Lecture 11 - Interactions and cross sections

What's important:

- contributions of interactions to scattering and decay processes
- conservation laws
- cross sections
- strength of interactions

Text: PHYS 120 on-line *Modern Physics: from Quarks to Galaxies*

Interactions in Reactions and Decays

How do the four fundamental forces - strong, electromagnetic, weak and gravitational influence decays or reactions? Consider the scattering of a pion against a proton:

$$\pi^+ + p \rightarrow \pi^+ + p. \qquad (11.1)$$

The interactions involved in this scattering process are:
(i) gravity, since the particles each have mass
(ii) weak
(iii) electromagnetic, since they are charged
(iv) strong, since they are hadrons
All four interactions are present, but the strong interaction dominates.

Now, suppose that the pion is replaced by an electron e^-:

$$e^- + p \rightarrow e^- + p. \qquad (11.2)$$

As electrons are leptons, the strong interaction is missing here. But Eq. (11.2) isn't just weak, because electrons are charged, so the electromagnetic force dominates.

The following "rules of thumb" apply to both reactions and decays:

1. A reaction or decay is dominated by the strongest interaction that is common to all of the participants.

2. If a photon is present in a reaction, then the strong interaction cannot dominate. The dominant interaction is electromagnetic or weak.

3. If a neutrino is present, then neither the strong nor electromagnetic interactions can dominate. The dominant interaction is the weak interaction.

Rules 1 - 3 also apply to decays. Suppose there are competing reactions that can take place in a decay. For example, the decay of the rho meson can occur via both

$$\rho \rightarrow \pi + \pi \qquad (11.3)$$

$$\rho \rightarrow \gamma + \gamma \qquad (11.4)$$

Using Rules 1 - 3, we would argue that (11.3) is dominated by the strong interaction and (11.4) is dominated by the electromagnetic interaction. In Nature, both of these decays occur, but it is decay (11.3) that occurs a thousand times more frequently than (11.4), because (11.3) is a strong interaction decay.

Conservation Laws

The energy, momentum and quantum numbers of *individual* particles can change during a reaction or decay. If a characteristic of the system *as a whole* does not change during a reaction, then the characteristic is said to be conserved. Scalar quantities like charge or lepton number (but not spin) add algebraically. For example, in the scattering process

$$A + B \rightarrow C + D$$

conservation of charge reads

$$Q_A + Q_B = Q_C + Q_D.$$

Example Consider the possible decay of a proton *via*

$$p \rightarrow \pi^o + e^+.$$

Initial state: $Q = +1$ $B = +1$ $L_e = 0$

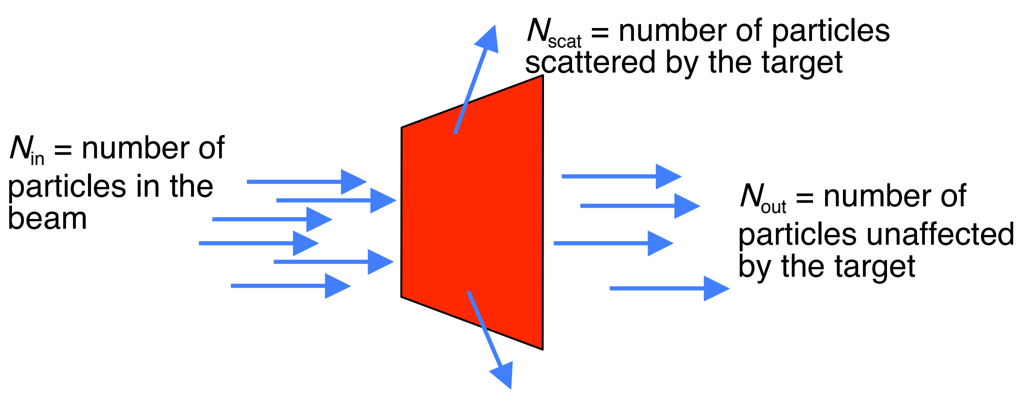
Final state: $Q = 0 + 1 = 1$ $B = 0 + 0 = 0$ $L_e = 0 + (-1) = -1.$

This decay mode violates conservation of B and L_e.

Experimental tests:
- electric charge is highly conserved: at least 1 part in 10^{24}
- lepton number is well conserved
- baryon number is usually not tested independently of lepton number. For example, the proton lifetime is at least 10^{31} years, but its decay violates conservation of B and L_e simultaneously.

Probabilities and cross sections

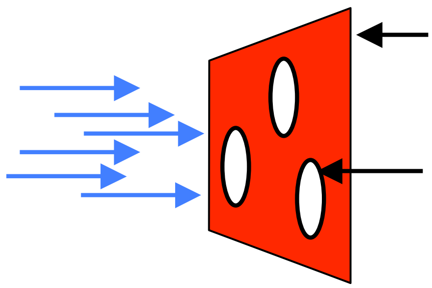
Our goal is to find the probability P that a given particle will scatter from a target particle. We consider a beam of identically prepared particles travelling in the same direction towards a target:



We measure N_{scat} to obtain

$$P = N_{scat} / N_{in}.$$

Theoretically, the probability of scattering is equal to the ratio of the effective area of the target objects (as seen by the incident beam of particles) compared to the total area of the target region exposed to the beam:



P = [number of target particles exposed to the beam] σ / A_T.

But, the number of target particles exposed to the beam is just the product of A_T with the number of particles per unit area n_T:

[*number of target particles exposed to the beam*] = $n_T A_T$.

Hence,

$$P = n_T\, A_T\, \sigma / A_T \quad \text{----->} \quad P = n_T\, \sigma. \qquad (11.5)$$

We don't need to know A_T (which is hard to measure), just n_T (which is easy to measure).

Mean reaction times

The probability P of interaction can be used to calculate the mean time between scattering for a particle traversing a target, which we define as t_{av}. We'll do this calculation in more generality later, but let's assume for now that the beam particles travel at a constant speed v. Now, n_T is the number of target particles per unit area, equal to the number of particles per unit volume N_T times the thickness of the target. For a given target density, P increases linearly with the target thickness. Suppose the target region were so thick that there is unit probability that a beam particle will interact. This thickness is equal to the velocity times the mean collision time, or vt_{av}. Thus, $P = n_T\,\sigma$ becomes for this thickness

$$1 = (N_T\, vt_{av})\sigma$$

which can be inverted to read

$$t_{av} = 1 / (N_T v\sigma). \qquad (11.6)$$

This equation tells us that:

the weaker the interaction, whence
the smaller the cross section
-> the longer the mean reaction time.

Interactions and cross sections

The strength of the interaction between particles is manifested in their scattering cross sections and lifetimes. For example, the electrostatic force between protons is much larger in magnitude than their gravitational attraction, meaning that, if two protons approach and scatter from each other, the contribution to the scattering probability from their electrostatic repulsion is much larger than the contribution from their gravitational attraction.

Similarly, if a decay produces a particle having only a weak interaction with the other products of the decay, then the process will be infrequent ("slow" decay means that particles decay infrequently - it doesn't mean that the physical process is slow). Thus, strong interactions correspond to large cross sections and short lifetimes, while weak interactions correspond to small cross sections and long lifetimes.

For example, pion decay
$\pi^o \rightarrow \gamma + \gamma$ is electromagnetic with lifetime of about 10^{-16} sec
$\pi^- \rightarrow \mu^- + \nu$ is weak with a lifetime of 10^{-4} sec.

Some typical examples are shown below, although each quantity covers a large range for any given process:

Interaction	Typical cross section	Typical lifetime
strong	10^{-30} m^2	10^{-24} sec
electromagnetic	10^{-36} m^2	10^{-16} sec
weak	10^{-42} m^2	10^{-8} sec

Note: cross sections are often measured in barns: 1 barn = 100 fm^2 = 10^{-28} m^2.

Why don't all measurements of a given target particle yield the same geometrical cross section independent of the fundamental interaction? Cross sections can be interpreted as $\sigma = \pi R^2$ only if the interaction is strong and short-ranged. Otherwise, σ is no more than a measure of interaction probability. Thus, even though the scattering of a neutrino from a proton has a lower cross section than that of a proton hitting another proton, it doesn't mean that the target proton has a different size: it just means that the interaction is weaker.

Example What is t_{av} for weak reactions in the Sun under the following approximations:

$T = 15 \times 10^6$ K

so that $v = (3k_B T/m)^{1/2} = (3 \cdot 1.38 \times 10^{-23} \cdot 1.5 \times 10^7 / 1.67 \times 10^{-27})^{1/2}$
$= 6.1 \times 10^5$ m/s

$N_T = 10^{57} / (4\pi [6.96 \times 10^8]^3 / 3) = 7.1 \times 10^{29}$ m^{-3}

$\sigma = 10^{-42}$ m^2

Hence

$t_{av} = 1 / (N_T v\sigma) = (6.1 \times 10^5 \cdot 7.1 \times 10^{29} \cdot 10^{-42})^{-1} = 2.3 \times 10^6$ s < 1 year.