Practice exam #1 Answer Key

Economics 808: Macroeconomic Theory

Fall 2000

1 Pricing complex bonds

Since the economy is in the steady state, the return on bonds is simply $\frac{1}{\beta}$. As a result, the present value at time t of the new security is:

$$p_t = \beta^3 + 5\beta^{200}$$

2 Overlapping generations with Leontief preferences

a)

b) Leontief preferences imply that $c_{1,t} = c_{2,t+1}$. Since $k_{t+1} = w_t - c_{1,t}$ and $c_{2,t+1} = r_{t+1}k_{t+1}$ we can find that

$$k_{t+1} = \frac{1}{1 + r_{t+1}} w_t$$

The savings rate is thus

$$s(r_{t+1}) = \frac{1}{1 + r_{t+1}}$$

which is clearly decreasing in the interest rate.

c) Since production is Cobb-Douglas, $w_t = (1-\alpha)Ak_t^{\alpha}$ and $r_t = \alpha A k_t^{\alpha-1}$. Substituting in, we get:

$$(1 + \alpha A k_{t+1}^{\alpha-1})k_{t+1} = (1-\alpha)Ak_t^{\alpha}$$

d) We find the steady state capital stock by setting $k_{t+1} = k_t = k_\infty$ in the above equation. This yields:

$$k_\infty = ((1-2\alpha)A)^{\frac{1}{1-\alpha}}$$

3 Proportional taxation in the AK model

a)

b) We find:

$$\gamma_t = \frac{\ln(\beta A(1-\tau))}{\sigma}$$

c) The marginal growth effect of taxes is:

$$\left.\frac{\partial \gamma_t}{\partial \tau}\right|_{\tau=0} = -\frac{1}{\sigma}$$

d) This tax increase will reduce consumption growth by $1/\sigma$ percentage points.

4 Increasing returns and the RA model

a) The consumer's problem is to select c_t, k_t, and b_t to maximize:

$$U = \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}$$

subject to the constraints:

$$\begin{aligned} k_{t+1} + c_t + b_{t+1} &\leq w_t + r_t k_t + R_t b_t + \pi_t \\ k_t &\geq 0 \\ k_0 &> 0 \quad \text{given} \\ b_0 & \quad\quad \text{given} \\ \lim_{t\to\infty} q_t b_t &= 0 \end{aligned}$$

where $q_t = q_{t-1}/R_t$, and $q_0 = 1$.

The firm's problem is to select $K_t(f)$ and $L_t(f)$ to maximize

$$\pi_t = AK_t(f)^\alpha L_t(f)^{1-\alpha} Y_t^\lambda - r_t K_t(f) - w_t L_t(f)$$

An equilibrium is a sequence of prices $\{w_t, r_t, R_t\}$ and allocations $\{k_t, b_t, c_t, K_t, L_t, Y_t, pi_t\}$ such that

1. Taking prices and firm profits as given, the allocations solve the consumer's problem.
2. Taking prices and Y_t as given, the allocations solve the firm's problem.
3. Markets clear, i.e., $L_t = 1$, $b_t = 0$, $K_t(f) = k_t = K_t$, and $Y_t = c_t + k_{t+1} = (AK_t)^{\frac{1}{1-\lambda}}$.

b) The aggregate production function is:

$$Y_t = (AK_t^{\alpha} L_t^{1-\alpha})^{\frac{1}{1-\lambda}}$$

c) The planner's problem is to select k_t and c_t to maximize:

$$U = \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}$$

subject to the constraints:

$$\begin{aligned} k_{t+1} + c_t &\leq (Ak_t^{\alpha})^{\frac{1}{1-\lambda}} \\ k_t &\geq 0 \\ k_0 &> 0 \qquad \text{given} \end{aligned}$$

d) The rental rate on capital is:

$$r_t = \alpha A^{\frac{1}{1-\lambda}} K_t^{\frac{\alpha+\lambda-1}{1-\lambda}}$$

e) The marginal productivity of capital is:

$$\frac{\alpha}{1-\lambda} A^{\frac{1}{1-\lambda}} K_t^{\frac{\alpha+\lambda-1}{1-\lambda}}$$

f) The equilibrium growth rate of consumption is:

$$g_E = \frac{1}{\sigma} \ln(\beta \alpha A^{\frac{1}{1-\lambda}})$$

g) The optimal growth rate of consumption is:

$$g_P = \frac{1}{\sigma} \ln(\beta \frac{\alpha}{1-\lambda} A^{\frac{1}{1-\lambda}})$$

and the difference is

$$g_P - g_E = -\frac{1}{\sigma} \ln(1-\lambda)$$