Exam #2 Answer Key

Economics 808: Macroeconomic Theory

Fall 2002

Questions 1 and 2 are worth 35 points each, question 3 is worth 30. "Extra credit" questions will get a small and variable number of points; don't take time away from the rest of the exam to answer them.

1 The welfare cost of business cycles

a) No. Notice that $\ln \bar{c}$ can be subtracted from both sides, giving:

$$0 = \ln(1+\lambda) + E\ln(1+\epsilon)$$

b)

$$v(\lambda) \approx \lambda$$

c)

$$w(\epsilon) \approx \epsilon - \frac{1}{2}\epsilon^2$$

d)

$$0 = \lambda + E\left(\epsilon - \frac{1}{2}\epsilon^2\right)$$

e)

$$\lambda = \frac{\sigma_\epsilon^2}{2}$$

f) The solution is $\lambda = 0.0000845$ or 0.008%.

g) The value to the representative consumer of eliminating all fluctuations in consumption is approximately 0.008 percent of GDP. If we believe the assumptions, the welfare cost of business cycles is very low.

2 Money in the utility function

a) The consumer's problem is to select $\{C_t, M_t, V_{t+1}, B_{t+1}\}$ to maximize:

$$U = \sum_{t=0}^{\infty} \beta^t \left(\ln C_t + b \ln \frac{M_t}{P_t} \right)$$

subject to the constraint

$$\frac{V_{t+1}}{P_t} + B_{t+1} + C_t + \frac{M_t}{P_t} \leq Y + \frac{M_{t-1}}{P_t} + \frac{D_t}{P_t} + R_t B_t + I_t \frac{V_t}{P_t}$$

plus the NPG condition on bonds and initial conditions $\{M_{-1}, B_0, V_0\}$.
An equilibrium is a set of prices $\{P_t, R_t, I_t\}$ and allocations $\{V_t, C_t, B_t, M_t, D_t\}$ such that:

1. Taking prices and D_t as given, the allocations solve the consumer's problem.
2. All markets clear, i.e.:

$$\begin{aligned} M_t &= M_{t-1} + D_t \\ D_t &= \pi M_{t-1} \\ C_t &= Y \\ B_t = V_t &= 0 \end{aligned}$$

b) The equilibrium real interest rate is $R_t = 1/\beta$.

c) Real money demand is given by:

$$\frac{M_t}{P_t} = bY \left(\frac{\frac{P_{t+1}}{P_t}}{\frac{P_{t+1}}{P_t} - \beta} \right)$$

d) The rate of inflation must therefore be π.

e) Real money demand is given by

$$\frac{M_t}{P_t} = bY \left(\frac{1+\pi}{1+\pi-\beta} \right)$$

f) The log money demand function is given by:

$$m_t - p_t = \ln b + y - \ln(1-\beta) - \frac{\beta}{1-\beta}\pi$$

where $m = \ln M$, etc.

g) It is decreasing in inflation.

h) First we use the data to estimate the money demand function. From the coefficient on inflation we can estimate β and from the intercept we can estimate b. Then we apply the same technique as used by Lucas. Find the λ that solves the equation:

$$\ln(1+\lambda)Y + b\left(\ln(1+\lambda)Y + \ln b - \ln(1-\beta) - \frac{\beta}{1-\beta}\pi\right) = \ln Y + b\left(\ln Y + \ln b - \ln(1-\beta)\right)$$

A first order Taylor series approximation and a little algebra gives us:

$$\lambda = \frac{b}{1+b}\frac{\beta}{1-\beta}\pi$$

As in the earlier problem, λ can be interpreted as the increase in average GDP needed to compensate the representative agent for an increase in the inflation rate of π.

3 Fun with VARs

a) Both employment and output respond immediately to money, and money responds to employment and output with a one-period lag. Employment responds to output immediately, while output responds to employment with a lag.

b) The answer is:

$$\begin{aligned}
\frac{\partial e_t}{\partial u_{mt}} &= c_{13} \\
\frac{\partial e_{t+1}}{\partial u_{mt}} &= b_{11}c_{13} + b_{12}c_{23} + b_{13}c_{33} \\
\frac{\partial y_t}{\partial u_{mt}} &= c_{23} \\
\frac{\partial y_{t+1}}{\partial u_{mt}} &= b_{21}c_{13} + b_{22}c_{23} + b_{23}c_{33}
\end{aligned}$$

c) First we need to recover c_{13}, c_{23}, and c_{33} from Σ. A little algebra gives us:

$$\begin{aligned}
c_{33} &= \sigma_m \\
c_{23} &= \frac{\sigma_{ym}}{\sigma_m} \\
c_{13} &= \frac{\sigma_{em}}{\sigma_m}
\end{aligned}$$

Substituting back in we get:

$$\begin{aligned}
\frac{\partial e_t}{\partial u_{mt}} &= \frac{\sigma_{em}}{\sigma_m} \\
\frac{\partial e_{t+1}}{\partial u_{mt}} &= b_{11}\frac{\sigma_{em}}{\sigma_m} + b_{12}\frac{\sigma_{ym}}{\sigma_m} + b_{13}\sigma_m \\
\frac{\partial y_t}{\partial u_{mt}} &= \frac{\sigma_{ym}}{\sigma_m} \\
\frac{\partial y_{t+1}}{\partial u_{mt}} &= b_{21}\frac{\sigma_{em}}{\sigma_m} + b_{22}\frac{\sigma_{ym}}{\sigma_m} + b_{23}\sigma_m
\end{aligned}$$

d) The answer is actually the same as before

$$\begin{aligned}
\frac{\partial e_t}{\partial u_{mt}} &= d_{13} \\
\frac{\partial e_{t+1}}{\partial u_{mt}} &= b_{11}d_{13} + b_{12}d_{23} + b_{13}d_{33} \\
\frac{\partial y_t}{\partial u_{mt}} &= c_{23} \\
\frac{\partial y_{t+1}}{\partial u_{mt}} &= b_{21}d_{13} + b_{22}d_{23} + b_{23}d_{33}
\end{aligned}$$

e) Again the results are the same:

$$\begin{aligned}
d_{33} &= \sigma_m \\
d_{23} &= \frac{\sigma_{ym}}{\sigma_m} \\
d_{13} &= \frac{\sigma_{em}}{\sigma_m}
\end{aligned}$$

Substituting back in we get:

$$\begin{aligned}
\frac{\partial e_t}{\partial u_{mt}} &= \frac{\sigma_{em}}{\sigma_m} \\
\frac{\partial e_{t+1}}{\partial u_{mt}} &= b_{11}\frac{\sigma_{em}}{\sigma_m} + b_{12}\frac{\sigma_{ym}}{\sigma_m} + b_{13}\sigma_m \\
\frac{\partial y_t}{\partial u_{mt}} &= \frac{\sigma_{ym}}{\sigma_m} \\
\frac{\partial y_{t+1}}{\partial u_{mt}} &= b_{21}\frac{\sigma_{em}}{\sigma_m} + b_{22}\frac{\sigma_{ym}}{\sigma_m} + b_{23}\sigma_m
\end{aligned}$$

f) There is in fact a general result here. If we are calculating the impulse response of the other variables in the VAR to a money shock, only the ordering of variables relative to the money shock matters. Changing their order with respect to one another does not change the IRF.