Exam #1

Economics 808: Macroeconomic Theory

Fall 2004

Time will be a factor, so be careful.

1 Time series

Suppose that x_t is an $AR(2)$ stochastic process:

$$x_t = ax_{t-2} + \epsilon_t$$

where ϵ_t is white noise with mean zero and variance σ_ϵ^2, and $a \in (0,1)$.

a) Find x_t as a function of current and past values of ϵ_t.

b) Calculate $E_t(x_{t+1})$, $var(x_t)$, $cov(x_t, x_{t+1})$ and $cov(x_t, x_{t+2})$

c) Plot the impulse response function $irf(k) \equiv \frac{\partial x_{t+k}}{\partial \epsilon_t}$ for $k = 0, 1, 2, 3, 4, 5$.

2 Population growth

We will modify our basic optimal growth model to include a growing population. Specifically, the population (labour force) grows at the exogenous rate g_L:

$$L_{t+1} = (1 + g_L)L_t$$

Aggregate variables are defined in both total and per capita terms, i.e., $C_t = c_t L_t$, $K_t = k_t L_t$. The social planner's objective function is given in terms of per-capita consumption:

$$U = \sum_{t=0}^{\infty} \beta^t \ln c_t$$

and the planner's constraints are given by:

$$K_{t+1} \leq K_t^\alpha L_t^{1-\alpha} - C_t$$

a) Define the social planner's problem, preferably in a form that is convenient to solve.

b) Write down a Bellman equation for this planner's problem. Remember, it will be easier to solve with only one state variable!

c) Write down the first order conditions, the Benveniste-Scheinkman equation, and the transversality condition for this Bellman equation.

d) Solve for the Euler equation.

e) Characterize the steady state and/or balanced growth path. Describe what is happening to k_t, K_t, c_t and C_t along the balanced growth path.

f) Is steady-state per-capita consumption increasing, decreasing, or unaffected by g_L? Explain why.

3 Optimal kleptocracy

Consider the following model of an economy with a government that wishes only to maximize its own tax revenue. The consumer is endowed with one unity of labor each period ($L_t = 1$) and has utility function:

$$\sum_{t=0}^{\infty} \beta^t u(c_t)$$

and budget constraint:

$$c_t + k_{t+1} + b_{t+1} \leq (1 - \tau^k) r_t k_t + (1 - \tau^w) w_t + (1 - \tau^b) R_t b_t + \pi_t$$

In addition, we assume that $u(.)$ has the usual properties, that $k_t \geq 0$, that $k_0 > 0$ is given, and the usual no-Ponzi-game condition holds. Notice that taxes are collected on income from physical capital, financial (bond) capital, and labor, and that these tax rates may differ. For the moment, simply take these tax rates as given.

The firm has Cobb-Douglas production function $y_t = A k_t^{\alpha} L_t^{1-\alpha}$.

a) Define the consumer's problem, firm's problem, and equilibrium in this economy.

b) Find equilibrium prices (r_t, w_t, and R_t) as a function of k_t and model parameters.

c) Find the Euler equation, and use it to solve for steady state capital as a function of model parameters.

d) Calculate steady state tax revenue, as a function of model parameters including tax rates.

e) Now suppose you work for the government, and are asked to help set tax policy. You are subject to the constraint $\tau^k, \tau^w, \tau^b \leq 1$; in other words you cannot collect taxes from someone who does not have the money. Note that tax rates can be negative (in other words, subsidies are allowed). What combination of tax rates maximizes steady-state government revenue?