SU(4) Explanation of the Narrow Resonances in e^+e^- Annihilation*

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The narrow resonances at $M_\psi = 3.105$ GeV and $M_{\psi'} = 3.695$ GeV are assigned, together with the familiar SU(3) nonet of vector mesons, as members of the $\underline{1}\oplus\underline{15}$ representation of SU(4). This leads to a solution consistent with the observed particle spectrum and accounts for the narrow widths of the ψ and ψ'. The masses of the charmed vector and pseudoscalar mesons are predicted.

A narrow resonance, ψ, at a mass of 3.105 GeV with a width $\Gamma_\psi \leq 1.3$ MeV was reported recently by the Stanford Linear Accelerator Center,[1] Brookhaven National Laboratory,[2] and Frascati[3] groups. A second narrow resonance, ψ', with a mass of 3.695 GeV has also been observed by the Stanford Linear Accelerator Center group.[4]

The narrowness of these resonances suggests that new selection rules are operating and that we have to consider higher symmetry groups such as SU(4) and SU(8).[5] In the following, we shall investigate the consequences of assigning the ψ and ψ' to the $\underline{1}\oplus\underline{15}$ representation of SU(4).

To accommodate the ψ and ψ' in SU(4), we assume that there exists a new semistrong interaction[6] which breaks hypercharge and the new quantum number $W = \frac{1}{3}X - \frac{1}{4}N$ such that the total charge and isospin in the extended Gell-Mann–Nishijima formula[7]

$$Q = I_3 + \tfrac{1}{2}Y + \tfrac{1}{3}X - \tfrac{1}{4}N \qquad (1)$$

are conserved. This interaction allows the single production of charmed particles as well as their decay into noncharmed ($W=0$) particles. We suppose that the ψ' is almost pure $|c\bar{c}\rangle$, while the ψ is almost entirely $|s\bar{c}\rangle + |c\bar{s}\rangle$. The new semistrong interaction causes mixing between these latter states and those with $Y=0$ and $I=0$ (i.e., the ω, φ, and ψ'), thus allowing the physical ψ to couple to a virtual photon in e^+e^- annihilation. The present framework provides a natural explanation of the photon decay modes of the ψ[3] and the observed small decay[8] $\psi' \to \psi + 2\pi$.

In order to explore these ideas more fully, we will employ a simple effective-Lagrangian model to describe the vector- and pseudoscalar-meson mass spectra. The point of this exercise is not to obtain detailed predictions or fits to data; rather, it is to demonstrate the consistency of our scheme with the present experimental situation. The simplest *effective* Hamiltonian with the desired properties is given by

$$H_{\text{int}} = T_8 + aT_{15} + bT_{13}', \tag{2}$$

where T_8 and T_{15} are the 8 and 15 components of one $\underline{15}$ representation of SU(4) and T_{13}' is the 13 component of a *different* $\underline{15}$ representation. The 13 component T_{13} of the same representation as T_8 and T_{15} can of course be rotated away to lowest order.[9] T_{15} separates the SU(4) multiplets, while T_8 splits the masses within the SU(3) submultiplets and mixes the particles of the same hypercharge and charm (e.g., ω, φ mixing). The T_{13}' mixes particles of equal isospin but different hypercharge and X subject to the condition

$$\Delta Y = -\tfrac{2}{3}\Delta X. \tag{3}$$

For the fractionally charged quark model of SU(4) described in Ref. 7, T_{13}' conserves charge.[10]

The squared-mass matrix for the $\underline{1} \oplus \underline{15}$ representation of the vector mesons V_i $(i = 0, 1, \ldots, 15)$ can be written as

$$\begin{aligned}
(M^2)_{ij} &= \overline{M}^2\delta_{ij} + A(d_{i8j} + ad_{i15j}) + bA'd_{i13j},\\
(M^2)_{0i} &= B(\delta_{8i} + a\delta_{15i}) + bB'\delta_{13i},\\
(M^2)_{00} &= \overline{M}_0{}^2.
\end{aligned} \tag{4}$$

$\overline{M}^2$ and $\overline{M}_0{}^2$ are the SU(4)-invariant squared masses of the regular representation $\underline{15}$ and the singlet representation, respectively, and A, A', B, and B' are the reduced matrix elements. To obtain a solution for the parameters of the mass matrix, and for the physical eigenstates, in terms of the known masses of the ρ, K^*, ω, φ, ψ, and ψ' we shall make several simplifying assumptions. We take $A/B = A'/B'$, a relationship which is suggested by SU(8) symmetry. In addition, as a rough guess, we set $A' = A$. A numerical analysis then leads to a solution

$$\begin{aligned}
&\overline{M}^2 = 5.0\ \text{GeV}^2, \quad A = A' = -0.23\ \text{GeV}^2,\\
&a = 46.0, \quad \overline{M}_0{}^2 = 3.0\ \text{GeV}^2,\\
&B = B' = -0.12\ \text{GeV}^2, \quad b = 0.05.
\end{aligned} \tag{5}$$

The physical eigenstates ω, φ, ψ, and ψ' are given by

$$\begin{aligned}
\omega &= 0.509V_8 + 0.338V_{15} + 0.792V_0,\\
\varphi &= 0.861V_8 - 0.001V_{13} - 0.197V_{15} - 0.469V_0,\\
\psi &= 0.001V_8 + 0.999V_{13} - 0.001V_{15},\\
\psi' &= -0.003V_8 + 0.001V_{13} + 0.920V_{15} - 0.391V_0.
\end{aligned} \tag{6}$$

Instead of the usual SU(3) mixing angle θ there will now be three angles. We see from Eq. (6) that the quark contents of ψ and ψ' are predominantly $|s\bar{c}\rangle + |c\bar{s}\rangle$ and $|c\bar{c}\rangle$, respectively. It follows that decays such as $\psi \to K + \overline{K}$, etc., will be essentially forbidden and hence Γ_ψ will be very small,[11] in agreement with the experimental data.[1-3] It is also observed from (5) that the size of the T_{13}' breaking is very small compared with the breaking caused by T_{15} and T_8, so that the conservation of the quark spins[5] J_s and J_c is still true, to a good approximation, suppressing decays like $\psi' \to K^* + \overline{K}$.

Since ψ' is not completely pure $|c\bar{c}\rangle$ and ψ is not pure $|s\bar{c}\rangle + |c\bar{s}\rangle$ the observed decay $\psi' \to \psi\pi\pi$ can take place.[12]

The values of the parameters in (5) predict the mass for the charmed-vector-meson isodoublet ξ^* to be

$$M_{\xi^*} = 3.04\ \text{GeV}. \tag{7}$$

The pseudoscalar mesons are also assigned[13] to the $\underline{1} \oplus \underline{15}$ representations of SU(4). Then, using the same values of a and b in the mass matrix for the pseudoscalar mesons, we predict

$$\begin{aligned}
M_{\xi_P} &= 3.08\ \text{GeV},\\
M_{\psi_P} &= 3.11\ \text{GeV},\\
M_{\psi_P'} &= 4.07\ \text{GeV}.
\end{aligned} \tag{8}$$

The remaining parameters of the pseudoscalar mass matrix are $A = A' = -0.25\ \text{GeV}^2$, $B = B' = -0.12\ \text{GeV}^2$, $\overline{M}^2 = 4.85\ \text{GeV}^2$, and $\overline{M}_0{}^2 = 3.15\ \text{GeV}^2$. It is interesting to note that these vary by less than 8% from those in the vector-meson case. In particular, $\overline{M}^2$ varies by less than 3% in agreement with what one might expect from SU(8) considerations.

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[7] J. W. Moffat, Phys. Rev. 140, B1681 (1965). The quantum numbers Q, I_3, Y, and X of the four quarks are $u(\frac{2}{3}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4})$, $d(-\frac{1}{3}, -\frac{1}{2}, \frac{1}{3}, \frac{1}{4})$, $s(-\frac{1}{3}, 0, -\frac{2}{3}, \frac{1}{4})$, and $c(-\frac{1}{3}, 0, 0, -\frac{3}{4})$; N equals $\frac{1}{3}$ for all four quarks. Note that the usual quarks u, d, and s have "charm," $W=0$, and c has $W=-\frac{1}{3}$.

[8] G. Goldhaber, private communication.

[9] S. Coleman and S. Glashow, Phys. Rev. 134, B671 (1964).

[10] It should be pointed out that, if another narrow resonance were to be discovered in e^+e^- annihilation experiments, it could be accommodated in the present framework by the addition of a T_{11} interaction to Eq. (2). Such a contribution would mix ξ^{*0} with ω, φ, ψ, ψ', and ρ^0. With the quark quantum-number assignments of S. L. Glashow, J. Iliopoulos, and L. Maiani [Phys. Rev. D 2, 1285 (1970)] we have checked that ψ and ψ' can be accommodated by replacing T_{13}' by T_9' in Eq. (2). A third narrow e^+e^- resonance could not be fitted into the $\underline{1}\oplus\underline{15}$ representations in the latter scheme. See D. Boal *et al.*, to be published.

[11] This, of course, depends on the masses of the charmed mesons being large enough so that the decay of ψ into one of them (plus, say, a kaon) is energetically forbidden. The charmed-meson masses in our simple model have this property.

[12] In fact, if one takes the effective Lagrangian for $V' \rightarrow V + 2\pi$ to be $gV_\mu' V_\mu \vec{\pi} \cdot \vec{\pi}$ and uses the decay $\rho' \rightarrow \rho + 2\pi$ to estimate the size of g, one finds $\Gamma(\psi' \rightarrow \psi\pi\pi) \sim 0.02(g^2/4\pi)b^2$ MeV $\gtrsim 20$ keV [b is the strength parameter in Eq. (2)].

[13] This is based on the SU(8) decomposition, $\underline{8}\otimes\underline{8}^* = \underline{1}\oplus\underline{63}$. The $\underline{63}$ contains the $\underline{1}\oplus\underline{15}$ SU(4) representations of vector mesons and a $\underline{15}$ representation of pseudoscalar mesons. (See Ref. 7 for details.)