Problem Set #3

Economics 808: Macroeconomic Theory

Fall 2004

1 A Markov chain

A stylized fact that is familiar to macroeconomists is that stochastic process governing the U.S. Solow residual is well-modeled by a simple $AR(1)$ process Letting s_t be the quarterly Solow residual (in terms of deviations from its mean),

$$s_t = 0.95 s_{t-1} + \epsilon_t$$

where ϵ_t is white noise with standard deviation $\sigma_\epsilon = 0.007$.

a) Find $\mu_s \equiv E(s_t)$ and $\sigma_s(0) \equiv var(s_t)$.

b) Find the first autocovariance ($\sigma_s(1) \equiv cov(s_t, s_{t+1})$) and the first autocorrelation ($\rho_s(1) \equiv corr(s_t, s_{t+1})$) of the Solow residual.

c) Now suppose we would like to model this same stochastic process as closely as possible using a simple two-state Markov chain, with state space

$$\omega \equiv \begin{bmatrix} \delta \\ -\delta \end{bmatrix}$$

and transition matrix

$$P \equiv \begin{bmatrix} p & 1-p \\ 1-p & p \end{bmatrix}$$

where $0 < p < 1$. Find the invariant distribution π of this Markov chain.

d) Suppose the stochastic process x_t is described by this Markov chain, and that $\pi_0 = \pi$. Find $\mu_x \equiv E(x_t)$, $\sigma_x(0) \equiv var(x_t)$, $\sigma_x(1) \equiv cov(x_t, x_{t+1})$, and $\rho_x(1) \equiv corr(x_t, x_{t+1})$.

e) Find values for p and δ that will imply that

$$(\mu_s, \sigma_s(0), \sigma_s(1), \rho_s(1)) = (\mu_x, \sigma_x(0), \sigma_x(1), \rho_x(1))$$

2 Finding invariant distributions

Let $p, q \in (0, 1)$. Find the invariant distribution(s) for each of the following 2-state Markov transition matrices:

a)

$$P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

b)

$$P = \begin{bmatrix} 1 & 0 \\ 1-q & q \end{bmatrix}$$

c)

$$P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

d)

$$P = \begin{bmatrix} p & 1-p \\ 1-q & q \end{bmatrix}$$