Problem Set #5

Economics 808: Macroeconomic Theory

Fall 2004

1 Human and physical capital in the RA model

Consider the RA model with the following modification. The production function has two inputs, physical capital (k) and human capital (h).

$$F(k,h) = Ak^{\alpha}h^{1-\alpha}$$

Human capital is accumulated by investment and depreciates over time, just like physical capital. Let x_{kt} be investment in physical capital and let x_{ht} be investment in human capital. The accumulation of human and physical capital are described by the following two equations:

$$\begin{aligned} h_{t+1} &= (1-\delta_h)h_t + x_{ht} \\ k_{t+1} &= (1-\delta_k)k_t + x_{kt} \end{aligned}$$

and the consumer's budget constraint is now:

$$c_t + x_{ht} + x_{kt} + b_{t+1} \leq w_t h_t + r_t k_t + R_t b_t + \pi_t$$

Notice that the wage (w_t) is now the rental rate on human capital. There is an initial stock of human capital $h_0 > 0$. Finally, assume utility is Cobb-Douglas.

a) Define the consumer's problem, the firm's problem, and an equilibrium for this economy.

b) For the moment, assume $\delta_h = \delta_k = 1$. For $t > 0$, solve for the equilibrium ratio of human to physical capital $\frac{h_t}{k_t}$. Call this number $\hat{h}$.

c) Find the wage w_t and rental rate on capital r_t for $t > 0$.

d) What is the long-run growth rate of consumption?

For the remainder of this problem, assume that $\delta_h = \delta_k = \delta < 1$.

e) Suppose $\frac{h_0}{k_0} = \hat{h}$. Describe the time path of w_t, r_t, and $\frac{h_t}{k_t}$.

f) Now suppose $\frac{h_0}{k_0} < \hat{h}$. Describe (qualitatively) the time path of w_t, r_t, and $\frac{h_t}{k_t}$.

2 A small open economy

Suppose that we have a standard representative agent (RA) economy, with one exception. The worker can borrow or loan money on the international market at an exogenous (fixed) interest rate. Let v_{t+1} denote the worker's international bond holdings which are purchased for 1 unit of output at time t and pay off R units of output at time $t+1$. The consumer has Cobb-Douglas preferences and maximizes:

$$\sum_{t=0}^{\infty} \beta^t \ln c_t$$

subject to the usual constraints:

$$\begin{aligned} k_{t+1} + c_t + b_{t+1} + v_{t+1} &\leq r_t k_t + w_t + \pi_t + R_t b_t + R v_t \\ k_t &\geq 0 \\ k_0 &> 0 \qquad \text{given} \end{aligned}$$

plus the no-Ponzi-game condition on both types of bond.

The firm has a Cobb-Douglas production function.

a) Define the consumer's problem, the firm's problem, and an equilibrium in this economy.

b) Describe the behavior over time of the capital stock k_t, and the rate of return on capital r_t and domestic bonds R_t.

c) Find the growth rate of consumption in this economy.

d) Under what conditions will consumption reach a positive steady state?

e) If consumption has a positive steady state value, how long does it take to get there?

3 Two economies

Consider two economies who are identical except in terms of discount rates. Country A has discount rate β_A and country B has discount rate β_B. Without loss of generality, assume $\beta_A > \beta_B$.

The representative worker in Country A maximizes:

$$\sum_{t=0}^{\infty} \beta_A^t \ln c_{A,t}$$

subject to the usual constraints:

$$\begin{aligned} k_{A,t+1} + c_{A,t} + b_{A,t+1} &\leq r_{A,t} k_{A,t} + w_{A,t} + \pi_{A,t} + R_{A,t} b_{A,t} \\ k_{A,t} &\geq 0 \\ k_{A,0} &> 0 \qquad \text{given} \end{aligned}$$

plus the no-Ponzi-game condition.

The production technology is Cobb-Douglas, and the depreciation rate is 100 percent.

Country B is the same, with all of the subscripts changed.

For the moment, assume that the countries are not allowed to trade (a condition called autarky), so they can be treated separately.

a) Define the consumer's problem, the firm's problem, and an equilibrium in this economy.

b) Find the steady state capital stock in each of the two countries.

c) Which country has more capital in the steady state?

d) Now suppose that both economies are in their autarky steady state and the two economies are economically integrated. In other words, firms can rent labor and capital from both countries, so that all prices are the same across the two countries. Let r_t, w_t and R_t be these prices and let the aggregate quantities be $K_t = k_{A,t} + k_{B,t}$, $L_t = 2$, $C_t = c_{A,t} + c_{B,t}$, etc.

Define the consumer's problem, the firm's problem, and an equilibrium in this economy.

e) What is the market clearing rental rate on capital in terms of K_t? Don't forget that $L_t = 2$

f) Find the new steady state values of $k_{A,t}$, $k_{B,t}$, and K_t. Be careful.

g) This is an extra hard question (I won't take points off for not getting it). Suppose that the economies are not integrated (i.e., they each produce separately and they don't necessarily have the same prices). However, they can trade bonds at a market-clearing return R_t (a bond costs one unit of consumption today and pays R_{t+1} units tomorrow). Find the new steady state values of $k_{A,t}$, $k_{B,t}$, and K_t