Lecture 8 - Viscosity and drag

What's important:
• drag force depends on velocity
• terminal velocity under gravity

Demo:
• coin and feather, coffee filters, ball bearings in corn syrup

Velocity-dependent forces

Coin and feather demonstration shows two things:
• in a vacuum, the coin and feather drop at the same rate: the force on both objects from gravity is \( F = ma \), giving them the same acceleration
• in air, the feather falls more slowly, reaching a constant speed; because the speed is constant, the force it experiences MUST DEPEND ON SPEED: if the force were constant (like \( mg \)) then the acceleration would be constant and the feather would continue to accelerate, although at \( a < g \).

Thus, the drag forces in fluids are different than the friction force \( f_{\text{max}} = \mu N \), which is independent of velocity.

The power law for drag force varies with the speed of the object with respect to the medium. Common examples:

Low speeds  Stokes' Law (after Irish physicist George Stokes) for the force on a sphere of radius \( R \) moving with speed \( v \) through a medium with viscosity \( \eta \):
\[
F = 6\pi \eta R v
\]  
(smooth flow, no turbulence)

Higher speeds the viscous force may become quadratic in \( v \):
\[
F = \frac{1}{2} \rho A C_D v^2
\]  
(turbulence, does not depend on \( \eta \))

where \( \rho \) = density of medium, \( A \) is cross sectional area of object, \( C_D \) = drag coefficient:
• \( C_D = 0.43 \) for BMW roadsters
• \( C_D = 0.37 \) for Mazda Miata

Drag + gravity

We look at a falling system subject to drag in order to demonstrate the power law behaviour of the drag force. The free-body diagram of an object subject only to gravity \( (F = mg) \) and drag \( (F_{\text{drag}} = c_1 v^i \) or \( c_2 v^j \), where the coefficients are expressed above) is
An object released from rest accelerates downwards because $mg$ is greater than $F_{\text{drag}}$, which is initially small because of the small velocity. As the object accelerates, the velocity increases and so does $F_{\text{drag}}$. Ultimately, the drag force may reach $mg$, but it can never exceed $mg$ or the object would accelerate upwards. Once the two forces are balanced, the object has reached its terminal velocity:

- **linear drag:**  
  \[ c_1 v^1 = mg \] 
  or  
  \[ v_{\text{term}} = \frac{mg}{c_1} \]

- **quadratic drag:**  
  \[ c_2 v^2 = mg \] 
  or  
  \[ v_{\text{term}} = \left(\frac{mg}{c_2}\right)^{1/2} \]

**Demo:** An example of the quadratic dependence on speed is the motion of falling coffee filters. If the mass of a nested set of coffee filters is increased by a factor of 4 (say 1 filter vs 4) the terminal speed increases by a factor of 2. Thus $v_{\text{term}} \sim m^{1/2}$ (4 filters cover twice the distance in the same time).

**Demo:** use linear drag to find viscosity of glycerine
Consider a steel ball-bearing of 0.5 cm radius in glycerine (this corresponds to a 3/8” diameter ball). Assume $C_0 = 0.5$ and use $\rho = 1260 \text{ kg/m}^3$. When are the linear and quadratic forces comparable for a ball bearing in glycerine?

\[ c_1 v = c_2 v^2 \]
\[ v = c_1 / c_2 = 5.1 \text{ m/s}. \]

Thus, **for this example**

\[ F_{\text{drag}} \]
\[ v (\text{m/s}) \]

By observation, $v$ is less than 1 m/s, so we assume we are dealing with linear drag (confirm this after the fact).

To find the mass of the ball bearing, use $\rho = 7700 \text{ kg/m}^3$ and $R = 0.0047 \text{ m}$, so that the
mass is
\[
m = \rho V = 7700 \times (4\pi/3) \times (0.0047)^3
\]
\[
= 0.0033 \text{ kg}
\]

Measurement:
\[
v = 20 \text{ cm} / 1.1 \text{ secs} = 0.18 \text{ m/s}.
\]

Solve for \(c_1\)
\[
c_1 \, v = mg \quad \Rightarrow \quad c_1 = \frac{mg}{v}
\]
\[
c_1 = \frac{0.0033 \times 9.8}{0.18} = 0.18 \text{ kg/s}.
\]

Solve for \(\eta\)
\[
c_1 = 6\pi\eta R \quad \Rightarrow \quad \eta = \frac{c_1}{6\pi R} = \frac{0.18}{(6\pi \times 0.0047)} = 2.0 \text{ kg/m\cdots}.
\]

Thus, \(\eta \sim 2 \text{ kg / m\cdots}\) for glycerine.

Fluid viscosities encompass a huge range, as shown in the table

<table>
<thead>
<tr>
<th>Fluid</th>
<th>(\eta) (kg/m\cdotsec at 20 °C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>1.8\times10^{-5}</td>
</tr>
<tr>
<td>Water</td>
<td>1.0\times10^{-3}</td>
</tr>
<tr>
<td>Mercury</td>
<td>1.56\times10^{-3}</td>
</tr>
<tr>
<td>Olive oil</td>
<td>0.084</td>
</tr>
<tr>
<td>Glycerine</td>
<td>1.34</td>
</tr>
<tr>
<td>Glucose</td>
<td>(10^{13})</td>
</tr>
</tbody>
</table>

For cells, the effective viscosity depends on the size of the object – larger objects experience more obstacles and are subject to a higher average viscosity. Typical values would be \(10^{-3}\) to \(10^{-2}\) kg/m\cdots, which is 1 to 10 times water. The apparent viscosity of the cell as a whole is in the range \(10^{+3}\) kg/m\cdots.