Problem Set #7 Answer Key

Economics 808: Macroeconomic Theory

Fall 2004

1 A very simple RBC model

a) Define the consumer's problem, the firm's problem, and equilibrium in this economy.

b) The system of 3 equations is:

$$\begin{aligned} K_{t+1} &= e^{z_t} K_t^{\alpha} - C_t \\ E_t\left(\frac{\alpha\beta e^{z_{t+1}} K_{t+1}^{\alpha-1}}{C_{t+1}}\right) &= \frac{1}{C_t} \\ Z_{t+1} &= Z_t^{\rho} e^{\epsilon_{t+1}} \end{aligned}$$

c) The nonstochastic steady state is given by:

$$\begin{aligned} K^* &= (\alpha\beta)^{\frac{1}{1-\alpha}} \\ C^* &= (\alpha\beta)^{\frac{\alpha}{1-\alpha}} - (\alpha\beta)^{\frac{1}{1-\alpha}} \\ Z^* &= 1 \end{aligned}$$

d) We get:

$$\begin{aligned} E_t\left(z_{t+1} + (\alpha-1)k_{t+1} - c_{t+1}\right) &= -c_t \\ E_t(z_{t+1}) &= \rho z_t \\ k_{t+1} &= \frac{1}{\alpha\beta} z_t + \frac{1}{\beta} k_t + \frac{\alpha\beta - 1}{\alpha\beta} c_t \end{aligned}$$

e) We have:

$$\begin{bmatrix} 1 & 0 & 0 \\ 1-\alpha & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} E_t k_{t+1} \\ E_t c_{t+1} \\ E_t z_{t+1} \end{bmatrix} = \begin{bmatrix} \frac{1}{\beta} & \frac{\alpha\beta-1}{\alpha\beta} & \frac{1}{\alpha\beta} \\ 0 & 1 & 0 \\ 0 & 0 & \rho \end{bmatrix} \begin{bmatrix} k_t \\ c_t \\ z_t \end{bmatrix}$$

f) We get:

$$\begin{bmatrix} k_{t+1} \\ c_{t+1} \\ z_{t+1} \end{bmatrix} = \begin{bmatrix} \frac{1}{\beta} & \frac{\alpha\beta-1}{\alpha\beta} & \frac{1}{\alpha\beta} \\ \frac{\alpha-1}{\beta} & \frac{\alpha^2\beta+1-\alpha}{\alpha\beta} & \frac{\alpha-1}{\alpha\beta} + \rho \\ 0 & 0 & \rho \end{bmatrix} \begin{bmatrix} k_t \\ c_t \\ z_t \end{bmatrix} + \begin{bmatrix} k_{t+1} - E_t k_{t+1} \\ c_{t+1} - E_t c_{t+1} \\ z_{t+1} - E_t z_{t+1} \end{bmatrix}$$

g) We have:

$$\Lambda = \begin{bmatrix} \frac{1}{\alpha\beta} & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \rho \end{bmatrix}$$

and:

$$Q = \begin{bmatrix} \frac{\alpha\beta-1}{1-\alpha} & 1 & 1 \\ 1 & \alpha & \rho \\ 0 & 0 & \rho-\alpha \end{bmatrix}$$

Taking inverses we get:

$$Q^{-1} = \begin{bmatrix} \frac{\alpha(1-\alpha)}{\alpha^2\beta-1} & \frac{\alpha-1}{\alpha^2\beta-1} & \frac{1-\alpha}{\alpha^2\beta-1} \\ \frac{\alpha-1}{\alpha^2\beta-1} & \frac{\alpha\beta-1}{\alpha^2\beta-1} & \frac{\rho\alpha\beta-\rho-1+\alpha}{\alpha^3\beta-\rho\alpha^2\beta-\alpha+\rho} \\ 0 & 0 & \frac{1}{\rho-\alpha} \end{bmatrix}$$

Since the first eigenvalue in Λ is greater than one in absolute value, this means the first row in $Q^{-1}X_t$ must be zero. This yields:

$$c_t = \alpha k_t + z_t$$

h) A variety of answers is acceptable here.

i) The answer here depends on your calibration.

j) The simplified model does poorly even in comparison to the baseline model. There is much less variation in output than in the baseline RBC model or in the data, and there is way too much contemporaneous correlation between the variables.