end{aligned}
$$

We have not specifically assumed[1] that $E(u|x = 1) = E(u|x = 0)$, or anything that implies it, so this estimator is biased.

d) Yes.

$$
\begin{aligned}
E(\bar{y}_s^T - \bar{y}_s^C) &= E(y|x=1, s) - E(y|x=0, s) \\
&= E(\beta_0 + \beta_1 x + u | x = 1, s) - E(\beta_0 + \beta_1 x + u | x = 0, s) \\
&= \beta_0 + \beta_1 + E(u|x=1, s) - \beta_0 - E(u|x=0, s) \\
&= \beta_1 + E(u|s) - E(u|s) \\
&= \beta_1
\end{aligned}
$$

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[1] You might feel the need to prove specifically that $E(u|x = 1) \neq E(u|x = 0)$. Well, you can't. Our assumptions are consistent with $E(u|x = 1)$ and $E(u|x = 0)$ being equal, and they are consistent with them being different. This brings up a subtle point about bias that you should make sure you understand. When we classify an estimator as unbiased we mean that it is unbiased for every probability distribution that satisfies our assumptions. For example there are some probability distributions (those in which $E(x) = 0$) in which $E(5\bar{x}) = E(x)$, but $5\bar{x}$ is still a biased estimator of $E(x)$.

This estimator is unbiased.

e) Let:

$$\hat{\beta}_1 = \frac{1}{S} \sum_{s=1}^{S} (\bar{y}_s^T - \bar{y}s^C)$$

We have the average of S independent random variables, all with mean β_1 and bounded variance (since y is a test score, it is bounded between "no correct answers" and "all correct answers", and so its variance must also be bounded). So by the law of large numbers,

$$\text{plim } \hat{\beta}_1 = \beta_1$$

f) Here's my answer: Schools that have a larger proportion of students in small classes may be different from schools that have a smaller proportion. The most important potential difference is in resources: it costs more money per student to provide smaller classes, so schools with more money may provide more smaller classes. In addition, parents prefer smaller classes, so schools where parents are more effective at lobbying for their children - generally educated parents with lots of free time - will tend to have smaller classes. Again, parents meeting this description will lobby for other school features, and will be able to provide their kids with more education at home. These factors all imply that a school with more small classes will tend to produce better test scores even if class sizes themselves have no effect. So the true benefit is likely to be smaller than this study finds.