CHAPTER 3

ENERGY, MOMENTUM AND MASS

Newtonian mechanics, with which the reader is familiar, provides a relationship between kinetic energy and momentum that applies to everyday objects with mass. But in Chapter 2, it is stated that there are elementary particles which, to the limit of experimental accuracy, have no mass. Does Newtonian mechanics apply to these objects? Einstein showed that not only was Newtonian mechanics inapplicable to massless objects, it also was only an approximation valid at velocities small compared to the speed of light.

The wave-like characteristics of light are well documented through a variety of classical optics experiments. But if light is actually a particle, then is light the only particle with wave-like characteristics? de Broglie proposed that all particles had wave-like features, and his proposal has been amply verified experimentally. In this chapter, we review both of these aspects of elementary particles: their energy-momentum relationship and their wave-like properties.

3.A Relativistic Energy Equation

In this section, we introduce some ideas about energy that are based on the special theory of relativity. The purist may want to skip to an introductory text on relativity to convince herself that the results from relativity quoted in this section are in fact correct. However, proving the equation that we seek involves a long intellectual detour, and so we will simply quote it here and use it in enough examples to feel comfortable with it.

In Newtonian mechanics (see Eq. A.18), the kinetic energy K of a particle is defined as

$$K \equiv mv^2/2 = p^2/2m, \tag{3.1}$$

where p, v and m are the momentum, velocity and mass of the particle, respectively. In his special theory of relativity, Einstein showed that this expression applies only at very small velocities compared to the speed of light c ($c = 3.0 \times 10^8$ m/s). The relationship between energy, momentum and mass that Einstein obtained is

$$E^2 = p^2c^2 + m^2c^4. \tag{3.2}$$

Note that pc and mc^2 have the same units as energy, and further that mc has units of momentum. Eq. (3.2) differs from (3.1) in several respects:

- E is the total energy of the object, not its kinetic energy K
- the total energy has two contributions, one of which is associated with the object's motion (through p) and the other of which is related to the object's mass, irrespective of movement
- the expression has E^2, rather than E, on the left hand side.

Einstein's expression is valid for any momentum. The equation can be massaged a bit to obtain an expression valid at low momentum:

$$E^2 = p^2c^2 + m^2c^4 = m^2c^4\left(1 + \frac{p^2}{m^2c^2}\right) \tag{3.3}$$

or

$$E = mc^2\left(1 + \frac{p^2}{m^2c^2}\right)^{1/2}. \tag{3.4}$$

At low momenta, $p << mc$, the expression for the square root looks like $(1+x)^n$ where x is a small number and $n = 1/2$. One can show that for small x

$$(1 + x)^n \approx 1 + nx. \tag{3.5}$$

[It is easy to convince oneself of this for n = 1, 2, 3...]. Applying (3.5) to (3.4) yields

$$E \approx mc^2\left(1 + \frac{1}{2}\frac{p^2}{m^2c^2}\right) = mc^2 + \frac{p^2}{2m} \qquad \text{for } p << mc. \tag{3.6}$$

Eq. (3.6) is only an approximation valid at small momenta, but it says that the total energy of a particle is equal to the sum of the mass energy of the particle, mc^2, and the term $p^2/2m$. At low velocities we can substitute $p = mv$ and we see that the term $p^2/2m$ is the same as $mv^2/2$. So the total energy E is a sum of the mass energy mc^2 and the kinetic energy K:

$$E \equiv K + mc^2. \tag{3.7}$$

Caution! Beware of the following:
(i) As written, Eq. (3.7) is valid for any momentum or velocity, but only at low velocity does $K = mv^2/2$ or $p^2/2m$.
(ii) It is also only at low velocities that $v = p/m$. The general relationship between velocity and momentum is

$$v = pc^2/E. \tag{3.8}$$

which becomes $v = p/m$ at small velocities where $E \approx mc^2$. Note that Eqs. (3.2) and (3.8) can be combined to show that the velocity of a particle cannot exceed the speed of light c. In these lectures, we will only concern ourselves with momenta and will not run across problems where it is necessary to calculate velocities.

The magnitude of the mass energy is immense compared to everyday kinetic energy scales. For example, suppose we have a proton travelling at 30 m/s, which is about 100 km/hr. For such a small velocity, the proton's kinetic energy K is given by the Newtonian expression

$$K = mv^2/2 = 1.67 \times 10^{-27}\ [\text{kg}] \times 30^2\ [\text{m}^2/\text{s}^2]\ /\ 2$$

$$= 7.5 \times 10^{-25}\ \text{J}. \tag{3.9}$$

In contrast, its mass energy mc^2 is

$$mc^2 = 1.67 \times 10^{-27} \text{ [kg]} \times (3.0 \times 10^8 \text{ [m/s]})^2$$

$$= 1.5 \times 10^{-10} \text{ J.} \qquad (3.10)$$

In other words, the ratio of the kinetic to the mass energy in this situation is 5×10^{-15}!

Example 3.1: *Calculate the total energy and the kinetic energy of a particle with momentum* mc, *where* m *is the particle's mass*.

The total energy E is given by

$$E^2 = p^2c^2 + m^2c^4 = (mc)^2c^2 + m^2c^4 = 2m^2c^4$$

or

$$E = \sqrt{2}\, mc^2.$$

From the total energy, we then calculate the kinetic energy as

$$K = E - mc^2 = (\sqrt{2} - 1)\, mc^2 = 0.414\, mc^2.$$

We finish with a reminder that even though $p = mc$, the particle's velocity is not equal to c. In fact, it is equal to $c/\sqrt{2}$ from Eq. (3.8).

To summarize, *relativistic* mechanics, which is true for any velocity, has

$$E^2 = p^2c^2 + m^2c^4 \qquad E \equiv K + mc^2 \qquad (3.11)$$

while *Newtonian* mechanics (or *non-relativistic* mechanics), which is only valid at small velocities $v \ll c$, has

$$K = mv^2/2 \qquad (3.12)$$

At zero kinetic energy, the total energy of a particle is just its mass energy mc^2. In fact, elementary particle masses are frequently quoted as an energy, mc^2. The mass energy of a particle in Joules is often cumbersome to deal with. For example, the proton mass energy in Eq. (3.10) is

1.5×10^{-10} J. It is conventional to quote mass energies in electron-volts (eV) or millions of electron-volts (1 MeV = 10^6 eV), where an electron-volt is the energy which an electron gains when accelerated through a potential difference of 1 volt. Using the conversion 1 eV = 1.6×10^{-19} J, the proton mass energy is 938 MeV.

3.B Momenta of Massless Particles

Radio waves, light and x-rays are all examples of electromagnetic radiation. At the microscopic level, electromagnetic radiation is thought to be particulate, with the *photon* as its elementary particle (see Sec. 2.C). The currently available limit on the photon mass is given in Table 3.1, and one can see that it is indeed exceedingly small - less than 2×10^{-63} kg! There are other particles which appear to have a small or zero mass as well. There are several types of neutrinos (called ν_e, ν_μ, ν_τ; see Sec. 2.C) with small mass limits, although there is continuing experimental controversy as to whether the masses are actually zero.

Suppose that these particles really do have zero mass. What is their dynamics? From Sec. 3.A, the relativistic energy-momentum relationship reads

$$E^2 = p^2c^2 + m^2c^4. \qquad (3.13)$$

This equation is perfectly general, and applies to any physical value of p or

Table 3.1 Experimental mass limits for very light particles, quoted in kg, and as mass energy equivalents in MeV (1 MeV = 10^6 eV).

Particle	Symbol	Mass limit	
photon	γ	$< 2 \times 10^{-63}$ kg	$< 1 \times 10^{-33}$ MeV
electron neutrino	ν_e	$< 3 \times 10^{-35}$ kg	$< 1.7 \times 10^{-5}$ MeV
muon neutrino	ν_μ	$< 4.8 \times 10^{-31}$ kg	< 0.27 MeV
tao neutrino	ν_τ	$< 6.2 \times 10^{-29}$ kg	< 35 MeV

m. Even if a particle is massless, then Eq. (3.13) states that the particle has momentum by virtue of its energy

$$p = E/c \qquad \text{(for } m = 0\text{).} \qquad (3.14)$$

This result is counter-intuitive at first sight. In Newtonian mechanics we are told that $p = mv$ so that a massless particle should have no momentum. But we now know that Newtonian mechanics only applies at low velocities, $v << c$, so we must first determine the speed of a massless particle before we try to apply Newtonian mechanics to it. The general definition of velocity from Eq. (3.8) is $v = pc^2/E$. For massless particles, $p = E/c$ according to Eq. (3.14), so that $v = c$ according to Eq. (3.8). Thus, the speed of a massless particle is always equal to c, independent of the particle's momentum. So Newtonian mechanics does not apply to photons, and Eq. (3.14) really does give the photon momentum correctly.

Example 3.2: *The energy of a photon of green light is 2.5 eV. What is such a photon's momentum?*

First, change units using the conversion 1 eV = 1.6×10^{-19} J:

[*photon energy*] = E = $2.5 \times 1.6 \times 10^{-19} = 4 \times 10^{-19}$ J.

Next, use Eq. (3.14) to determine the momentum:

$p = E/c = 4.0 \times 10^{-19} / 3.0 \times 10^8 = 1.3 \times 10^{-27}$ kg-m/s.

So the momentum of a green photon, $\approx 10^{-27}$ kg-m/s, is hardly enough to knock one off one's feet.

Eq. (3.14) can be tested experimentally in a scattering experiment. Arthur Compton (1892-1962) was the first to make the measurement in 1922-23, so the experiment is referred to as Compton scattering. In the experiment a photon of known energy is scattered from an atom. The energy of the scattered photon is then measured as a function of scattering angle. Using the laws of conservation of energy and momentum one can test whether Eq. (3.14) holds. To within experimental error, it does, convincing us that a massless particle can indeed have momentum.

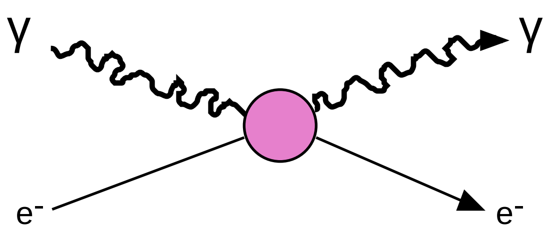

Fig. 3.1. Schematic representation of Compton scattering showing the scattering of a photon from an electron.

More is known about photons than just their mass. At least as far back as the 1600's, it was appreciated that light had a wavelength associated with it, and that the wavelength can be measured experimentally. In these lectures, wavelength is given the symbol λ and is defined as the distance between successive crests on a wave. The frequency of the wave, f, is the number of wave crests passing a fixed point in a given period of time.

Let's examine water waves for a moment and determine the relationship between f, λ and the speed of the wave v. A typical wave on the sea has a wavelength of several meters, let's use λ = 5 m as an example. If we count how many waves crash on the shore every minute, then a typical number might be 4, so that the frequency is f = 4 min^{-1}. Suppose now that we are sitting on a rock watching the waves go by and we want to measure their speed. We start timing the waves with the passing of a wavecrest which we call wave #1. By the time wave #4 has completely passed us, 1 minute has elapsed. By this time, wave #1 is four wavelengths away, or 4 x 5 = 20 m. The speed of the waves is then 20 m/min = 0.333 m/s = 1.2 km/hr.

To put into symbols what we just said in words:

$$[speed] = [frequency] \times [wavelength] \qquad (3.15)$$

or

$$v = f\lambda. \qquad (3.16)$$

Although Eq. (3.16) was derived for water waves, it is applicable to any wave. Note that the units are as expected: f has units of $[time]^{-1}$ and λ has

Table 3.2. The spectrum of electromagnetic waves.

Common name	Wavelength (m)	Frequency (s^{-1})
long radio waves	$\geq 10^3$	$\leq 10^5$
AM radio waves	10^2	10^6
short radio waves (including FM and TV)	0.1 to 10	10^7 to 10^9
microwaves	10^{-3} to 10^{-2}	10^{10} to 10^{12}
infrared	10^{-5} to 10^{-4}	10^{12} to 10^{14}
visible light	10^{-6}	10^{14}
ultraviolet	10^{-9} to 10^{-7}	10^{15} to 10^{17}
x-rays	10^{-11} to 10^{-9}	10^{18} to 10^{19}
gamma rays	$\leq 10^{-12}$	$\geq 10^{20}$

units of [*length*] so speed must be [*length*] / [*time*].

The colour of visible light depends upon its wavelength. The wavelengths of visible light are in the range 400 nm (blue to ultraviolet) to 700 nm (red). It is now known that light is just one component of a broad category of radiation including radio waves, microwaves *etc.* which are collectively referred to as electromagnetic radiation. Conventionally, the electromagnetic *spectrum* is broken down into broad ranges of wavelength and frequency. The names typically applied to these ranges are shown in Table 3.2.

3.C Photoelectric Effect

It was known in the 1800's that light is in fact electromagnetic radiation. In setting up the general equations of electricity and magnetism, Maxwell discovered that the equations had a solution corresponding to a travelling electric and magnetic field. Using experimentally measured values for the parameters which govern the electric and magnetic interactions [for example, the constant k in Eq. (2.2)] he calculated the speed at which the electromagnetic wave travelled and found it was the same as the speed of light. But Maxwell's equations did not provide a link between the frequency (or wavelength) of the wave and its energy.

The relationship was found several decades later in Einstein's interpretation of the photoelectric effect, an effect originally discovered by Hertz in 1887. A simplified description of the photoelectric effect is as follows. When light hits the surface of some metals, it can dislodge electrons bound in the metal. By 1900 it was known how the energy of the ejected electrons (called *photoelectrons*) varied with the intensity and frequency of the light. For light of a given frequency, the photoelectrons are emitted with a range of kinetic energies, but there is always a well-defined maximum kinetic energy. The following observations are found:

i) as the intensity of the light is increased (i.e., the light source is made brighter) but the frequency f of the light is not changed (i.e., the colour of the light is not changed) there are more photoelectrons produced, but their energy remains the same (Lenard, 1900).

ii) as the frequency of the light is increased (blue light instead of red) but the intensity is held fixed, the energy of the photoelectrons increases.

iii) even at low intensity (i.e., dim light), there are always some electrons emitted, and they are emitted as soon as the light is switched on.

iv) there is a cutoff frequency below which no photoelectrons are observed. The cutoff frequency varies with the metal used to make the sample.

These observations can be summarized in an equation relating the maximum kinetic energy of the emitted photoelectron K_{max} to the frequency of the incident beam of light f:

$$K_{max} = hf - W. \qquad (3.17)$$

Two constants appear in Eq. (3.17), h and W. The first constant h is universal in the sense that it is observed for all metals, whereas the second constant W is material-specific and is referred to as the work function of the material. If $hf < W$, then no photoelectrons are observed. In other words, W is proportional to the cut-off frequency.

Now, these observations did not agree with what one expected from the theory of electromagnetism as it was known in 1900. In the classical theory of electromagnetism, the force that an electromagnetic wave could exert on a charged object depended on the intensity of the light, not its colour or frequency. So, as the intensity increases, the force on the metal's electrons should increase and the photoelectrons should be emitted with higher kinetic energy. This prediction is contradicted by observation (i). Further, at low intensity, the classical theory predicts that there may not be enough energy in the light to knock out the electrons immediately, so that there may be a time delay at low intensity in which the electrons are slowly accelerated and finally break free of the metal in which they are bound. This prediction is contradicted by observation (iii). Finally, the electric force of the electromagnetic radiation does not depend on colour or frequency in the classical picture, so observations (ii) and (iv) are not expected in the classical theory.

The observations of the photoelectric effect remained a puzzle for some years until Einstein proposed in 1905 that electromagnetic radiation is composed of massless elementary particles or *photons* which possess a wavelength or frequency. The energy of the photon depends on the frequency according to

$$E = hf \tag{3.18}$$

where f is the frequency of the photon and h is Planck's constant. Planck's constant was not something new at the time Einstein proposed Eq. (3.18), it had been introduced by Planck in 1900 in a model for the emission of radiation in a high temperature cavity. Numerically, Planck's constant is equal to 6.626×10^{-34} J-s.

Einstein's explanation of the photoelectric effect is as follows. The intensity of a beam of light is proportional to the number of photons, each of which carries a fixed amount of energy. Observation (i) is then explained by the number of photoelectrons being proportional to the number of photons striking the metallic surface. Since the energy of the photons depends on their frequency, then the kinetic energy of the photoelectrons must increase with the photon frequency. Hence observation (ii). Observation (iii) follows since even in dim light there are still incoming photons and each of them has energy hf. However, if the energy hf is too low to overcome the binding energy of the electrons in the metallic surface, then no photoelectrons will be observed. In this picture, the work function W is interpreted as a measure of the electron's binding energy.

Example 3.3: *Find the energy of green light with a wavelength of 500 nm*.

First, we have to calculate the frequency of the light. The speed of a wave [c in the case of light] is equal to the frequency f times the wavelength λ according to Eq. (3.16). Here, λ is equal to 500 nm = 5.0×10^2 [nm] x 10^{-9} [m/nm] = 5.0×10^{-7} m. Hence, the frequency is

$$f = c/\lambda = 3.0 \times 10^8 / 5.0 \times 10^{-7} = 6.0 \times 10^{14}\ s^{-1}.$$

Using Einstein's relation for the photon energy Eq. (3.18), we find

$$E = hf = (6.63 \times 10^{-34}) \times (6.0 \times 10^{14}) = 4.0 \times 10^{-19}\ J = 2.5\ eV.$$

Is Einstein's relationship (3.18) consistent with our qualitative understanding of atomic binding? In Example 3.3, we show that the energy of a photon of green light is 2.5 eV. Typical electron binding energies are a few eV. Hence, at a qualitative level we expect that light in the visible range of frequencies should be capable of ejecting electrons from some metals, as is observed in the photoelectric effect.

3.D Wavelengths of Massive Particles

Light consists of elementary particles called photons which have a wavelength and which carry momentum. By suitable manipulation of Eqs. (3.14), (3.16) and (3.18) the photon's wavelength λ can be related to its momentum p through

$$p = E/c = (hf)/c = h\,(f/c) = h/\lambda. \qquad (3.19)$$

Hence, a photon's momentum is inversely proportional to its wavelength. Now, Einstein's energy-momentum relation has been shown to apply to all particles regardless of their mass. Is the same true of Eq. (3.19): does a massive particle have a wavelength?

Count Louis de Broglie proposed that massive particles do indeed have a wavelength which obeys Eq. (3.19):

$$\lambda = h / p. \tag{3.20}$$

The wavelength in Eq. (3.20) is commonly referred to as the *de Broglie wavelength*. de Broglie's hypothesis has been confirmed experimentally in many situations on the atomic scale. For example, it is known that mono-energetic x-rays scatter from a regular crystal lattice and produce a scattering pattern which depends on the x-ray wavelength and the properties of the crystal lattice. A beam of mono-energetic neutrons also produces a scattering pattern, confirming the neutron's wavelike characteristics. The wavelength of the neutrons can be extracted from the experiments and are found to obey Eq. (3.20). In our everyday world, the de Broglie wavelengths of objects are so small that their effects are undetectable.

Example 3.4: *Find the de Broglie wavelength of a 1 kg box of cookies moving at 1 m/s*.

The box of cookies is travelling at a very small speed compared to the speed of light (3×10^8 m/s), so it is perfectly safe to use the expression $p = mv$ for momentum. The momentum of the box is then trivially

$$p = 1 \text{ [kg]} \times 1 \text{ [m/s]} = 1 \text{ kg-m/s}$$

so that the de Broglie wavelength is

$$\lambda = h/p = 6.63 \times 10^{-34} \text{ [J-s]} / 1 \text{ [kg-m/s]}$$

$$= 6.63 \times 10^{-34} \text{ m.}$$

Summary

The general relationship between the total energy E of a particle and its mass m and momentum p is given by Eq. (3.2),

$$E^2 = p^2c^2 + m^2c^4,$$

where $c = 3.0 \times 10^8$ m/s is the speed of light. The total energy is also equal to the sum of the kinetic energy K and mass energy mc^2, as in Eq. (3.7):

$$E \equiv K + mc^2.$$

It is only at low velocity v that $K = mv^2/2$ or $K = p^2/2m$. The mass energy mc^2 is often quoted in units of MeV = 10^6 eV, where 1 eV = 1.6×10^{-19} J.

A massless particle travels at the speed of light and has a momentum p that is related to its energy E via

$$p = E/c \qquad \text{(at } m = 0\text{)}.$$

For light or any other wave, the velocity of the wave v is related to its frequency f and wavelength λ via Eq. (3.16):

$$v = f\lambda.$$

In the photoelectric effect, electrons are emitted from a metallic surface when light is shone on the surface. The main characteristics of the photoelectric effect are:

i) as the intensity of the light is increased at fixed frequency, there are more photoelectrons produced, but their energy is unchanged.

ii) as the frequency of the light is increased at fixed intensity, the energy of the photoelectrons increases. The maximum kinetic energy of the photoelectrons at a specific frequency f is given by Eq. (3.17),

$$K_{max} = hf - W,$$

where $h = 6.63 \times 10^{-34}$ J-s is called Planck's constant and W is the work function of the metal.

iii) even at low intensity, there are always some electrons emitted, and they are emitted as soon as the light is switched on.

iv) there is a cutoff frequency which depends on the sample material and below which no photoelectrons are observed.

In explaining the photoelectric effect, Einstein proposed that light comes in packets called *photons* and each photon has an energy of

$$E = hf,$$

according to Eq. (3.18), and a momentum p given by Eq. (3.19)

$$p = h/\lambda.$$

de Broglie proposed that all particles have a wavelength, now referred to as the de Broglie wavelength, which is related to the particle's momentum by Eq. (3.20):

$$\lambda = h/p.$$

Further Reading

A. Einstein, *Relativity* (Methuen, London, 1920), Part I.

P. A. Tipler, *Physics for Scientists and Engineers* (Worth, New York, ed. 3, 1991), Chaps. 13, 29, 34, 35.

H. D. Young, *University Physics* (Addison-Wesley, Reading, ed. 8, 1992), Chaps. 40, 41.

Problems

1. What is the kinetic energy of:
(a) a 1 tonne car travelling at 100 km/hr?
(b) a 1 kg chicken flying with a momentum of 10^8 kg-m/s?

2. Calculate (in kg-m/s) the momentum of an electron with a kinetic energy of 1 MeV (1 MeV = 10^6 eV) using the full relativistic equation and its Newtonian approximation.

*3. What is the speed of an object whose kinetic energy is one-half of its mass energy? Use Eq. (3.8) and quote your answer in terms of c.

*4. From Eq. (3.2) and (3.8), show that the velocity of a particle can never exceed the speed of light.

5. A photon of red light has an energy of 3 x 10^{-19} J. (a) What is its momentum? (b) How many photons would a red laser have to send at a 10 g bullet travelling towards the laser at 300 m/s in order to overcome the momentum of the bullet? Assume that the laser light is completely absorbed by the bullet.

*6. Show that $E = pc + m^2c^3/2p$ for an ultra-relativistic particle with $p >> mc$.

7. Calculate the wavelengths of photons with energy (a) 1 eV, (b) 1 MeV and (c) 1 GeV.

8. A flashlight radiates a beam of light at 2 J/s with an average wavelength of 600 nm.
(a) What is the energy of a single photon with λ = 600 nm?
(b) How many photons leave the flashlight per second?
(c) How much momentum is carried by the photons?

9. When the Sun is directly overhead, 0.14 J of energy in the form of light reaches an area 1 cm^2 on the Earth's surface every second.
(a) Find the total change in momentum (per second) which this energy imparts to the Earth. [Assume that the light is absorbed and not reflected by the Earth.]
(b) Compare this momentum change with that of the Earth in its orbit

around the Sun.

10. A 1 tonne car is travelling towards a 1000 watt laser (1 watt = 1 J/s) at 100 km/hr. How long must the laser shine on the car in order to overcome the car's momentum and bring it to a stop? [Assume that the light is absorbed and not reflected by the car.]

11. The work function of a metal is 2.1 eV. Find the maximum velocity of a photoelectron emitted when (a) blue light of wavelength 400 nm, (b) red light of wavelength 700 nm, strikes the metal's surface. [Use the non-relativistic expression for the kinetic energy.]

12. The maximum velocity of a photoelectron ejected from a particular metallic surface by blue light (λ = 400 nm) is observed to be 9 x 10^5 m/s. What is the work function of the metal (in eV)?

13. Calculate the momentum and kinetic energy of an electron with a de Broglie wavelength equal to (a) 1.0 nm and (b) 0.2 nm.

14. You wish to make a device for telling whether an Okanagan apple is red or green. Light is shone on the apple and reflected into a photocell. Find the work function of the photocell such that it generates a current when the apple is green (550 nm), but not when it is red (650 nm).

*15. In scattering experiments from regular crystal lattices, a scattering pattern emerges when the wavelength of the incoming particle is comparable to the spacing between the atoms in the crystal.
(a) Calculate the kinetic energies of photons and neutrons whose de Broglie wavelength is 0.2 nm.
(b) The average kinetic energy of particles in an ideal gas is on the order of k_BT, where k_B is Boltzmann's constant (1.38 x 10^{-23} J/K^o) and T is the temperature in Kelvin. Using the kinetic energies from part (a), assign temperatures to the photons and neutrons. What do the temperatures tell you about the sources you would have to use to produce the photons or neutrons?

16. Compare the de Broglie wavelengths of
(a) the Earth in its orbit around the Sun
(b) a 40 g stone travelling at 2 m/s
(c) an electron travelling at 0.1*c*.

*17. Consider the motion of a particle in one dimension between two perfectly reflecting walls separated by a distance L. Suppose that the de Broglie wavelength of the particle is required to be $2L$, so that there is a node (i.e. a point of no motion) in the particle's "wave" at each wall. Find the corresponding non-relativisitic kinetic energies of
(a) an electron in a box with L = 0.2 nm
(b) a nucleon in a box with L = 3 fm
(c) a quark of mass energy 1550 MeV in a box with L = 0.5 fm.

18. Two accelerators now under construction will produce beams of nuclei with momenta $p = 100 m_A c$, where m_A is the mass of the nucleus. For our purposes, the nuclear mass $m_A = A \times 1.67 \times 10^{-27}$ kg.
(a) Calculate the kinetic energy (in J) of a nucleus with A = 200 in a beam from this accelerator.
(b) Calculate the kinetic energy per unit volume of the nucleus, using R from Eq. (1.5), in J/m^3.
(c) For comparison, calculate the kinetic energy density of a 0.1 kg ball of radius 5 cm travelling at 100 km/hr. Again, quote your answer in J/m^3.

19. To use high energy particles to probe the inner structure of a nucleus, the wavelength λ of the particle must be less than the nuclear radius R.
(a) If the probe particle is a *photon*, what energy must it have in order to study an A = 16 nucleus at $\lambda = R$, where λ is the photon wavelength?
(b) If the probe particle is a *proton*, what kinetic energy must it have to study the same A = 16 nucleus at $\lambda = R$, where λ is now the proton's de Broglie wavelength.
Quote both of your answers in MeV, and use Eq. (1.5) for the nuclear radius.