Lecture 16 - Stellar lifetimes

What's Important:
- time evolution of H-R diagram
- estimates of stellar lifetimes
- energy released in gravitational collapse

Text:

Time evolution of H-R diagram

The luminosity of a star depends on its energy output and radius:
- the energy output increases with the mass and interior temperature
- the radius increases with the mass.

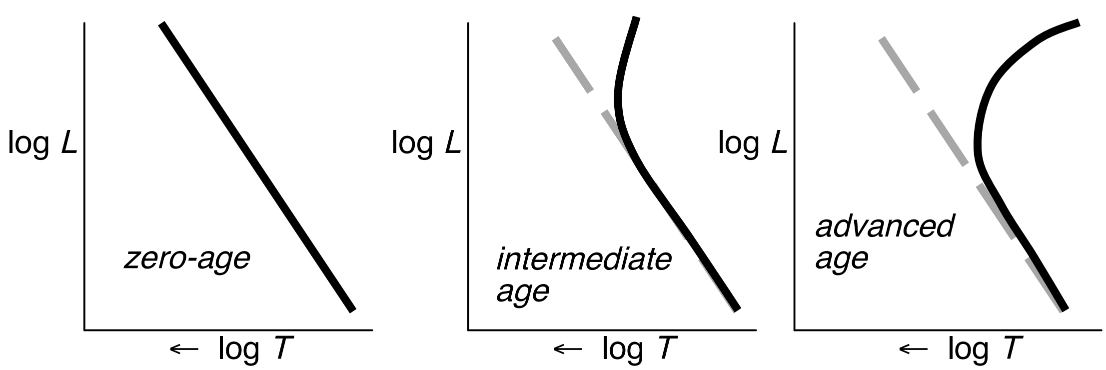
It is observed that the luminosity increases as the fourth power of the mass:

$$L \propto M^4 \qquad \text{(observed range is 3.5 to 4.0)}$$

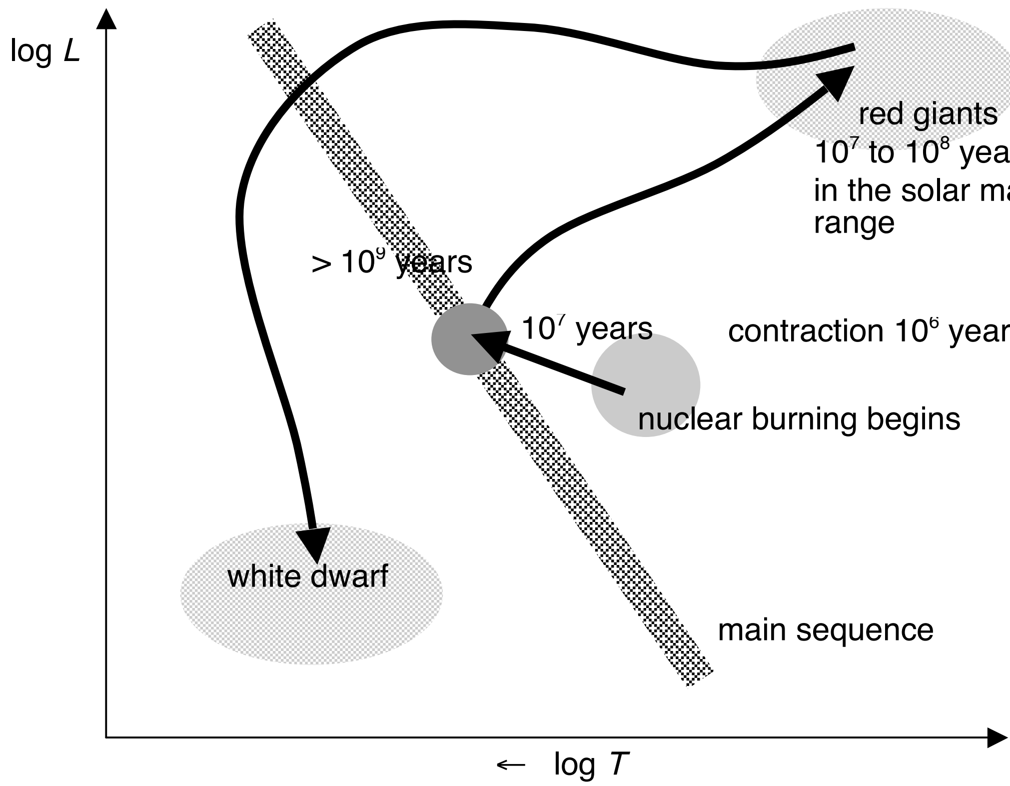
Now, a star is driven by nuclear reactions, and the main fuel for these reactions in stars like the Sun is hydrogen (75% by weight for material from the Big Bang). That is, the fuel is proportional to the mass of the star. If a star shines with constant luminosity L, then the lifetime of the star T_E is just

$$T_E = [\textit{mass}] \;/\; [\textit{mass burned per unit time}] = M / L \propto M / M^4 \propto M^{-3}.$$

In other words, the more massive the star, the shorter it lifetime. Our Sun should have a lifetime of about 10 billion years, but a massive star with ten times the Sun's mass would be predicted to burn out a thousand times sooner, just 10 million years. Given that the Milky Way is at least as old as the Sun, the most massive stars initially present in our galaxy are probably long gone. As far as the H-R diagram is concerned, we would expect the hottest (largest L) stars would evolve fastest towards the red giant phase (low surface temperature) marking the end of their lives:



Once the star has burned up most of its hydrogen fuel in the red giant phase, it may follow one of several scenarios depending upon its mass. For stars similar to the Sun, the core of the star will collapse under gravity, throwing off a planetary nebula from the outer region. The star then settles into a white dwarf stage, with a similar luminosity as the Sun, but only a hundredth the radius (e.g., Sirius B has the same mass as the Sun, but 3% L_{sun}, 0.8% R_{sun} and T = 27,000 K). The overall evolution of a typical star on the H-R diagram would then look like:



Estimates of stellar lifetimes

Stars like the Sun derive their energy from the burning of hydrogen in nuclear reactions. We can estimate the lifetimes of stars knowing the energy released in the conversion of hydrogen to helium, and a scaling law relating the mass and luminosity of stars.

Suppose we have a star of mass M whose initial fraction by weight of hydrogen is X_H. The mass of hydrogen in such a star is

[*mass of hydrogen*] = $X_H M$.

Now, each kilogram of hydrogen releases ε = 6.4 x 10^{14} J of energy through nuclear reactions, so that the total energy obtainable from the star as a whole is

[*total energy*] = $\varepsilon X_H M$.

Suppose that a fraction f of a star's core burns through to helium in time T_f while it is on the main sequence. Then the energy released in this time is [*energy released over* T_f] = $f\varepsilon X_H M$.

Assuming a constant luminosity L over most of this time gives

T_f = [*energy released over* T_f] / L = $f\varepsilon X_H M / L$.

Now, we can normalize these quantities to solar quantities by writing

$T_f = (f\varepsilon X_H M_{sun} / L_{sun})\ (M / M_{sun}) / (L / L_{sun})$.

Lastly, substituting the observed scaling law that luminosity varies as the fourth power of a star's mass, or

$M / M_{sun} = (L / L_{sun})^{1/4}$,

we are left with

$T_f = (f\varepsilon X_H M_{sun} / L_{sun})\ (L / L_{sun})^{-3/4}$.

Now, most of the first group of quantities are all known constants, including $X_H = 0.75$. The unknown is f, the fraction of the core burned while on the main sequence, estimated at 15% for stars like the Sun.

Hence:

$T_f = 12 \times 10^9\ (L / L_{sun})^{-3/4}$ years.

Aside: this tells us that the Sun should spend about 12 billion years as a main sequence star. Given that the Sun is already 5 billion years old, it has another 7 billion years before it will evolve away from the main sequence.

Gravitational contraction

The contraction of a star like the Sun from a cloud of hydrogen and helium occurs over a time frame of about 10^6 years. The energy released during this phase is impressive. One can show from integral calculus that the change in the gravitational potential energy ΔU_{grav} of a spherical object of mass M, radius R, and uniform density is

$$\Delta U_{grav} = \frac{3}{5} G \frac{M^2}{R}$$

While this could be the binding energy of a low-temperature solid, but is not the binding energy of a high temperature star, which has a kinetic energy of half the magnitude of its potential energy. Let's evaluate ΔU_{grav} of the Sun, using

$M = 1.99 \times 10^{30}$ kg $\qquad R = 6.96 \times 10^8$ m.

Then

$$\Delta U_{grav} = \frac{3}{5} 6.67 \times 10^{-11} \frac{(1.99 \times 10^{30})^2}{6.96 \times 10^8} = 2.3 \times 10^{41}\,\mathrm{J}$$

Even when converted to an energy per particle, this energy is impressively large. The

Sun has about 10^{57} protons, so ΔU_{grav} per proton is about 1 x 10^7 K. This is far more energy than is needed to ionize hydrogen, and the energy release raises the temperature of the star into the low end of the nuclear reaction regime.

Note that this is not the binding energy of the star - some of this energy will be lost to the star, while another fraction (half) will be converted to the thermal energy of the star's nuclei and electrons.