Problem Set #1

Economics 808: Macroeconomic Theory

Fall 2004

1 The Cobb-Douglas production function

Suppose that the world is described by the Solow model, and that the production function is:

$$F(K, L) = AK^{\alpha}L^{1-\alpha}$$

where $0 < \alpha < 1$. As you will see in your microeconomics course, this is known as the Cobb-Douglas production function. It has some very handy properties.

a) Show that this production function is "neoclassical," in the sense that it obeys the six conditions I outlined in lecture.

b) In the formulation above, technological progress is neutral. Show that with a suitable transformation of A we can rewrite the function so that technological progress is labor augmenting.

c) Find the equilibrium wage (w_t).

d) Find the equilibrium rental rate on capital (r_t).

e) Poor countries have a low capital-to-labour ratio, where rich countries have a high capital-to-labour ratio. What does this model imply about relative wages and interest rates (assume the same level of technology in both countries) in rich and poor countries?

f) What is labour's share of output $\left(\frac{w_t L_t}{Y_t}\right)$?

g) What is capital's share of output $\left(\frac{r_t K_t}{Y_t}\right)$?

2 Constant and increasing returns in the Solow model

In the Solow model, markets are competitive, and production exhibits constant returns to scale. As a result, factors of production are rented by firms at marginal revenue product, and firm profits are zero. Here's a simple problem to see why.

Suppose you are a firm with production function $Y = aL$. You buy labour at wage w, and sell your output on the open market at a price of p. You are a price taker in all markets. Without loss of generality, we normalize prices so that $p = 1$.

a) For a given wage rate w, find the:

1. Profit-maximizing output Y.
2. Profit-maximizing labour demand L.
3. Total profit

Note: "Infinity" is a potentially valid answer.

b) Now suppose that labour markets are competitive. Find the:

1. Market-clearing wage
2. Firm profits at that wage

c) Now we allow for increasing returns to scale. The production function is $Y = aL^2$. For a given wage rate w, find the:

1. Profit-maximizing output Y.
2. Profit-maximizing labour demand L.
3. Total profit

Will there be a market-clearing wage?

3 More on constant returns

Prove that, if the production function exhibits constant returns, all firms will use the same proportion of capital to labour.

Notice that this result means that the analysis in the previous question (which ignores capital) applies to the case where both capital and labor are used.

4 Growth accounting

We are going to do a little quantitative research on economic growth. Pick a country (Canada, your home country, a place you've always wanted to visit, etc.) and a computer program for doing the calculations. A spreadsheet such as Microsoft Excel will do fine for this particular problem, but feel free to use anything you like.

Look at the Penn World Tables (PWT) to find annual capital and GDP series data for your chosen country. The PWT is the standard source of GDP data for international comparisons. If your chosen country is not available on the PWT, I suggest chosing a country that is. If you prefer, you can track down your own data. The web address for the PWT data set is

`http://datacentre.chass.utoronto.ca/pwt/index.html`

a) I would like to see the following things (the printouts and graphs should be clearly labeled):

1. A printout from your computer program showing annual time series on
   (a) Capital per worker
   (b) Real GDP per worker
   (c) The growth rates for capital per worker and real GDP per worker
   (d) The Solow residual (growth rate of TFP)
2. A graph (also from your computer program) showing the time series of
   (a) The growth rate of capital per worker
   (b) The growth rate of real GDP per worker
   (c) The Solow residual
3. For the entire time frame of available data:
   (a) The average annual growth rate in capital per worker
   (b) The average annual growth rate in real GDP per worker
   (c) The average annual growth rate in TFP
   (d) The percentage of growth in output per worker that can be attributed to technical progress.

In calculating the Solow residual, assume that capital's share of output is 0.36 and labor's share is 0.64.

b) Next I would like you to answer the following questions, based on your results:

1. The Solow residual is supposedly measuring "technological progress". Would you say your estimates imply that technology progresses steadily or follows a very unpredictable path?
2. Does your estimate of technological progress imply that your chosen country occasionally experiences a *decrease* in productivity (technical regress)?
3. In your opinion (there are no wrong answers to this) is the Solow residual a good measure of technical progress over a period of 20 years? Over a period of one year? Briefly explain why you think what you do.

c) Finally I would like you to find 3 classmates who have analyzed countries other than the one you have chosen. Write down their names, the countries they have studied, and whether their results are similar to yours or different. If they are different, say how they are different.

Hints:

- The easiest way to calculate a growth rate is $g_x = \frac{\ln x_s - \ln x_t}{s-t}$.
- The PWT doesn't have separate series for GDP, capital, and size of labour force. Instead, you need to use their series for GDP/worker (GDPW) and capital/worker (KAPW). These two will allow you to calculate the Solow residual, provided you apply a little bit of algebra to the formula I gave you. In order to do this you need to remember that $\ln \frac{x}{y} = \ln x - \ln y$.