Problem Set #3 Answer Key

Economics 808: Macroeconomic Theory

Fall 2004

1 A Markov chain

a)

$$\begin{aligned} \mu_s &= 0 \\ \sigma_s(0) &= 0.000503 \end{aligned}$$

b)

$$\begin{aligned} \sigma_s(1) &= 0.000477 \\ \rho_s(1) &= 0.95 \end{aligned}$$

c) The invariant distribution is $\pi = [0.5 \quad 0.5]'$

d)

$$\begin{aligned} \mu_x &= 0 \\ \sigma_x(0) &= \delta^2 \\ \sigma_x(1) &= \delta^2(2p-1) \\ \rho_x(1) &= 2p-1 \end{aligned}$$

e)

$$\begin{aligned} \delta &= 0.0224 \\ p &= 0.975 \end{aligned}$$

2 Finding invariant distributions

a)

$$\pi = [\pi_0 \quad 1-\pi_o]' \qquad \text{for all } \pi_0 \in [0,1]$$

b)

$$\pi = [1 \quad 0]'$$

c)

$$\pi = [0.5 \quad 0.5]'$$

Notice that this is an example of a Markov chain with a unique invariant distribution, but no limiting distribution - it cycles back and forth between the two states.

d)

$$\pi = \left[\frac{1-q}{2-q-p} \quad \frac{1-p}{2-q-p} \right]'$$