Exam #2

Economics 808: Macroeconomic Theory

Fall 2002

Questions 1 and 2 are worth 35 points each, question 3 is worth 30. "Extra credit" questions will get a small and variable number of points; don't take time away from the rest of the exam to answer them.

1 The welfare cost of business cycles

Lucas (1987) makes a simple calculation of the welfare cost of business cycles by asking the following question: how much of a decrease in average income would the representative consumer be willing to accept in exchange for the complete elimination of fluctuations in consumption?

Suppose that the representative consumer has a Cobb-Douglas expected utility function:

$$U = E_{-1} \sum_{t=0}^{\infty} \beta^t \ln c_t$$

The consumer has two choices.

Option 1 is to have completely predictable consumption:

$$c_t = \bar{c}$$

where $\bar{c}$ is a fixed and exogenous number.

Option 2 is to have slightly higher average consumption in exchange for increased variability in consumption:

$$c_t = \bar{c}(1+\lambda)(1+\epsilon_t)$$

where $\lambda > 0$ is exogenous, and ϵ_t is an IID exogenous random variable[1] with mean zero and standard deviation σ_ϵ.

Lucas' question is: at what value of λ will the consumer be indifferent between option 1 and option 2? In other words, for what value of λ will

$$E_{-1} \sum_{t=0}^{\infty} \beta^t \left(\ln\left(\bar{c}\right)\right) = E_{-1} \sum_{t=0}^{\infty} \beta^t \left(\ln\left(\bar{c}(1+\lambda)(1+\epsilon_t)\right)\right) \tag{1}$$

[1] We need to assume that the support of ϵ is bounded away from -1. If you don't know why, don't worry about it.

a) Does the solution depend on the value of $\bar{c}$ or β? Support your answer.

b) Let $v(\lambda) \equiv \ln(1+\lambda)$. Find the first-order Taylor series approximation to v in a neighborhood of $\lambda = 0$. Just in case you forgot, the first order Taylor series approximation is:

$$v(\lambda) \approx v(0) + v'(0)\lambda$$

c) Let $w(\epsilon) \equiv \ln(1+\epsilon)$. Find the second-order Taylor series approximation to w in a neighborhood of $\epsilon = 0$. Just in case you forgot, the second order Taylor series approximation is:

$$w(\epsilon) \approx w(0) + w'(0)\epsilon + \frac{1}{2}w''(0)\epsilon^2$$

d) Substitute your two approximations into equation (1) and display the result.

e) Take expectations of this result, and solve for λ in terms of σ_ϵ.

f) The standard deviation of log real quarterly consumption in the United States is approximately 0.013. Assuming $\sigma_\epsilon = 0.013$, what value of λ solves equation (1)? This question can be answered without a calculator, just be careful to get the right number of decimal places.

g) State your result in words. If we believe the assumptions of this model, do business cycles have a large welfare cost?

2 Money in the utility function

Consider the following simple money-in-the-utility function model.

To keep things as simple as possible, we will ignore production and assume that the representative agent receives Y units of the consumption good each period. The good is nonstorable, so it must be consumed in that period. The agent's utility function is:

$$U = \sum_{t=0}^{\infty} \beta^t \left(\ln C_t + b \ln \frac{M_t}{P_t} \right)$$

Money enters the economy by a "helicopter drop." In other words, the government simply gives a sum of money D_t to the consumer at the beginning of period t. This is added to the consumer's existing stock of money, which starts out at M_{-1}. The consumer's budget constraint (in real terms) is given by:

$$\frac{V_{t+1}}{P_t} + B_{t+1} + C_t + \frac{M_t}{P_t} \leq Y + \frac{M_{t-1}}{P_t} + \frac{D_t}{P_t} + R_t B_t + I_t \frac{V_t}{P_t}$$

where V is the consumer's holdings of dollar-denominated bonds and B is the consumer's holdings of consumption-denominated bonds. Finally, the money supply grows at a constant rate π, which implies that $D_t = \pi M_{t-1}$. Of course, the representative agent takes the time path of D_t as given when choosing money holdings.

a) Define a competitive equilibrium in this economy.

b) Find the equilibrium real interest rate R_t.

c) Find real money demand (M_t/P_t) as a function of P_{t+1}/P_t, Y, and the model parameters.

d) In the steady state of the economy (which will be reached immediately), real money demand will be constant, i.e.,

$$\frac{M_t}{P_t} = \frac{M_{t+1}}{P_{t+1}}$$

Rather than proving this, I'll ask you to simply accept it as fact and derive its implications. What does this imply about the rate of inflation?

e) Find real money demand as a function of Y, π, and model parameters. Is money demand increasing/decreasing/constant in real income?

f) The inflation rate enters nonlinearly into the money demand function. Take a first order Taylor series approximation to the *log* money demand function (i.e., $\ln M_t/P_t$) in a neighborhood of $\pi = 0$ and report the results. Is money demand increasing/decreasing/constant in inflation? Nominal interest rates?

g) Is the utility of the representative agent increasing/decreasing/constant in inflation?

h) Extra credit: Suppose that you have a cross country data set on per capita real GDP, inflation rates, and per capita real balances. Also suppose that the crazy assumptions of this model hold (constant income, Cobb-Douglas preferences, etc.) and that the behavioral parameters β and b are the same across all agents. Construct an empirical strategy for estimating the welfare costs of inflation.

3 Fun with VARs

A macroeconomist (call him Larry) proposes the following structural VAR:

$$\begin{bmatrix} e_t \\ y_t \\ m_t \end{bmatrix} = B \begin{bmatrix} e_{t-1} \\ y_{t-1} \\ m_{t-1} \end{bmatrix} + C \begin{bmatrix} u_{et} \\ u_{yt} \\ u_{mt} \end{bmatrix}$$

where e_t is the employment rate, y_t is real per capita GDP, and m_t is the real money supply. The structural shocks (u_{et}, u_{yt}, u_{mt}) are white noise with mean zero and covariance matrix I (the identity matrix).

The structural parameters are given by:

$$B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

and

$$C = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ 0 & c_{22} & c_{23} \\ 0 & 0 & c_{33} \end{bmatrix}$$

The reduced form VAR has the form

$$\begin{bmatrix} e_t \\ y_t \\ m_t \end{bmatrix} = B \begin{bmatrix} e_{t-1} \\ y_{t-1} \\ m_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{et} \\ \epsilon_{yt} \\ \epsilon_{mt} \end{bmatrix}$$

where the reduced form shocks $(\epsilon_{et}, \epsilon_{yt}, \epsilon_{mt})$ are white noise with mean zero and covariance matrix Σ:

$$\Sigma \equiv \begin{bmatrix} \sigma_e^2 & \sigma_{ey} & \sigma_{em} \\ \sigma_{ey} & \sigma_y^2 & \sigma_{ym} \\ \sigma_{em} & \sigma_{ym} & \sigma_m^2 \end{bmatrix}$$

Remember that if Larry's structural model is correct, then $\Sigma = CC'$.

a) Interpret the identifying assumptions in the structural model. Which variables respond to shocks to the other variables immediately and which respond with a one-period lag?

b) Calculate the following impulse response functions to a money supply shock in terms of elements of the B and C matrices:

$$\frac{\partial e_t}{\partial u_{mt}}, \frac{\partial e_{t+1}}{\partial u_{mt}} \frac{\partial y_t}{\partial u_{mt}}, \frac{\partial y_{t+1}}{\partial u_{mt}},$$

c) Now suppose that you have estimates of the reduced form parameter matrices B and Σ. Find the above impulse response functions based on those estimates.

d) Another macroeconomist (by the name of Marty) comes along and says that Larry's identifying assumption is mistaken. In particular Marty believes that employment shocks affect output immediately but output shocks affect employment only with a lag, where Larry believes the opposite. Larry and Marty agree that money shocks affect both output and employment immediately. In other words, Marty believes that

$$\begin{bmatrix} e_t \\ y_t \\ m_t \end{bmatrix} = B \begin{bmatrix} e_{t-1} \\ y_{t-1} \\ m_{t-1} \end{bmatrix} + D \begin{bmatrix} u_{et} \\ u_{yt} \\ u_{mt} \end{bmatrix}$$

where

$$D = \begin{bmatrix} d_{11} & 0 & d_{13} \\ d_{21} & d_{22} & d_{23} \\ 0 & 0 & d_{33} \end{bmatrix}$$

Calculate the following impulse response functions to a money supply shock in terms of elements of the B and D matrices:

$$\frac{\partial y_t}{\partial u_{mt}}, \frac{\partial y_{t+1}}{\partial u_{mt}}, \frac{\partial e_t}{\partial u_{mt}}, \frac{\partial e_{t+1}}{\partial u_{mt}}$$

e) Now suppose that you have estimates of the reduced form parameter matrices B and Σ. Find the above impulse response functions based on those estimates and Marty's identifying assumption.

f) Extra credit: what general principle do these results suggest?