Problem Set #6

Economics 808: Macroeconomic Theory

Fall 2004

1 Overlapping generations with Cobb-Douglas production

One potential criticism of the proof I did in class that the OLG model did not necessarily have Pareto efficient equilibria was that I used a crazy production function that didn't fit the neoclassical conditions. However, the result can be proved even if we restrict attention to neoclassical production functions, it's just a little harder.

Suppose that we have an overlapping generations model and utility is:

$$U_i = \ln c_{1t} + \beta \ln c_{2t+1}$$

and the production function is

$$F(k_t, L_t) = Ak_t^{\alpha} L_t^{1-\alpha}$$

a) Find the appropriate first order conditions

b) Solve for the worker's savings rate.

c) Solve for the golden rule level of the capital stock.

d) Solve for the steady-state savings rate needed to maintain the golden rule level of the capital stock.

e) Under what conditions on the model parameters will the savings rate exceed the golden rule level?

f) Prove that these conditions imply that the equilibrium is inefficient by finding a Pareto superior allocation.

g) Put this result in words. I suggest something of the form "Dynamic inefficiency is more likely if capital's share is (low?/high?) and if young people care (more?/less?) about their future consumption."

h) Will this model exhibit multiple equilibria? How do you know this?

i) Will this model exhibit history-dependence? How do you know this?

2 Overlapping generations with linear preferences

Suppose that we have an overlapping generations model in which agents have *linear* preferences over consumption

$$U_t = c_{1,t} + \beta c_{2,t+1}$$

The production function is Cobb-Douglas.

a) Define the consumer's problem, the firm's problem, and equilibrium.

b) Find the savings rate of the young worker as a function of r_{t+1}. Is savings increasing or decreasing in the interest rate?

c) Find a difference equation expressing k_{t+1} as a function of k_t.

d) Find the steady state capital stock.

3 Overlapping generations with Leontief preferences

Suppose that we have an overlapping generations model in which agents have *Leontief* preferences over consumption

$$U_t = \min\{c_{1,t}, c_{2,t+1}\}$$

Production is Cobb-Douglas, with $\alpha < 0.5$.[1]

a) Define the consumer's problem, the firm's problem, and an equilibrium for this economy.

b) Find the savings rate of the young worker as a function of r_{t+1}. Is savings increasing or decreasing in the interest rate?

c) Find a difference equation expressing k_{t+1} as a function of k_t.

d) Find the steady state capital stock.

[1] If you don't know why this is necessary, don't worry about it.