Exam #2

Economics 808: Macroeconomic Theory

Fall 2004

Problem #1 is worth 35 points, problem #2 is worth 65 points.

1 Identifying VARs through heteroskedasticity

Some recent work by Roberto Rigobon and Brian Sack (for example, in the May 2003 *Quarterly Journal of Economics*) has developed an alternative scheme for identifying structural VARs which exploits heteroskedasticity in the shocks. We will investigate an example inspired by this work.

Suppose that we are interested in characterizing the relationship between the stock market (say, returns on the S&P 500) and monetary policy as indicated by the federal funds rate. Suppose we have fairly high-frequency data, and that we have in mind the structural VAR:

$$X_t = AX_t + BX_{t-1} + Cv_t$$

in matrix notation, or

$$\begin{bmatrix} f_t \\ s_t \end{bmatrix} = \begin{bmatrix} 0 & a_2 \\ a_1 & 0 \end{bmatrix} \begin{bmatrix} f_t \\ s_t \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} f_{t-1} \\ s_{t-1} \end{bmatrix} + \begin{bmatrix} c_{11} & 0 \\ 0 & c_{22} \end{bmatrix} \begin{bmatrix} v_{ft} \\ v_{st} \end{bmatrix}$$

where $E_{t-1}(v_t) = 0$ and $E_{t-1}(v_t v_t') = I$. We could identify this structural VAR from the data if we assume either $a_1 = 0$ or $a_2 = 0$. Unfortunately, financial market variables like the stock market and the federal funds rate can respond to events in the economy another almost instantaneously, so we are not willing to make either assumption.

However, we have another piece of information which can aid in identification. We know that the Fed has regularly scheduled policy-making meetings, and these meetings often lead to clear shifts in interest rate policy. In other words, the shock to the federal funds rate may be conditionally heteroskedastic, with higher variance immediately following an FOMC meeting.

To model this we will assume that we have access to a binary time series m_t where $m_t = 1$ if there is an FOMC meeting concluding during period t, and $m_t = 0$ otherwise. To keep things simple we will assume that m_t is nonstochastic and its only effect is to produce a more variable shock. Specifically, we assume that:

$$c_{11} = c_{11}^m m_t + c_{11}^n (1 - m_t)$$

where c_{11}^m is the standard deviation of the federal funds rate shock in a period when there is a meeting, and c_{11}^n is the standard deviation of the shock when there is no meeting.

The associated reduced form regression is:

$$X_t = \Gamma_1 X_{t-1} + \epsilon_t$$

where $E_{t-1}(\epsilon_t | m_t = 1) = E_{t-1}(\epsilon_t | m_t = 0) = 0$ and:

$$\begin{aligned} E_{t-1}(\epsilon_t \epsilon_t' | m_t = 1) &= \Omega^m = \begin{bmatrix} \omega_{11}^m & \omega_{12}^m \\ \omega_{12}^m & \omega_{22}^m \end{bmatrix} \\ E_{t-1}(\epsilon_t \epsilon_t' | m_t = 0) &= \Omega^n = \begin{bmatrix} \omega_{11}^n & \omega_{12}^n \\ \omega_{12}^n & \omega_{22}^n \end{bmatrix} \end{aligned}$$

If you don't remember how to invert a matrix, I'll just give you a result:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

a) Find the value of each element in Ω^m and Ω^n as a function of the structural parameters.

b) Use your answer above to find the structural parameter of interest a_1 (the immediate effect of shocks to the federal funds rate on the stock market) as a function of the reduced form coefficients.[1] Assume that $a_1 a_2 \neq 1$ and $c_{11}^m \neq c_{11}^n$.

2 The big push

Suppose that the economy is described by the following two-period model. There are a large number of sectors, each of which produces a distinct good. We will model this by assuming that there is a continuum of infinitesimal sectors indexed by $q \in [0, 1]$.

The representative consumer supplies L units of labor inelastically per period and receives all firm profits. Her utility is given by:

$$\int_0^1 \ln c_1(q) dq + \beta \int_0^1 \ln c_2(q) dq$$

where $c_t(q)$ is her consumption of good q in period t. We make labour the numeraire good, so the wage in each period is exactly one. The consumer also can buy or sell one period bonds that cost one wage unit in period one and pay R wage units in period two. As a result the consumer's budget constraints are:

$$\begin{aligned} \int_0^1 p_1(q) c_1(q) dq &\leq C_1 \equiv L + \pi_1 - b \\ \int_0^1 p_2(q) c_2(q) &\leq C_2 \equiv L + Rb + \pi_2 \end{aligned}$$

where C_t is total consumption expenditure in period t.

[1] Note that this parameter is overidentified, so that there are multiple distinct solutions to this question. You only need to find one.

In each sector q, there is a competitive group of "traditional" firms with a simple constant returns to scale technology:

$$c_t^T(q) = L_t^T(q)$$

In addition, in each sector q there is a single "high-tech" firm. The high-tech firm has monopoly access to a more efficient production technology, subject to some limitations. Specifically, the high-tech firm cannot produce in period one. In order to produce in period two, it must use $L_1^H(q) = F$ units of labour in period one (think of this as the cost of installing the new technology). If it invests in period one, the high-tech firm can produce in period two according to the production function

$$c_2^H(q) = aL_2^H(q)$$

where $a > 1$. If it invests, the firm finances the investment through bonds.

Total output in sector q is given by:

$$c_t(q) = c_t^T(q) + c_t^H(q)$$

and total labour supply must satisfy:

$$\int_0^1 L_t^T(q) + L_t^H(q)dt \leq L$$

a) Find the consumer's demand curve $c_t(q)(p_t(q), C_t)$ for a given good q.

b) Suppose for the moment that the high-tech firms do not invest in period one. Find equilibrium quantity and price for each goods in both periods, and the equilibrium interest rate.

c) Suppose that a particular high-tech firm has invested in period one. Find the firm's profit-maximizing price, sales, and net operating revenue (revenue minus variable costs) in period 2, as a function of the consumer's total period-two consumption expenditure C_2.

d) Suppose that high-tech firm q is the *only* high-tech firm that chooses to invest. Find that firm's net operating revenue in period two, and its period two bond payment, in terms of model parameters. Under what conditions on parameters will the firm find the investment profitable?

e) Now suppose that all high-tech firms invest in period one. What is total consumption expenditure C_t in each of the two periods? What is the interest rate?

f) Find the profit maximizing price, period two operating profits, and period two bond payment for a representative high-tech firm when all firms have invested in period one. Under what conditions will the firm find its investment profitable?

g) If it is profitable for each firm to invest when every other firm invests, then "all firms invest" is a Nash equilibrium. If it is unprofitable for each firm to invest when no other firms invest, then "no firms invest" is a Nash equilibrium. Is it possible for both situations to be Nash equilibria at the same time?