Lecture 2: Acceleration

What's important:
• relationships between displacement, velocity, acceleration
• permutations of two basic equations for motion in one dimension
• vectors

Demonstrations: none

Acceleration

We can go to higher order rates of change by looking at the rate of change of the velocity (since $s = |v|$, then we will deal with velocities rather than speeds).

$[\text{average acceleration}] = \bar{a} = \frac{v_2 - v_1}{t_2 - t_1}$ (independent of path)

$[\text{instantaneous acceleration}] = a = \frac{\Delta v}{\Delta t}$ as $\Delta t \to 0$ (which becomes $a = \frac{dv}{dt}$)

In terms of graphs:

For example, suppose that the motor fails on a boat heading into the wind:

Constant acceleration in one dimension

To obtain $x$ from $v$, or $v$ from $a$ (that is, to proceed in the opposite direction from the rates), one takes areas under the curves of time-evolution graphs. For example, a car travelling at a constant speed of 100 km/hr ($s$) covers a distance of 100 km ($st$) in a time of 1 hour ($t$). The distance is the product of $s$ and $t$, and is the area under the $s$ vs $t$ graph. We apply this to constant acceleration in one dimension.
The area under the curve gives the change in \( v \) (that is, \( \Delta v = v - v_o \)), NOT \( v \) itself. From the graph of constant acceleration vs. \( t \),

\[
\Delta v = \text{area under } a \text{ vs } t \\
\implies v - v_o = at \\
\implies v = v_o + at \tag{1}
\]

Eq. (1) shows that the \( v \) vs. \( t \) curve should be a straight line with a y-intercept of \( v_o \).

\[
\Delta x = \text{area under } v \text{ vs } t = (v - v_o) t / 2 + v_o t \\
\implies x - x_o = (v + v_o) t / 2 \\
\implies x = (v + v_o) t / 2 \quad \text{(if } x_o = 0) \tag{2}
\]

Although (2) looks like a linear equation in time (whereas the \( x \) vs. \( t \) is anything but linear), in fact \( v \) contains time dependence. Substituting (1) into (2) to show the explicit time-dependence gives

\[
x = (v_o + at + v_o) t / 2 = (2v_o + at) t / 2 \\
\implies x = v_o t + (1/2)at^2 \tag{3}
\]

To confirm that the form of Eq. (3) is correct, take derivatives to obtain \( v \) and \( a \).

Equations (1) and (2) can be written in other ways as well. For example, since the plot of \( v \) vs. \( t \) is linear, then the average velocity \( v_{av} \) is just \( (v + v_o)/2 \). Hence, Eq. (2) also can be written as

\[
x = v_{av} t \tag{4}
\]

Alternatively, one could invert (1) to find \( t \),

\[
t = (v - v_o) / a
\]

and substitute the result into (2) to obtain

\[
x = (v^2 - v_o^2) / 2a \tag{5}
\]
Variable acceleration in one dimension

Eqs. (2) - (5) only hold if the acceleration is constant. If it is not constant, the area under the $a$ vs. $t$ curve must be determined by some analytic or numerical means:

![Graph showing area under acceleration vs. time curve]

Vectors

In this course, we are concerned with motion in three dimensions, and this requires us to use vectors.

Scaler: a quantity with magnitude only, e.g. distance.

Vector: a quantity with magnitude and direction, e.g. position.

Denote position vector as $\vec{R}$. This is called the displacement of the object with respect to the origin.

Addition:
The addition of vectors is not the same procedure as the addition of scalers:

Add $\vec{A}$ to $\vec{B}$

Addition rule: put tip of $\vec{A}$ to tail of $\vec{B}$, resultant runs from tail of $\vec{A}$ to tip of $\vec{B}$.

Note that the order in which the vectors are added doesn’t matter.
Subtraction:

\[ \vec{A} - \vec{B} \]

Form negative of \( \vec{B} \), then add as usual.

\[ \vec{A} + (\vec{-B}) \Rightarrow \vec{C} \]

Magnitude:

The length of vector \( \vec{A} \) is denoted by \( |\vec{A}| \).

Scaler times vector:

\[ a \vec{A} = \vec{A} + \vec{A} + \vec{A} \ldots \text{ “a” times.} \]

Multiplication:

There are three products that one can form from vectors, two of which (the dot and cross product) are needed in this course. The dot product of two vectors is a scalar quantity, and hence the dot product also is called the scalar product. The notation for the dot product, and its operation, are:

\[ \vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta \]

This operation is equivalent to taking the projection of one vector times the length of the other:

Note that the dot product of a vector with itself is just the square of the vector’s length

\[ \vec{A} \cdot \vec{A} = |\vec{A}| |\vec{A}| \cos(0) = \vec{A}^2 \]

It is often useful to represent vectors in terms of their Cartesian components:

\[ \vec{A} = (a_x, a_y, a_z) \]

Then

\[ \vec{A} + \vec{B} = (a_x+b_x, a_y+b_y, a_z+b_z) \]

and

\[ \vec{A} \cdot \vec{B} = (a_xb_x, a_yb_y, a_zb_z) \].