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SU(4) symmetry and the decays of the new hadrons*

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The masses and mixing of the 0^- and 1^- mesons are calculated in an SU(4) model assuming that $\psi(3095)$ is a pure $|c\bar{c}\rangle$ state. The meson masses are predicted using the masses of the usual hadrons and the $\psi(3095)$ as input. The hadronic and radiative decays of the mesons are calculated using three proposed quark models.

The recent discovery of the narrow resonances $\psi(3095)$ (see Ref. 1) and $\psi(3684)$ (see Ref. 2) has promoted a renewed interest in SU(4) and SU(8) schemes.[3,4] Borchardt, Mathur, and Okubo[3] derived predictions for meson and baryon masses based on the SU(4)-symmetry-breaking interaction

$$H' = T_8 + aT_{15}\,, \tag{1}$$

where T_8 and T_{15} belong to the same 15-dimensional representation of SU(4).

The $\psi(3095)$ is assumed to have $J^P = 1^-$ and $I = Y = 0$ and is assigned, together with the usual vector mesons ρ, K^*, ω, and ϕ, to the $\underline{4}\otimes\underline{4}^* = \underline{15}\oplus\underline{1}$ representations of SU(4). The pseudoscalar mesons are assigned to another $\underline{15}\oplus\underline{1}$ representation. The decomposition of the $\underline{15}$-plet in SU(3) is $\underline{15}\supset\underline{8}\oplus\underline{3}\oplus\underline{3}^*\oplus\underline{1}$. The representation $\underline{3}$ contains an $\overline{\text{SU}(2)}$ doublet D (D^*) and singlet F (F^*) of pseudoscalar (vector) mesons which have nonzero "charm."

The ψ' [$=\psi(3684)$] is assumed to belong to another $\underline{1}\oplus\underline{15}$ representation of SU(4) together with the $\rho'(1600), \omega', \phi' \cdots$. In an alternative classification[4] the ψ and ψ' were both assigned to the same $\underline{1}\oplus\underline{15}$ representation.

In the following, we shall reanalyze the mass-diagonalization problem[5] for the mesons in the radially excited SU(4) model of the $\psi(3095)$ in the light of more recent data on the radiative[6] and hadronic[7] decays of the ψ and the possible discovery of a charmed meson at SPEAR[8] with a mass ~2.4 GeV.

The mass-squared matrix for the $\underline{1}\oplus\underline{15}$ representation of the vector mesons V_i ($i = 0, 1, \ldots, 15$) can be written as[9]

$$\begin{aligned}(M^2)_{ij} &= \bar{M}^2\delta_{ij} + A(d_{8ij} + a d_{i\,15\,j})\,,\\ (M^2)_{0i} &= B(\delta_{8i} + a\delta_{15\,i})\,,\\ (M^2)_{00} &= \bar{M}_0^{\,2}\,.\end{aligned} \tag{2}$$

$\bar{M}^2$ and $\bar{M}_0^{\,2}$ are the SU(4)-invariant squared masses of the regular representation $\underline{15}$ and the singlet representation, respectively, and A and B are the reduced matrix elements. In terms of the known masses of the ρ, K^*, ω, ϕ, and ψ a numerical analysis leads to a solution

$$\begin{aligned}&\bar{M}^2 = 2.98\ \text{GeV}^2,\quad A = -0.23\ \text{GeV}^2,\quad a = 24.0\,,\\ &\bar{M}_0^{\,2} = 2.95\ \text{GeV}^2,\quad B = -0.16\ \text{GeV}^2\,.\end{aligned} \tag{3}$$

The quark content of the vector mesons corresponding to this solution is

$$\begin{aligned}\rho^0 &= \frac{1}{\sqrt{2}}\,(u\bar{u} - d\bar{d})\,,\\ K^{*0} &= d\bar{s}\,,\\ \omega &= 0.707(u\bar{u} + d\bar{d}) - 0.037s\bar{s} + 0.0039c\bar{c}\,,\\ \phi &= -0.026(u\bar{u} + d\bar{d}) - 0.9993s\bar{s} - 0.0033c\bar{c}\,,\\ D^* &= \{u\bar{c}, d\bar{c}\}\,,\\ F^* &= s\bar{c}\,,\\ \psi &= 0.0029(u\bar{u} + d\bar{d}) + 0.0032s\bar{s} - 0.9999c\bar{c}\,,\end{aligned} \tag{4}$$

where u, d, and s are the usual $I = \frac{1}{2}$ and 0 SU(3) triplet quarks and c is the charmed quark.

For the pseudoscalar mesons, the mass-squared matrix takes the same form as Eq. (2), but the parameters are now

$$\begin{aligned}&\bar{M}^2 = 2.92\ \text{GeV}^2,\quad A = -0.28\ \text{GeV}^2,\quad a = 24.0\,,\\ &\bar{M}_0^{\,2} = 3.10\ \text{GeV}^2,\quad B = -0.17\ \text{GeV}^2\,.\end{aligned} \tag{5}$$

The physical eigenstates for the pseudoscalar mesons are

$$\begin{aligned}\pi^0 &= \frac{1}{\sqrt{2}}\,(u\bar{u} - d\bar{d})\,,\\ K^0 &= d\bar{s}\,,\\ \eta &= 0.569(u\bar{u} + d\bar{d}) - 0.593s\bar{s} + 0.0066c\bar{c}\,,\\ X^0 &= 0.419(u\bar{u} + d\bar{d}) + 0.805s\bar{s} + 0.026c\bar{c}\,,\\ D &= \{u\bar{c}, d\bar{c}\}\,,\\ F &= s\bar{c}\,,\\ \eta_c &= 0.015(u\bar{u} + d\bar{d}) + 0.017s\bar{s} - 0.996c\bar{c}\,.\end{aligned} \tag{6}$$

We see that this solution leads to a physical ψ which is a very pure $|c\bar{c}\rangle$ state. In Table I we show the masses obtained from the diagonalization of the

TABLE I. Predicted masses obtained from the diagonalization of the mass-squared matrix.

	Particle	Predicted mass (MeV)	Experimental mass (MeV)
$J^P = 1^-$	ρ^0	771	770 ±10 [a]
	K^{*0}	891	892.2 ±0.5 [a]
	ω	786	782.7 ±0.6 [a]
	ϕ	1001	1019.7 ±0.3 [a]
	D^{*0}	2273	...
	F^*	2316	...
	ψ	3111	3095 [b]
$J^P = 0^-$	π^0	129	135 [a]
	K	509	495.7 [a]
	η	522	548.8 ±0.6 [a]
	X^0	977	957.6 ±0.3 [a]
	D^0	2362	...
	F	2413	...
	η_c	3259	...

[a] See Particle Data Group, Phys. Lett. 50B, 1 (1974).
[b] See Refs. 1 and 2.

TABLE II. Strong decay widths. The $\rho \to \pi\pi$ width was used as input to determine g_{VPP}.

Decay	Theoretical widths	Experimental widths
$\rho \to \pi\pi$	160 MeV	150±10 MeV [a]
$\phi \to K\bar{K}$	3.00 MeV	3.41 ±0.36 MeV [a]
$\psi \to K\bar{K}$	~60 eV	≲35 eV [b]
$\eta_c \to \psi\pi$	~0	...
$\eta_c \to K^*\bar{K} + \bar{K}^*K$	~20 keV	...

[a] See Particle Data Group, Phys. Lett. 50B, 1 (1974).
[b] See Ref. 7.

mass-squared matrix. The predicted masses of the D^* and F^* ($J^P = 1^-$) mesons are close to 2.4 GeV. This prediction would agree with the recent discovery at SPEAR of a new particle with a mass ~2.4 GeV, which decays into lepton pairs ν_μ, μ^+ and $\bar{\nu}_e, e^-$ and could be identified with either the D^* or F^* mesons. If any of the μe events represent two-body decays of charmed particles, then they must arise from D^* (and/or F^*)$\to l + \bar{\nu}_l$ since $D(F) \to l + \bar{\nu}_l$ should be greatly suppressed because of the helicity structure of the conventional weak currents. In fact, the μe events can be attributed to charmed-meson decays only if, as in the present case, the masses of the D^* and F^* are *lower* than their pseudoscalar counterparts; otherwise, the decays $D^* \to D + \gamma$ ($F^* \to F + \gamma$) would be the dominant modes.

It is important to note that *unlike in the work of Okubo et al.*[5] the predicted mass of the η_c, 3260 MeV, lies above that of the ψ. This places it considerably higher than a recently discovered[10] candidate at ~2800 MeV. A higher-lying η_c could, perhaps, be identified with one of the newly observed[10,11] χ states.

The hadronic decays of the mesons are calculated using the SU(4)-invariant effective Lagrangian

$$\mathcal{L}_{VPP} = g_{VPP} f_{ijk} V_i^\mu P_j \partial_\mu P_k \,, \qquad (7)$$

where the V_i (P_i) are the components of the vector (pseudoscalar) meson 15-plet and the f_{ijk} are the SU(4) structure constants. The decay widths are obtained from Eq. (7) by making use of the physical eigenstates in (4) and (6). The results[12] are displayed in Table II. The prediction for $\Gamma(\psi \to K\bar{K})$ is higher than the experimental upper bound, but this is not a serious problem.[13]

We shall now consider the radiative decays $V \to P + \gamma$ and $P \to V + \gamma$. They will be calculated by using the magnetic dipole moment ($M1$) transitions within the broken-SU(8) quark model. As will be seen below, the quark model appears to have some difficulty[14] in describing even the more familiar radiative decays such as $\rho \to \pi + \gamma$. Nevertheless, we shall use it to get rough order-of-magnitude estimates for the radiative decay widths of the new particles. The width for the transition $a \to b + \gamma$ is given by[15]

$$\Gamma = 2.226 \left(\frac{M_a^2 - M_b^2}{M_a M_p} \right)^3 \sum_f \left\langle \frac{\mu_{fi}}{\mu_q} \right\rangle^2 (\text{MeV}) \,, \qquad (8)$$

where μ_q is the quark magnetic moment and M_p is the proton mass. The transition moment μ_{ba} ($\equiv \mu_{fi}$) is defined by

$$\begin{aligned} \mu_{ba} &= \left\langle a \middle| \sum_i \mu_i \middle| b \right\rangle \\ &= \mu_q \left\langle a \middle| \sum_i Q_i \sigma_{iz} \middle| b \right\rangle \,, \end{aligned} \qquad (9)$$

where Q_i and σ_{iz} are the charge and the z component of the Pauli spin matrix of the ith quark. From this equation we see that different quark models will predict different decay rates for transitions between particles which have charmed quark components. In Ref. 16 the charge of the charmed quark is $Q_c = -\frac{1}{3}$. This will be referred to as model I. The scheme in which $Q_c = \frac{2}{3}$ (model II) was devised[17] in order to suppress strangeness-changing neutral-current effects in hadronic weak interactions. We will also consider, for the sake of comparison, the case[18] $Q_c = -\frac{4}{3}$ (model III). The results of the radiative-decay calculations are presented in Table III for the three models.

The predictions in Table III are in considerable disagreement with recent measurements[19-21] of the $\rho \to \pi + \gamma$, $K^{*0} \to K^0 + \gamma$, and $\phi \to \eta + \gamma$ decay widths. If these measurements are corroborated by future

TABLE III. Predicted radiative decay widths of vector and pseudoscalar mesons in keV. Models I, II, and III are referred to in Refs. 16, 17, and 18.

Decay	Theoretical widths Model I	 Model II	 Model III	Experimental widths (keV)
$\Gamma(\rho\to\pi\gamma)$	125	125	125	35 ±10 [a]
$\Gamma(\rho\to\eta\gamma)$	94.8	94.8	94.8	...
$\Gamma(K^{*+}\to K^{+}\gamma)$	70.1	70.1	70.1	<80
$\Gamma(K^{*0}\to K^{0}\gamma)$	281	281	281	75 ±35 [b]
$\Gamma(\omega\to\pi\gamma)$	1180	1180	1180	870 ±80 [c]
$\Gamma(\omega\to\eta\gamma)$	11.0	11.0	11.0	<50 [c]
$\Gamma(\phi\to\pi\gamma)$	3.6	3.6	3.6	5.9 ±2.1 [d]
$\Gamma(\phi\to\eta\gamma)$	168	168	167	65 ±15 [d]
$\Gamma(\phi\to X^{0}\gamma)$	1.3	1.3	1.3	...
$\Gamma(\psi\to\pi^{0}\gamma)$	1.3	1.3	1.3	<0.7 [e]
$\Gamma(\psi\to\eta\gamma)$	3.3	3.0	29.0	...
$\Gamma(\psi\to X^{0}\gamma)$	15.9	74.4	275	...
$\Gamma(X^{0}\to\rho\gamma)$	37.0	37.0	37.0	<270 [c]
$\Gamma(X^{0}\to\omega\gamma)$	4.1	4.1	4.1	<80 [c]
$\Gamma(\eta_c\to\rho\gamma)$	33.7	33.7	33.7	...
$\Gamma(\eta_c\to\omega\gamma)$	7.7	0.3	24.7	...
$\Gamma(\eta_c\to\phi\gamma)$	5.4	16.4	0.3	...
$\Gamma(\eta_c\to\psi\gamma)$	39.4	158	630	...
$\Gamma(F\to F^{*}\gamma)$	8.27	2.07	51.7	...
$\Gamma(D^{++}\to D^{*++}\gamma)$	...	...	6.4	...
$\Gamma(D^{+}\to D^{*+}\gamma)$	1.59	1.59	39.7	...
$\Gamma(D^{0}\to D^{*0}\gamma)$	6.4	25.4	...	...

[a] See Ref. 19.

[b] See Ref. 20.

[c] See Particle Data Group, Phys. Lett. 50B, 1 (1974).

[d] See Ref. 21.

[e] See Ref. 6.

experiments, then it is clear that the present method of calculating these widths will have to be modified.[14] However, note that it is the conservation of quark lines,[22] guaranteed in the present approach, which leads to the satisfactory suppression of the $\phi\to\pi^0+\gamma$ and $\psi\to\pi^0+\gamma$ rates.[13]

Since π^0 does not contain any charmed quarks, models I–III all predict the same rate for $\psi\to\pi^0+\gamma$. From Table III it is evident that the best means for discriminating between models is provided by the decays $\psi\to X^0+\gamma$ and $\eta_c\to\psi+\gamma$. Obviously, if the η_c were a pure $c\bar{c}$ state the $\psi\to X^0+\gamma$ width would be equal to zero in the present approach; this width is very sensitive to the degree of impurity of the η_c.

It is also important to realize that an η_c at ~2800 MeV would result in an unacceptably large width for $\psi\to\eta_c+\gamma$ in all models.[23] For example, in model II, $\Gamma(\psi\to\eta_c(2800)+\gamma)\approx 600$ keV. It would be hard to understand how the quark-model estimate (even with all of its assumptions) of this width could be wrong by three to four orders of magnitude.

The above results show that it is possible to construct a radially excited SU(4) model of the ψ and ψ' which leads to masses and decay rates in reasonable agreement with recent data. There is some evidence in $\bar{\nu}$ interactions of a charmed-particle threshold effect which favors models I and III.[24] The question of which quark model underlies the SU(4) hadron spectroscopy would, of course, be unambiguously settled by the charges of the charmed mesons F, F^* and the charmed baryons. It is interesting to note that in a dynamical quark-model calculation of the leptonic decay widths of the vector mesons, the "natural" choice for the quark-antiquark internal wave function at the origin, $\psi(0)\sim m^{3/2}$, leads to the predictions

$$\Gamma_\rho/m_\rho : \Gamma_\omega/m_\omega : \Gamma_\phi/m_\phi : \Gamma_\psi/m_\psi = 9:1:2:2$$

in model I. This is consistent with the latest experimental value

$$\Gamma(\psi\to e^+e^-)=4.8\pm0.6\ \text{keV}$$

(see Ref. 7).

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$$\Gamma(\psi \to \pi^0\gamma) < 0.13\Gamma(\psi \to e^+e^-) \sim 0.7 \text{ keV}.$$

[7] G. S. Abrams, in *Proceedings of the 1975 International Symposium on Lepton and Photon Interactions at High Energies, Stanford, California*, edited by W. T. Kirk (SLAC, Stanford, 1976), p. 25.

[8] M. Perl, Lectures at McGill Summer School of Physics, 1975 (unpublished); G. Feldman, in *Proceedings of the 1975 International Symposium on Lepton and Photon Interactions at High Energies, Stanford, California* (Ref. 7), p. 39. The present data favor a three-body decay for the anomalous μe events found at SLAC; this suggests the existence of a heavy lepton. Only $\lesssim 20\%$ of the μe events could, perhaps, correspond to charmed-meson decays.

[9] See e.g. S. Borchardt, V. S. Mathur, and S. Okubo (Ref. 3).

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[12] The calculations of the hadronic and radiative decay widths have been carried out using the observed masses where possible.

[13] The predicted ψ decay width can easily be reduced by making the ψ an even purer $c\bar{c}$ state. This purification of ψ will not significantly change other predicted widths or masses.

[14] A more general treatment of these decays within the framework of SU(3) will be reported in a forthcoming article by the present authors. It is found that a much better fit to the data is achieved by taking quark-model couplings with a somewhat lower over-all scale, together with vector and pseudoscalar meson mixing angles derived from linear mass formulas. Despite its possible shortcomings, the quark model should give predictions for the radiative decays of the new particles which are accurate to within an order of magnitude.

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[23] Our calculations have been based on the simplest assumption for the quark magnetic moments, namely tha they are all equal: $\mu_u = \mu_d = \mu_s = \mu_c = \mu \equiv \mu_{\text{proton}}$. Setting $\mu = e/2m_q$, one finds $m_q \approx 350$ MeV. If, instead, we take $\mu_c = e/2m_c$ with $m_c \approx 1500$ MeV, the width of $\psi \to \eta_c(2800) + \gamma$ is still several orders of magnitude too large.

[24] J. W. Moffat, University of Toronto report, 1975 (unpublished).