Lecture 27 - Evolution on the main sequence

What's Important:

- zones of nuclear reactions
- advanced nucleosynthesis

Text: Carroll and Ostlie, Chap. 13

In this lecture, we collect results from our previous lectures to paint the theoretical big picture of the formation and evolution of a conventional star on the main sequence.

Initiation

From Lec. 26, the time for the free-fall collapse of a molecular cloud is

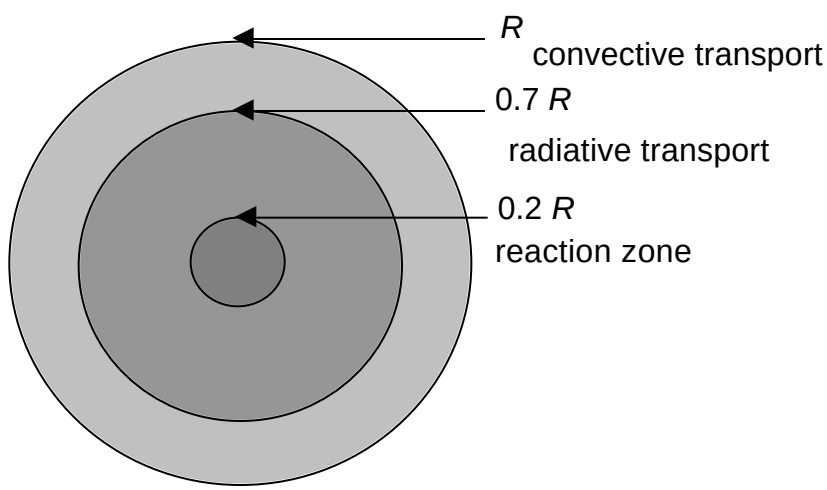
$$t_{ff} = \left(\frac{3\pi}{32 G \rho_o}\right)^{1/2}$$

The time taken for a typical cloud to collapse to a star (independent of stellar mass, according to this equation) is of the order 10^5 years, for initial densities ρ_o large enough to be of interest. The energy released during collapse raises the temperature of the proto-star to the initiation range for nuclear reactions.

Main sequence burning

Numerical calculations based upon the physics presented in the preceding lectures have been made for stellar interiors. The ingredients include a star's
radius, energy output, surface temperature.

The most intensively studied star is, of course, the Sun. Typical results show (from Carroll and Ostlie, Fig. 11.9):



To find the amount of mass encompassed by each zone, note that:

- the volume is proportional to the radius cubed
- the density is not at all uniform.

Detailed calculations show that a little more than half of the Sun's mass is contained within 0.3 of its radius.

There are two limits on stellar sizes:

- To reach nuclear temperatures in its core, a star's mass must be more than about 0.08 of a solar mass (surface temperature of 2700 K)
- Stars with masses greater than 90 solar masses are unstable (surface temperature of 53,000 K).

Helium burning

Hydrogen-based reactions in a star are the most efficient because of the small Coulomb barrier between participants. As the hydrogen supply is exhausted, other reaction pathways must be found to keep the star burning. The next most likely reactions involve the burning of helium nuclei, again because of the Coulomb barrier. The first set of reactions are the conversion of three ^{4}He nuclei to produce ^{12}C, in a two step process:

$$^{4}He + {}^{4}He \leftrightarrow {}^{8}Be \quad \text{(unstable)}$$

The lifetime of ^{8}Be against decay into two helium nuclei is long (by strong interaction standards) at 2.6×10^{-16} s. Although this reaction is not favourable, it does permit the buildup of a small abundance of ^{8}Be. As described further in Clayton, the ratio of ^{8}Be to helium at 10^{8} K is about 10^{-9}. This small abundance provides a pathway to produce ^{12}C, through the addition reaction

$$^{8}Be + {}^{4}He \rightarrow {}^{12}C \qquad \text{(liberates a little more than 7 MeV).}$$

It turns out that the energy released through this reaction increases very rapidly with temperature, about T^{40}, similar to what one might find in an explosion.
These reactions have the lowest Coulomb barriers because they involve $Z = 2$ and $Z = 4$ partners. However, if the temperature is sufficiently high, a sequence of reactions involving ^{4}He becomes accessible:

$$^{4}He + {}^{12}C \rightarrow {}^{16}O + \gamma \qquad \text{(energy release = 7.16 MeV)}$$

$$^{4}He + {}^{16}O \rightarrow {}^{16}Ne + \gamma \qquad \text{(energy release = 4.73 MeV)}$$

$$^{4}He + {}^{20}Ne \rightarrow {}^{24}Mg + \gamma \qquad \text{(energy release = 9.31 MeV)}$$

$$^{4}He + {}^{24}Mg \rightarrow {}^{28}Si + \gamma ...$$

It is no coincidence that the most abundant elements in the solar system have the order

H > He > O > C > {Ne, N} > Mg > Si ...

or that the Earth is composed largely of iron and magnesium silicates (SiO_4) - they are the natural products in the advanced burning stages of stars at high temperatures (about 10^{8} K or more).

Advanced burning stages

At ever higher temperatures, it is possible to have reactions with ever larger Coulomb barriers. For example, the reaction of two carbon nuclei has a significant Coulomb barrier, and can produce a variety of products

$^{12}C + {}^{12}C \rightarrow {}^{24}Mg + \gamma$ (energy release = 13.93 MeV)

$\rightarrow {}^{23}Na + p$ (energy release = 2.24 MeV)

$\rightarrow {}^{20}Ne + \alpha$ (energy release = 4.62 MeV)

These reactions require temperatures exceeding 10^8 K. As the temperature approaches 10^9 K, sufficient kinetic energy is available to overcome the Coulomb barrier between oxygen nuclei. In principle, one could keep on going to ever heavier nuclei, except that temperatures of 10^9 K are so high that nuclei themselves become subject to photodisintegration, just as we saw in our discussion of the early universe:

$$T = 10^9 \text{ K} \rightarrow k_B T = 1.38 \times 10^{-23} \bullet 10^9 / 1.6 \times 10^{-19} = 86{,}000 \text{ eV} = 0.086 \text{ MeV}.$$

In other words, the photon bath at this temperature is hot enough to disintegrate vulnerable nuclei. One of the first casualties is ^{20}Ne:

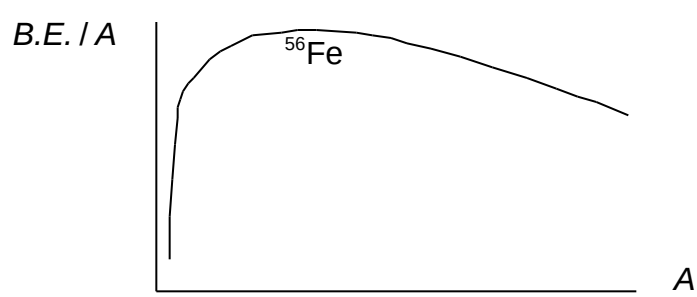
$$^{20}Ne + \gamma \rightarrow \alpha + {}^{16}O.$$

The presence of the photon bath drives the star towards equilibrium:

- low temperatures - abundances determined by collisions across Coulomb barriers, starting with H + He reactants
- very high temperatures - abundances driven towards their equilibrium values, approaching from the low-*A* side of the binding energy per nucleon curve

This is referred to as the **photodisintegration rearrangement** process or **e-process**.

Schematically, the binding energy per nucleon curve looks like



In other words, this process drives nuclei towards the production an equilibrium distribution centered around ^{56}Fe, which is the most deeply bound nucleus. While the natural abundance of ^{56}Fe (for example in the Earth's core) testifies to the importance of the e-process, the temperature reached in such an environment cannot be too hot,

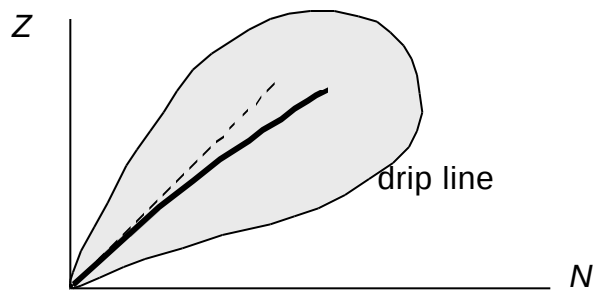
or there would also be a high yield of nearby isotopes such as ^{54}Fe.

Detailed calculations have been made of stars which evolve through this environment, including the effects of subsequent beta-decays *etc.*, and are described at length in Clayton.

Production of heavy nuclei

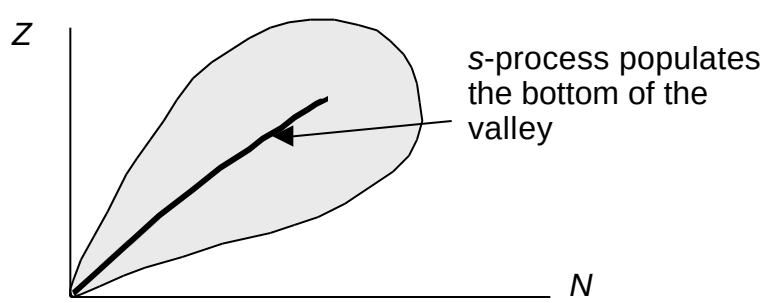
The e-process by itself cannot account for the observed abundances of the heavy elements - even raising the temperature to account for the lower binding energy per nucleon at large *A* does not broaden out the distribution sufficiently (because of photodisintegration, and the large Coulomb barrier against large *Z*). An alternative mechanism is the capture of neutrons (or, less likely, protons) produced in the disintegration of lighter nuclei.

The time scale for the capture process influences the distribution of the product nuclei. A schematic of the nuclear stability region against *Z* and *N* has the appearance:

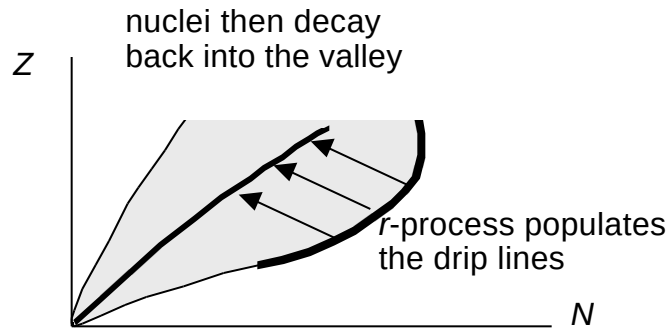


The most stable nuclei fall close to $Z = N$, tending towards larger *N* as Coulomb repulsion within the nucleus becomes important. Encompassing the minimum of the "valley of beta-stability" are the drip lines, beyond which nuclei are unstable against **nucleon** decay, even though they are still bound.
The capture of individual neutrons is classified according to time scale:

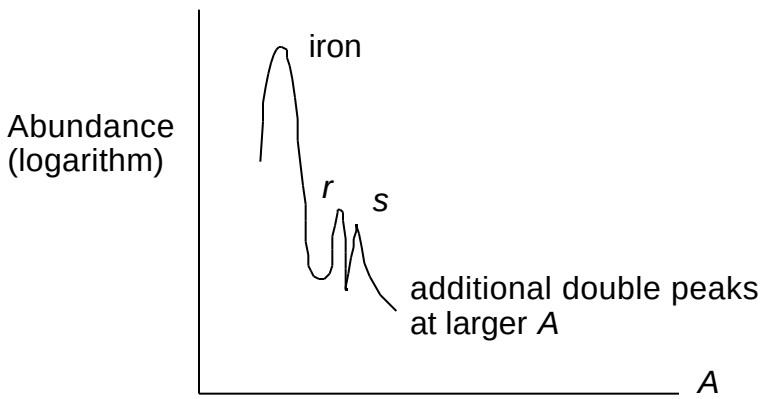
s-process Slow capture, with plenty of time for new nuclei to decay if they lie to the side of the valley (i.e., if their binding energy is not optimal). This will produce nuclei near the minimum in the valley, that is, nuclei having the greatest binding energy. Because beta-decay times may range up to about 10^3 hours (in this mass region), the capture times would have to be correspondingly longer, perhaps a thousand years.



r-process Fast capture of neutrons, with a rapid buildup to populate the neutron drip line. Of course, these nuclei aren't stable, and they decay back down to the bottom of the valley.



These processes result in unique, and very different distributions of elements. Evidence for both processes is found in the mass distributions from cosmic rays.



Time scales for advanced burning stages

This is from a simulation of the evolution of a 25 solar mass star (from Kaufmann and Freedman, p. 549):

Stage	Duration
H-burning	7×10^6 years
He-burning	7×10^5 years
C-burning	600 years
Ne-burning	1 year
O-burning	0.5 years
Si-burning	1 day
core collapse	0.25 seconds
core bounce	10^{-3} seconds
explosion	10 seconds