Problem Set #8 Answer Key

Economics 435: Quantitative Methods

Fall 2011

Please use WebCT to turn in both the Word document and R code for question (1) and turn in your answer to question (2) in class.

1 Public sector unionization and the size of government: Part V

Please see the file http://www.sfu.ca/~bkrauth/econ435/restricted/psu5.pdf. The R code used to generate these results is available at http://www.sfu.ca/~bkrauth/econ435/restricted/qunionization5.r

2 Fixed effects with measurement error

a)

$$\begin{aligned}
\text{plim } \hat{\beta}^{OLS} &= \text{plim } \frac{\hat{cov}(y_{it}, \tilde{x}_{it})}{\hat{var}(\tilde{x}_{it})} \\
&= \frac{cov(y_{it}, \tilde{x}_{it})}{var(\tilde{x}_{it})} \\
&= \frac{cov(a_i + \beta x_i + \beta v_{it} + u_{it}, x_i + v_{it} + \epsilon_{it})}{var(x_i + v_{it} + \epsilon_{it})} \\
&= \frac{cov(a_i, x_i) + \beta var(x_i) + \beta var(v_{it})}{var(x_i) + var(v_{it}) + var(\epsilon_{it})} \\
&= \frac{\beta(\sigma_x^2 + \sigma_v^2) + \sigma_x \sigma_a \rho_{a,x}}{(\sigma_x^2 + \sigma_v^2) + \sigma_\epsilon^2}
\end{aligned}$$

b)

$$\begin{aligned}
\text{plim } \hat{\beta}^{FD} &= \text{plim } \frac{\hat{cov}(\Delta y_{it}, \Delta \tilde{x}_{it})}{\hat{var}(\Delta \tilde{x}_{it})} \\
&= \frac{cov(\Delta y_{it}, \Delta \tilde{x}_{it})}{var(\Delta \tilde{x}_{it})} \\
&= \frac{cov(\beta \Delta v_{it} + \Delta u_{it}, \Delta v_{it} + \Delta \epsilon_{it})}{var(\Delta v_{it} + \Delta \epsilon_{it})} \\
&= \frac{\beta var(\Delta v_{it})}{var(\Delta v_{it}) + var(\Delta \epsilon_{it})} \\
&= \beta \frac{\sigma_v^2}{\sigma_v^2 + \sigma_\epsilon^2}
\end{aligned}$$

c) If there is no fixed effect, then the probability limit of the OLS estimator is:

$$\text{plim } \hat{\beta}^{OLS} = \beta \frac{(\sigma_x^2 + \sigma_v^2)}{(\sigma_x^2 + \sigma_v^2) + \sigma_\epsilon^2}$$

Since $\frac{(\sigma_x^2+\sigma_v^2)}{(\sigma_x^2+\sigma_v^2)+\sigma_\epsilon^2}$ is closer to one than $\frac{\sigma_v^2}{\sigma_v^2+\sigma_\epsilon^2}$ is, the OLS estimator has smaller asymptotic bias.

d) If the fixed effect is uncorrelated with x_i, then the probability limit of the OLS estimator is:

$$\text{plim } \hat{\beta}^{OLS} = \beta \frac{(\sigma_x^2 + \sigma_v^2)}{(\sigma_x^2 + \sigma_v^2) + \sigma_\epsilon^2}$$

Since $\frac{(\sigma_x^2+\sigma_v^2)}{(\sigma_x^2+\sigma_v^2)+\sigma_\epsilon^2}$ is closer to one than $\frac{\sigma_v^2}{\sigma_v^2+\sigma_\epsilon^2}$ is, the OLS estimator has smaller asymptotic bias.

e) If there is no measurement error, then the probability limit of the OLS estimator is:

$$\text{plim } \hat{\beta}^{OLS} = \frac{\beta(\sigma_x^2 + \sigma_v^2) + \sigma_x \sigma_a \rho_{a,x}}{(\sigma_x^2 + \sigma_v^2)}$$

and the probability limit of the FD estimator is

$$\text{plim } \hat{\beta}^{FD} = \beta$$

Since the FD estimator is consistent and the OLS estimator is not, the FD estimator obviously has smaller asymptotic bias.

f) The first statement is the correct one.