

Practice exam #1

Economics 808: Macroeconomic Theory

Fall 2000

1 Pricing complex bonds

Suppose that an economy is in the steady state of the standard RA model, and that a new type of security (bond) is created. This security pays one unit of output at time $t + 3$ and 5 units of output at time $t + 200$. All agents are free to issue this security or purchase it, though in equilibrium no one will trade this security. What is the equilibrium selling price (p_t) of this security at time t ? (If you find yourself doing a lot of algebra on this problem, you are going down the wrong path).

2 Overlapping generations with Leontief preferences

Suppose that we have an overlapping generations model in which agents have *Leontief* preferences over consumption

$$U_t = \min \{c_{1,t}, c_{2,t+1}\}$$

Production is Cobb-Douglas, with $\alpha < 0.5$.¹

- a) Define the consumer's problem, the firm's problem, and an equilibrium for this economy.
- b) Find the savings rate of the young worker as a function of r_{t+1} . Is savings increasing or decreasing in the interest rate?
- c) Find a difference equation expressing k_{t+1} as a function of k_t .
- d) Find the steady state capital stock.

3 Proportional taxation in the AK model

Consider an RA model in which the government taxes income at a constant rate τ and throws the money away.² The consumer has a CRRA utility function, the firm has an "AK" production function, and the depreciation rate is 100%.

- a) Define the consumer's problem, the firm's problem, and an equilibrium for this economy.

¹If you don't know why this is necessary, don't worry about it.

²The value of τ is exogenous - i.e., it's not like the tax rate we used in class which adjusted to achieve a desired revenue level.

- b) Let $\gamma_t = \ln\left(\frac{c_{t+1}}{c_t}\right)$. In other words, γ_t is the approximate growth rate of consumption. Find γ_t .
- c) Find $\frac{\partial \gamma_t}{\partial \tau}$, evaluated at $\tau = 0$.
- d) Suppose that you are advising the prime minister, who is considering a one percentage point increase in the tax rate (assume that $\tau = 0$ right now). By approximately how many percentage points will this reduce the growth rate of consumption?

4 Increasing returns and the RA model

Suppose that the world is described by an RA model in which each firm has the following production function:

$$Y_t(f) = AK_t(f)^\alpha L_t(f)^{1-\alpha} Y_t^\lambda$$

where $Y_t(f)$ and $K_t(f)$, are output and capital of the firm and Y_t is the total output of all firms in the economy. This production function exhibits spillovers much like those found in Paul Romer's model of increasing returns that I discussed in lecture. Firms choose $K_t(f)$ and $L_t(f)$ to maximize profits ($\pi_t(f) = Y_t(f) - r_t K_t(f) - w_t L_t(f)$). They take Y_t , A , and prices as given, meaning they don't take into account the fact that their decisions affect prices and the value of Y_t . Let K_t denote the total amount of capital in the economy. Assume that consumers have CRRA preferences and that the depreciation rate is 100%.

- a) Define the consumer's problem, the firm's problem, and equilibrium.
- b) Find the *aggregate* production function ($Y_t = F(K_t, L_t)$).
- c) Define the associated planner's problem.
- d) Find r_t .
- e) Find the marginal productivity of capital ($\frac{\partial Y_t}{\partial K_t}$) in the economy as a whole.

For the rest of the problem, assume that $\lambda + \alpha = 1$.

- f) Find the equilibrium growth rate of consumption.
- g) Find the growth rate of consumption in the corresponding planner's problem, and the difference in the two growth rates.