Lecture 4 - Centripetal acceleration

What’s important:
• uniform circular motion
• centripetal acceleration
Demo: Tennis ball on string

This material does not take a full lecture; combine with adjacent lectures.

Circular Motion

As a simple, but often confusing, application of vectors, we consider circular motion. In the diagram, an object moves in a circular path with radius \( R \), in a clockwise direction (as indicated by the red arrow):

Let’s look at the displacement and velocity vectors at four different positions, labelled 1 ... 4 in the diagram.

We can fill in the various intermediate positions to see how the change in position and change in velocity themselves change in time.
During the period $T$ for one complete revolution,

\[
\text{Total distance covered} = \sum |\Delta R| = 2\pi R
\]

\[
\therefore \text{speed} = |v| = \text{distance} / \text{time} = 2\pi R / T
\]

\[
\text{Total change in velocity} = \sum |\Delta v| = 2\pi v
\]

\[
\therefore \text{acceleration} = 2\pi v / T = 2\pi v \left(\frac{v}{2\pi R}\right) = \frac{v^2}{R}
\]

At first sight, this may be a surprising result: the speed is constant, but there is an acceleration $a = \frac{v^2}{R}$. Of course, even though the speed is not changing, the velocity does change because it is changing direction.

*Demo: use tennis ball on a string to illustrate how $v$ varies inversely with $R$.*

Finally, note that $\Delta v$ is in the opposite direction to $R$.

\[
\Delta v_1
\]

Further, since $\vec{a}$ is parallel to $\Delta v$, then $\vec{a}$ must be in the opposite direction to $R$ as well:

\[
\vec{a}
\]

We call $\vec{a}$ the centripetal (or centre-seeking) acceleration.