Exam #2 Answer Key

Economics 808: Macroeconomic Theory

Fall 2004

Problem #1 is worth 35 points, problem #2 is worth 65 points.

1 Identifying VARs through heteroskedasticity

a) I have:

$$\begin{aligned}
\omega_{11}^m &= \frac{{c_{11}^m}^2 + a_2^2 c_{22}^2}{(1-a_1a_2)^2} \\
\omega_{12}^m &= \frac{a_1{c_{11}^m}^2 + a_2 c_{22}^2}{(1-a_1a_2)^2} \\
\omega_{22}^m &= \frac{a_1^2{c_{11}^m}^2 + c_{22}^2}{(1-a_1a_2)^2} \\
\omega_{11}^n &= \frac{{c_{11}^n}^2 + a_2^2 c_{22}^2}{(1-a_1a_2)^2} \\
\omega_{12}^n &= \frac{a_1{c_{11}^n}^2 + a_2 c_{22}^2}{(1-a_1a_2)^2} \\
\omega_{22}^n &= \frac{a_1^2{c_{11}^n}^2 + c_{22}^2}{(1-a_1a_2)^2}
\end{aligned}$$

b) Possible answers include:

$$\begin{aligned}
a_1 &= \frac{\omega_{12}^m - \omega_{12}^n}{\omega_{11}^m - \omega_{11}^n} \\
a_1 &= \frac{\omega_{22}^m - \omega_{22}^n}{\omega_{12}^m - \omega_{12}^n}
\end{aligned}$$

2 The big push

a) The demand curve is:

$$c_t(q) = \frac{C_t}{p_t(q)}$$

b) For all goods q we have $p_t(q) = 1$ and $c_t(q) = C_t = L$. The interest rate is β^{-1}.

c) Because the demand curve has unit price elasticity, the firm will set prices as high as possible. However, the firm cannot set the price above the price in the competitive market. So $p_2(q) = 1$ and $c_2(q) = C_2$. Net operating revenue is $C_2 \frac{a-1}{a}$.

d) If the firm is the only high-tech firm that invests, then $C_2 = L$, and its net operating revenue is $L\frac{a-1}{a}$. Its period two bond payment is F/β. The firm will find the investment profitable if

$$L\frac{a-1}{a} > \frac{F}{\beta}$$

or

$$(a-1) > \frac{aF}{\beta L}$$

e) In the first period, spending is $C_1 = L - F$. In the second period, spending is $C_2 = aL$. The interest rate is $R = \frac{aL}{\beta(L-F)}$.

f) Again the profit maximizing price is $p_2(q) = 1$. Since $C_2 = aL$, the operating profit of the representative firm will be $(a-1)L$. The bond payment will be $F\frac{aL}{\beta(L-F)}$. The firm will find its investment profitable if:

$$(a-1) > \frac{aF}{\beta(L-F)}$$

g) In this case, no. This is a simplified variation on the "big push" model of Murphy, Shleifer, and Vishny (JPE 199?). In that model there are parameter values such that both "everyone invests" and "no one invests" are Nash equilibria. They use this to get at some classic ideas in development economics; basically the idea is that in a less developed country, it may be unprofitable for a single company to modernize because they will experience insufficient demand to cover their capital costs unless other firms have also modernized. This is known as an "aggregate demand externality", and can yield to multiple Pareto-rankable equilibria.

Here, notice that the condition for profitability is actually more stringent when there are more firms investing. The reason for this is that the consumer has a fairly strong preference for consumption-smoothing across the two periods. When a large number of firms invest, this drives up interest rates quite a bit. This model would yield multiple equilibria if we were to generalize the preferences a little bit.